International Journal of Basic and Applied Sciences, 1 (4) (2012) 363-373

www.sciencepubco.com/index.php/IJBAS

Predator-prey system with seasonally varying additional food to predators

Banshidhar Sahoo

Department of Mathematics, Daharpur A.P.K.B Vidyabhaban
Paschim Medinipur, West Bengal, India.
Email:banshivu@gmail.com

Abstract

We have proposed a three species food chain model with additional food to predators. We have studied the dynamics of this predator-prey model with seasonally varying additional food to predators. Bifurcation analysis of the proposed model is done with respect to quality and quantity of additional food, amplitude of oscillation and angular frequency of oscillations. The bifurcation analysis of our model plays a vital role of seasonality parameters in the controllability of the predator-prey system. Our analysis predicts that seasonality parameters are very important to determine the quality and quantity of additional food for controlling the dynamics of a real food chain model.

Keywords: *Predator-prey, Additional food, Seasonal variation, Periodic oscillations, Bifurcation, Chaos.*

1 Introduction

The study of ecological systems subject to seasonal variation is important for both theoretical and experimental ecologists. Most of the models are assumed with constant environment. The constant environment assumption is rarely the case in real life. It is natural to identify the functional role that seasons play on the behavior of population communities and to understand the relationship between the magnitude of the seasonal variations and the complexity of the ecosystem. There were several studies which investigated the interactions between seasonality and internal biological rhythms of simple predator-prey systems [1-3]. These studies show that seasonality has impotant consequences, such as the existence of multiple attractors, catastrophes and chaos. Some studies [4, 5] suggest that chaos should serve as a representation of how real world ecosystems evolve. However, efforts by field ecologists [6] to observe chaos in natural systems have brought negative results. In these studies, the effects of seasonal variation of critical parameters on the dynamical behavior of the systems were investigated. But, the control of chaotic behaviour using seasonal variation in critical parameters of the systems was not examined.

Hastings and Powell (HP) [7] proposed a chaotic tri-trophic food chain model with Holling type-II functional response. After the work of HP [7], many researchers explored their model by including various ecological factors to obtain regular behaviour from the system. In this paper, we introduce a three species predator-prey model with additional (alternative) food to predators. These additional food is assumed to be either non-reproducing prey or some food source. Many experimentalists and theoreticians investigated the effects of supplying alternative food to predators in a predator-prey system [8-13]. Srinivasu et al. [14], observed that, for a chosen quality and quantity of the additional food the asymptotic state of a solution of the system can either be an equilibrium or a limit cycle. Sahoo [15] applied the concept of additional food on a predator-prey model with different growth rates and different functional response for showing stabilize effects on the system. Recently, Sahoo [16] reported that for biological conservation, additional food plays an important role for servival of consumer species in an ecosystem.

In this paper, we have studied a predator-prey system with additional food for predators considering seasonal variation of quality of additional food. We have analysed the behaviour of the proposed model through bifurcation analysis. We have done bifurcation analysis of our model with respect to quality and quantity of additional food, amplitude of oscillations and angular frequency of oscillations of quality of additional food.

2 Model Formulation

The famous HP [7] model with pairwise interactions between three species, namely, X, Y, Z, which incorporates a Holling type-II functional interactions in both consumer species is the following

$$\begin{aligned}
\frac{dX}{dT} &= R_0X(1-\frac{X}{K_0}) - C_1A_1\frac{XY}{B_1+X} \\
\frac{dY}{dT} &= A_1\frac{XY}{B_1+X} - A_2\frac{YZ}{B_2+Y} - D_1Y \\
\frac{dZ}{dT} &= C_2A_2\frac{YZ}{B_2+Y} - D_2Z
\end{aligned} \tag{1}$$

Here X are the numbers of species at lowest level of the food chain, Y the size of the species that preys upon X and Z the size of the species that preys upon Y. Here T is time. The constant R_0 is the "intrinsic growth rate"and the constant K_0 is the "carrying capacity"of the species X. The constant C_1^{-1} and C_2 are conversion rates of prey to predators for species Y and Z respectively; D_1 and D_2 are constant death rates for species Y and Z respectively. The constants A_i and B_i for $i = 1, 2$ are maximal predation rate and half saturation constants for Y and Z respectively.

If h_1 and e_1, e_2 are constants representing handling time of the predators Y, Z per prey item and ability of the predators to detect the prey. Then A_i and B_i, represent the maximum predation rate and half saturation values of the predators Y, Z, to be $1/h_1$ and $1/e_1h_1$, $1/e_2h_1$ respectively. Hastings and Powell [7] demonstrated that chaos is possible for a simple biologically reasonable, continuous-time, three species food chain model in certain region of parametric space.

Now, we modify the model (1) by introducing "additional food"to predators population. We make the following assumptions:

(a) Predators are provided with additional food of constant biomass A which is distributed uniformly in the habitat.
(b) The number of encounters per predator with the additional food is proportional to the density of the additional food.
(c) The proportionality constant characterizes the ability of the predator to identify the additional food.
(d)The handaling time of the both predators per unit quantity of additional food are same.

With the above assumptions, HP model (1) takes the following form:

$$\begin{aligned}
\frac{dX}{dT} &= R_0X(1-\frac{X}{K_0}) - C_1A_1\frac{XY}{B_1+\alpha\mu A+X} \\
\frac{dY}{dT} &= A_1\frac{(X+\mu A)Y}{B_1+\alpha\mu A+X} - A_2\frac{YZ}{B_2+\alpha\nu A+Y} - D_1Y \\
\frac{dZ}{dT} &= C_2A_2\frac{(Y+\nu A)Z}{B_2+\alpha\nu A+Y} - D_2Z
\end{aligned} \tag{2}$$

If h_2 represents the handaling time of both the predators Y, Z per unit quantity of additional food and e_3, e_4 respectively represent the ability for the predators Y, Z to detect the additional food, then we have $\mu = e_3/e_1$, $\nu = e_4/e_2$ and $\alpha = h_2/h_1$. The terms μA and νA represent effectual additional food for the predators Y and Z respectively.

We nondimensionalize the system (2) with $x = \frac{X}{K_0}$, $y = \frac{Y}{K_0}$, $z = \frac{Z}{K_0}$, $t = R_0 T$ and obtain the following system

$$\begin{aligned}
\frac{dx}{dt} &= x(1-x) - \frac{a_1 x}{1+\alpha\xi+b_1 x} y \\
\frac{dy}{dt} &= \frac{\beta(x+c\xi)}{1+\alpha\xi+b_1 x} y - \frac{a_2 y}{1+\alpha\eta+b_2 y} z - d_1 y \\
\frac{dz}{dt} &= \frac{\gamma(y+e\eta)}{1+\alpha\eta+b_2 y} z - d_2 z
\end{aligned} \tag{3}$$

where $a_1 = \frac{C_1 A_1 K_0}{R_0 B_1}$, $a_2 = \frac{A_2 K_0}{B_2 R_0}$, $b_1 = \frac{K_0}{B_1}$, $b_2 = \frac{K_0}{B_2}$, $\beta = \frac{A_1 K_0}{B_1 R_0}$, $\gamma = \frac{C_2 A_2 K_0}{R_0 B_2}$, $c = \frac{B_1}{K_0}$, $e = \frac{B_2}{K_0}$, $\xi = \frac{\mu A}{B_1}$, $\eta = \frac{\nu A}{B_2}$, $d_1 = \frac{D_1}{R_0}$, $d_2 = \frac{D_2}{R_0}$.

Here α represents the “quality”of the additional food (ratio between predator's handling time towards additional food and prey item) and ξ, η represent the “quantity”of the additional food for the intermediate predators and top-predators respectively. The parameters α, ξ and η are the paramaters which characterize the additional food.

Most natural environments are variable, and in response, birth rates, death rates, and other vital parameters vary greatly in time. Therefore, a constant supply of additional food to predators in a system is not relistic in ecology. When the environmental fluctuation is taken into account, a model must be more difficult to analyze in general. So, due to seasonal variation, the supply of additional food to predators must be variable. Here, we assume that the quantity of additional food ξ and η are varied periodically for seasonal reason. We use $\xi = \xi_0(1+\delta cos(\omega_1 t))$ and $\eta = \eta_0(1+\delta cos(\omega_2 t))$, where ξ_0 and η_0 are constants. Here δ is the amplitude of oscillations and ω_1, ω_2 are the angular frequency of oscillations of ξ and η respectively. Therefore, with above assumption, the model (3) becomes

$$\begin{aligned}
\frac{dx}{dt} &= x(1-x) - \frac{a_1 x}{1+\alpha\xi_0(1+\delta cos(\omega_1 t))+b_1 x} y \\
\frac{dy}{dt} &= \frac{\beta(x+c\xi_0(1+\delta cos(\omega_1 t)))}{1+\alpha\xi_0(1+\delta cos(\omega_1 t))+b_1 x} y - \frac{a_2 y}{1+\alpha\eta_0(1+\delta cos(\omega_2 t))+b_2 y} z - d_1 y \\
\frac{dz}{dt} &= \frac{\gamma(y+e\eta_0(1+\delta cos(\omega_2 t)))}{1+\alpha\eta_0(1+\delta cos(\omega_2 t))+b_2 y} z - d_2 z
\end{aligned} \tag{4}$$

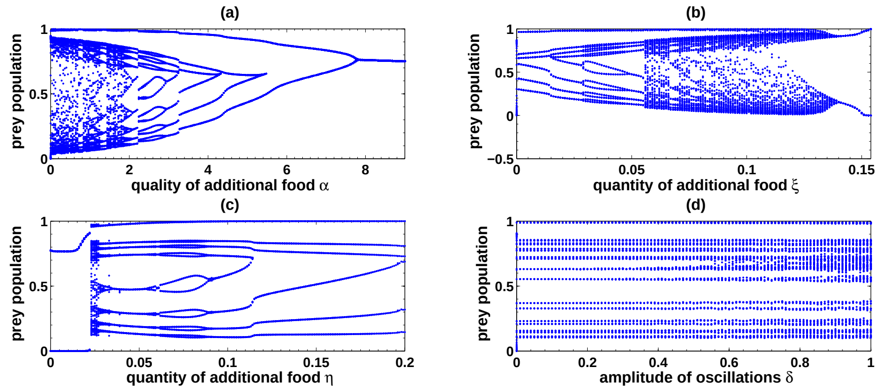

Figure 1: Bifurcation diagram of prey population with respect to (a) quality of additional food $\alpha \in [0, 9]$; (b) quantity of additional food $\xi \in [0, 0.154]$; (c) quantity of additional food $\eta \in [0, 0.2]$; (d) amplitude of oscillations $\delta \in [0, 1]$ of the system (4).

Now, we shall analyze the dynamics of the system (4) under the variation of quality and quantity of additional food, amplitude and frequency of oscillations of quantity of additional food. The system (4) has to be analyzed with the following initial conditions: $x(0) > 0$, $y(0) > 0$, $z(0) > 0$.

3 Bifurcation Analysis

We have done bifurcation analysis of the system (4) with the parameter values $a_1 = 5.0$, $a_2 = 0.1$, $b_1 = 3$, $b_2 = 2.0$, $c = 0.95$, $e = 0.85$, $\beta = 4.6$, $\gamma = 0.08$, $d_1 = 0.4$, $d_2 = 0.01$, $\omega_1 = 2$, $\omega_2 = 1.2$ which remains unchanged throughout simulations. The remaining parameters α (quality of additional food), ξ and η (quantity of additional food), δ (amplitude of oscillations) are varying.

3.1 Bifurcation analysis of the prey population

Bifurcation diagrams of prey population with respect to quality of additional food α, quantity of additional food ξ, quantity of additional food η, amplitude of oscillations δ are shown in figure 1. Figure 1(a) is the bifurcation diagram of the prey population with respect to quality of additional food α taking fixed $\xi = 0.05$, $\eta = 0.05$, $\delta = 0.02$. Figure 1(a) shows that the system has chaotic attractor without additional food α. With the increase of quality of additional food the system dynamics becomes periodic and after

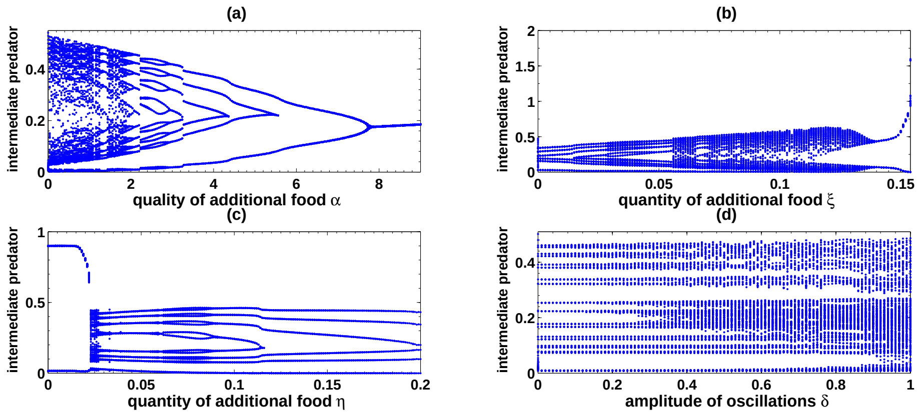

Figure 2: Bifurcation diagram of intermediate predator with respect to (a) quality of additional food $\alpha \in [0,9]$; (b) quantity of additional food $\xi \in [0,0.154]$; (c) quantity of additional food $\eta \in [0,0.2]$; (d) amplitude of oscillations $\delta \in [0,1]$ of the system (4).

$\alpha \geq 7.8$, it reaches the steady state. Figure 1(b) is the bifurcation diagram of the prey population with respect to quantity of additional food ξ taking $\alpha = 2.0$, $\eta = 0.05$, $\delta = 0.02$ fixed. Figure 1(b) shows that the system (4) has period-3 behaviour within $0 \leq \xi \leq 0.015$. After $\xi > 0.02$ it has high periodic oscillations and goes to chaotic region for $0.048 < \xi < 0.14$. Within $0.14 < \xi < 1.5$, it shows limit cycle behaviour. Therefore a period-doubling casecade is observed in figure 1(b). Bifurcation diagram of the prey population with respect to quantity of additional food η taking fixed $\alpha = 2$, $\xi = 0.05$, $\delta = 0.02$ is shown in figure 1(c). From figure 1(c) it is clear that the system has limit cycle oscillations for $0 \leq \eta \leq 0.02$. But, in between $0.02 \leq \eta \leq 0.04$ either periodic oscillations or chaotic bands are occuring. Finaly, the system (4) settles down to period-3 cycle. Therefore a period-doubling cycle is shows in figure 1(c). Figure 1(d) is the bifurcation diagram of the prey population with respect to amplitude of oscillations δ taking fixed $\alpha = 2$, $\xi = 0.05$, $\eta = 0.05$. Figure 1(d) shows that the system has high periodic and chaotic bands within $0 \leq \delta \leq 1$. However, from these bifurcation diagrams, we have observed that proper choice of parameters α, ξ, η and δ can control the system dynamics.

3.2 Bifurcation analysis of intermediate predator

The bifurcation diagrams of the intermediate predator with respect to quality of additional food α, quantity of additional food ξ, quantity of additional food η, amplitude of oscillations δ are shown in figure 2. Figure 2(a) is the bifurcation diagram of the intermediate predator with resprct to quality

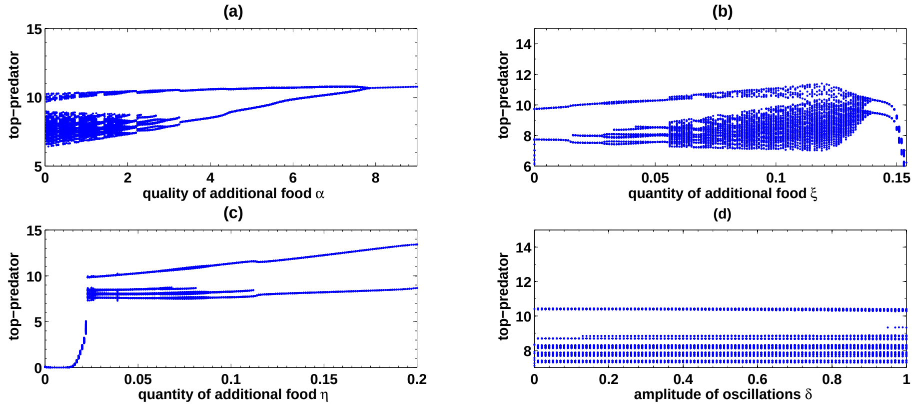

Figure 3: Bifurcation diagram of top-predator with respect to (a) quality of additional food $\alpha \in [0, 9]$; (b) quantity of additional food $\xi \in [0, 0.154]$; (c) quantity of additional food $\eta \in [0, 0.2]$; (d) amplitude of oscillations $\delta \in [0, 1]$ of the system (4).

of additional food α taking fixed $\xi = 0.05$, $\eta = 0.05$, $\delta = 0.02$. Figure 2(a) shows that the system has chaotic attractor without any quality of additional food α. The system shows period-8, period-4, period-3, peiod-2, limit cycle oscillations for increase strength of additional food $\alpha \geq 2.0$. But, after $\alpha \geq 7.8$ it settles down to steady state. Figure 2(b) is the bifurcation diagram of the intermediate predator with resprct to quantity of additional food ξ taking fixed $\alpha = 2$, $\eta = 0.05$, $\delta = 0.02$. From figure 2(b) we observe that the system (4) has period-3 behaviour within $0 \leq \xi \leq 0.015$. After $\xi > 0.02$ it has high periodic oscillations and then goes to chaotic region for $0.048 < \xi < 0.14$. Within $0.14 < \xi < 1.5$, it shows limit cycle behaviour. A period-doubling behaviour is predicted in figure 2(b). Figure 2(c) is the bifurcation diagram of the intermediate predator with respect to quantity of additional food η taking fixed $\alpha = 2$, $\xi = 0.05$, $\delta = 0.02$. The diagram shows that system has limit cycle oscillations within $0 \leq \eta \leq 0.02$. But, in between $0.02 \leq \eta \leq 0.04$ either periodic oscillations or chaotic bands are there. We observe period-3, period-4, period-6 dynamical behaviour. At last it settles down to period-3 oscillation. Figure 2(d) is the bifurcation diagram of the intermediate predator with resprct to amplitude of oscillations δ taking $\alpha = 2$, $\xi = 0.05$, $\eta = 0.05$ fixed. Figure 2(d) shows that the system shows high periodic oscillations and chaotic bands within $0 \leq \delta \leq 1$. Therefore from the bifurcation diagrams we conclude that suitable choice of parameters α, ξ, η and δ are needed to control the dynamics of the system.

3.3 Bifurcation analysis of top-predator

The bifurcation diagrams of top-predator with respect to quality of additional food α, quantity of additional food ξ, quantity of additional food η, amplitude of oscillations δ are shown in figure 3. Figure 3(a) is the bifurcation diagram of the top-predator with resprct to quality of additional food α taking fixed $\xi = 0.05$, $\eta = 0.05$, $\delta = 0.02$. Figure 3(a) shows that the system has chaotic attractor without additional food α. When we increase of additional food α after $\alpha \geq 3.2$, it has limit cycle oscillation and it settles down to steady state from $\alpha \geq 7.8$. Figure 3(b) is the bifurcation diagram of the top-predator with resprct to quantity of additional food ξ taking fixed $\alpha = 2$, $\eta = 0.05$, $\delta = 0.02$. From figure 3(b) we observe that the system (4) has limit cycle oscillation within $0 \leq \xi \leq 0.015$. After $\xi > 0.02$ it has more periodic oscillations and goes to chaotic region for $0.048 < \xi < 0.14$. Within $0.14 < \xi < 1.5$, it shows limit cycle behaviour. From figure 3(b) we observe that top-predator has extinction risk for higher quantity of additional food ξ. Figure 3(c) is the bifurcation diagram of the top-predator with respect to quantity of additional food η taking fixed $\alpha = 2$, $\xi = 0.05$, $\delta = 0.02$. The diagram shows that figure 3(c) has limit cycle oscillations within $0 \leq \eta \leq 0.02$. But, in between $0.02 \leq \eta \leq 0.11$ either periodic oscillations or chaotic bands are there. Lastly, it settles down to limit cycle oscillation. Figure 3(d) is the bifurcation diagram of the top-predator with resprct to amplitude of oscillations δ taking fixed $\alpha = 2$, $\xi = 0.05$, $\eta = 0.05$. Figure 3(d) shows that the system has high periodic cycles and chaotic bands within $0 \leq \delta \leq 1$. Therefore we conclude that proper choice of parameters are very important to control the dynamics of the system.

3.4 Bifurcation analysis with respect to frequency of oscillations

In this section, we have analysed the bifurcation of prey population, intermediate predator and top-predator with respect to frequency of oscillations ω_2 of the system (4). Here, we take the ratio $\frac{\omega_1}{\omega_2}$ as rational as well as irrational numbers. The figure 4, figure 5 are respectively the bifurcation diagram of prey population, intermediate predator and top-predator with respect to $\omega_2 \in [0, 1.5]$ taking the ratio $\frac{\omega_1}{\omega_2}$=Golden number=$\frac{\sqrt{5}+1}{2}$. From the figure 4, figure 5 we observe that the system (4) has chaotic attractor within $0 \leq \omega_2 \leq 0.45$. After $\omega_2 > 0.45$, it shows high periodic cycles. Similar behaviour is obtained when the ratio $\frac{\omega_1}{\omega_2}$ is a rational number. Therefore, for high frequency of oscillations of quantity of alternative food the system shows periodic behaviour.

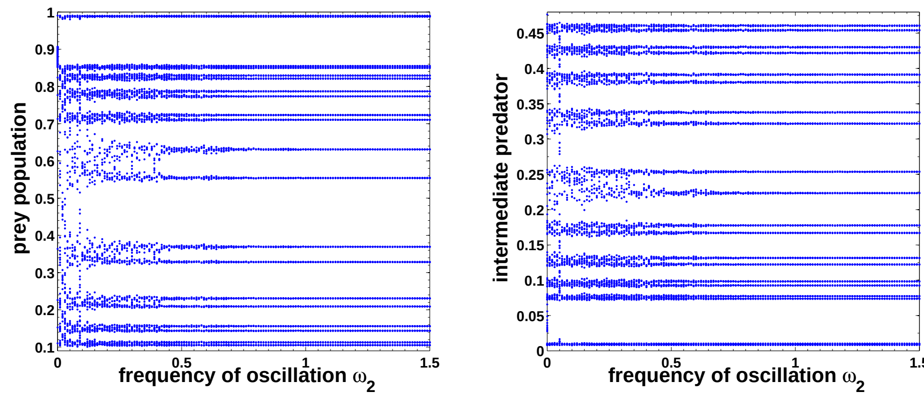

Figure 4: Bifurcation diagram of prey population and intermediate predator with respect to frequency of oscillation $\omega_2 \in [0, 1.5]$, where $\omega_1 = \frac{\sqrt{5}+1}{2}\omega_2$ keeping fixed $\alpha = 2$, $\xi = 0.05$, $\eta = 0.05$, $\delta = 0.02$ of the system (4)

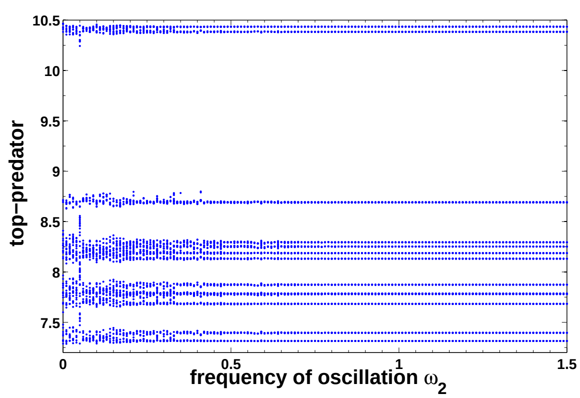

Figure 5: Bifurcation diagram of top-predator with respect to frequency of oscillation $\omega_2 \in [0, 1.5]$, where $\omega_1 = \frac{\sqrt{5}+1}{2}\omega_2$ keeping fixed $\alpha = 2$, $\xi = 0.05$, $\eta = 0.05$, $\delta = 0.02$ of the system (4)

4 Conclusions

We have proposed a three species predator-prey model with additional food to predators. We assume the periodic variation of quantity of additional food. The bifurcation analysis of proposed model is done with respect to quality of additional food α, quantity of additional food ξ and η, amplitude of oscillations δ and angular frequency of oscillations separately. From the bifurcation diagrams, we have observed that for suitable choice of parameters the system dynamics can be controlled. Our analysis confirms that if quantity of additional food is season dependent, then system dynamics may not be controlled through variation of quantity and quality of additional food. We can reach our target state from the system with periodically varying quantity of additional food only if we have knowledge of frequency as well as amplitude of oscillation of the quality of additional food. Therefore we conclude that together with quality and quantity of additional food, the frequency and amplitude of oscillation of quantity of additional food are key finding parameters to control the dynamics of a three species predator-prey system. Therefore for pest managment and biological conservation consideration of seasonal variation of quantity of additional food is urgently required.

References

[1] A. Gragnani, S. Rinaldi, A universal bifurcation diagram for seasonally perturbed predator-prey models, Bull. Math. Biol. 57 (1995) 701-712 .

[2] A.R. Ives, K. Gross, V.A.A. Jansen, Periodic mortality events in predatorprey systems, Ecology 81 (2000) 3330-3340.

[3] S. Rinaldi, S. Muratori, Y. Kuznetsov, Multiple attractors, catastrophes and chaos in seasonally perturbed predator-prey communities, Bull. Math. Biol. 55 (1993) 15-35.

[4] L.F. Olsen, H. Degn, Chaos in biological systems, Q. Rev. Biophys. 18 (1985) 165-225.

[5] E. Ott, Chaos in Dynamical Systems, Cambridge University Press: Cambridge, 1993.

[6] E. Ott, Strange attractors and chaotic motions of dynamical systems, Rev. Mod. Phys. 53 (1981) 655-671.

[7] A. Hastings, T. Powell, Chaos in a Three-Species Food Chain. Ecology 72 (1991) 896-903.

[8] J.D. Harwood, J.J. Obrycki, The role of alternative prey in sustaining predator populations, In: HoddleMS (ed) Proc. second int. symp. biol. control of arthropods. 2 (2005) 453-462.

[9] M.V. Baalen, M. Kivan, P.C.J. van Rijn, M.W. Sabelis, Alternative food, switching predators, and the Persistence of Predator-Prey Systems, Am Nat. 157 (2001) 512-524.

[10] P.C.J. van Rijn, Y.M. von Houten, M.W. Sabelis, How plants benefit from providing food to predators even when it is also edible to herbivores. Ecology, 83 (20010 2664-2679.

[11] M.W. Sabelis, P.C.J. van Rijn, When does alternative food promote biological pest control? In: Hoddle MS (ed) Proc. second int. symp. biol. control of arthropods 2 (2005) 428-437.

[12] T. Bilde, S. Toft, Quantifying food limitation of arthropod predators in the field, Oecologia 115 (1998) 54-58.

[13] B. Sahoo, Effects of Additional Food on Predator-Prey Systems with Monotonic and Non-Monotonic Functional Response: A Comparative Study, International Journal of Basic and Applied Sciences, 1 (2012) 124-136.

[14] P.D.N. Srinivasu, B.S.R.V. Prasad, M. Venkatesulu, Biological control through provision of additional food to predators: a theoretical study, Theor Popul Biol. 72 (2007) 111-120.

[15] B. Sahoo, Predator-prey Model with Different Growth Rates and Different Functional Responses: A Comparative Study with Additional Food, International Journal of Applied Mathematical Research, 1 (2012) 117-129.

[16] B. Sahoo, Effects of Additional Foods to Predators on Nutrient-Consumer-Predator Food Chain Model, ISRN Biomathematics, doi:10.5402/2012/796783, 2012.