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Research Paper

Multiple-step stress accelerated life for Weibull Poisson distribution with type I censoring

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Abstract

This article presents multi-step stress accelerated life tests based on type I censoring. It is assumed that the lifetime at design stress has the Weibull Poisson distribution. The scale parameter of the Weibull Poisson failure time at constant levels is assumed to have an inverse power law of the stress levels. Under the assumption of a cumulative exposure model, the maximum likelihood method is used to obtain the estimators of the model parameters. The optimal design of the accelerated life tests is studied according to the A-optimality criterion to specify the optimal stress change time and the optimal censoring time. Finally, the numerical studies are performed to illustrate the proposed procedures.

Keywords: *Cumulative Exposure Model, Lifetime Distribution, Step Stress ALT, Optimal Test Plan, A-Optimality.*

1. Introduction

Accelerated life tests (ALTs) are used to estimate the lifetime of highly reliable products within a reasonable testing time. The test products are run at higher than usual levels of stress (which includes temperature, voltage, pressure, etc.) to obtain early failures. In ALT experiments, the researcher is often interested in the effects of varying stress levels on the lifetimes of experimental units.

One important way in ALT is step-stress accelerated life test (SSALT). There are mainly two types of SSALTs, a simple and a multiple SSALT. In the simple SSALT there is a single change of stress during the test. Nelson [14] originally proposed the simple SSALT, in which only one change of stress occurs with a cumulative exposure (CE) model for Type-I and Type-II censored data. Miller and Nelson [10] obtained the optimal simple SSALT plans for the case where test products have exponentially distributed lives and are observed continuously until all test products fail. Xiong [1] has studied an exponential CE model with a threshold parameter in the simple SSALT. Lu and Rudy [9] dealt with the Weibull CE model with the inverse power law in the simple SSALT. For more research on SSALTs (see for example, Xu and Fei [5], Nelson [16] and Wu et al. [12].

On the other hand, in the multiple-step SSALT there is more than one change of stress level. Khamis and Higgins [6] considered the optimum three step SSALT for the exponentially distributed Type I censored data. Khamis [7] proposed an optimal m level SSALT design with multiple stress variables and investigated it on three level SSALT data.

Yeo and Tang [8] have investigated three-level SSALT in an exponential CE model. McSorley et al [2] have shown the properties of the maximum likelihood estimators (MLEs) of parameters in the Weibull CE model with a log-linear function of stress on three-level SSALT data. Aly and Bleed [4] presented estimation and derivation of optimum test plan for generalized logistic distribution. Also, Saxena et al. [11] dealt with the problem of designing an optimum step stress accelerated life test for Rayleigh distribution.

The modeling and analysis of lifetimes is an important aspect of statistical work in a wide variety of scientific and technological fields. Several distributions have been proposed in the literature to model lifetime data by compounding some useful life distributions. Lu and Shi [13] introduced a new compounding distribution named the Weibull Poisson (WP) distribution. The failure rate function of WP distribution has various shapes; it can be increasing, decreasing and upside down bathtub shaped or unimodal. The property of various shapes of the failure rate function encourages practitioners to adopt the WP for the inferences of life information and biological study. The probability density function (pdf) of Weibull Poisson distribution takes the following form:

$$f(t;\alpha,\beta,\lambda)=\frac{\alpha\beta\lambda t^{\alpha-1}}{1-e^{-\lambda}}\,.\,e^{-\lambda-\beta t^{\alpha}+\lambda exp\,(-\beta t^{\alpha})},\qquad t>0,\qquad \alpha,\beta,\lambda>0, \tag{1}$$

Where α is the shape parameter and β is scale parameter. the corresponding cumulative distribution function (cdf) has the following form:

$$F(t;\alpha,\beta,\lambda)=\left(e^{\lambda exp^{-\beta t^{\alpha}}}-e^{\lambda}\right)\left(1-e^{\lambda}\right)^{-1},\qquad t>0. \tag{2}$$

Note that when $\alpha=1$, the Weibull Poisson distribution reduces to an exponential Poisson and as λ tends to zero, the Weibull Poisson distribution reduces to a two parameter Weibull distribution.

This article deals with the problem of designing an optimum multi-step stress accelerated life test for Weibull Poisson distribution based on type I censoring. The scale parameter of the distribution is assumed to have an inverse power law function of stress level. The optimum time for changing stress levels will be obtained using A-optimality criterion which is based on the trace of the Fisher information matrix. Also the optimum censoring time are obtained.

The layout of the article is as follows. The model and assumptions are described in details in Section 2. In Section 3 the maximum likelihood method is introduced to estimate the model parameters under type I censoring data. In the same section, the approximate confidence intervals are obtained for the MLEs. In Section 4 the optimum time of changing stress levels and the optimum censoring time are presented. The simulation technique is presented in Section 5. Finally conclusion is presented in Section 6.

2. Assumptions and test procedure

During the multiple SSALT, units are subjected to successively higher levels of stress. After a unit is used at a normal level of stressV_0, it is subjected to an initial level of stress V_1 for a predetermined time τ_1 at the first stage in the test. If it does not fail, it is subjected to a higher level of stress V_2 for a predetermined time τ_2 at the next stage. In analogy, it is repeatedly subjected to higher levels of stress until it fails. The other units are tested similarly. The pattern of stress levels and time intervals is the same for all units. The model assumptions for multiple SSALT procedure will be described as follows:

i. There are multiple (k) levels of high stress, $V_j, j=1,2,\ldots,k$ in the experiment, and V_0 is the design stress under normal use conditions, where, $V_0<V_1<\cdots<V_k$.

ii. A random sample of N identical units is placed on test under the first stress V_1 and run until time τ_1 and then stress is changed to V_2 and run until time τ_2 and so on, the test is continued until all units are failed or it reached the censored time T (predetermined time).

iii. The number of failures in the sample is $\eta=\sum_{j=1}^{k}n_j$ where n_1 is the number of failures from first step under the first stress V_1, n_2 is the number of failures from second step under the second stress V_2 and n_k is the number of failures from last step under the last stress V_k, N are the total number of units run on the test.

iv. The failure times $t_{ij}, i=1,2,\ldots,n_j$ at stress levels $V_j, j=1,2,\ldots,k$ are assumed to be WP distribution at any stress; with pdf defined in (1).

v. The lifetimes of the units at each stress level are identically independent distributed (i.i.d).

vi. The WP scale parameter $\beta_j, j=1,2,\ldots,k$ of the underlying lifetime distribution is assumed to have an inverse power function of stress levels i.e, $\beta_j=CS_j^{p}, j=1,2,\ldots,k, C,p>0, S_j=\frac{V^*}{V_j}, V^*=\prod_{j=1}^{k}V_j^{b_j}, b_j=\frac{n_j}{\eta}$, C is the constant of proportionality and p is the power of the applied stress.

vii. The WP parameters λ and α are assumed to be constant, i.e. independent of stress.

viii. To analyze the data from multi-step stress ALT, a model is needed to relate the distribution under step stress to the distribution under constant stress. The most commonly used model is cumulative exposure model proposed by Nelson [14]. The basic idea of the CE model starts from the fact that a step stress ALT model must explain the cumulative effect of the applied stresses; the CE model assumes that the remaining test units are failed according to the cumulative density function of current stress level regardless of the history on previous stress levels. According to Nelson [15], the cumulative exposure model with k-step stress ALT is given by:

$$G(t)=\begin{cases}F_1(t) & ,0\le t<\tau_1\\ F_2\big((t-\tau_1)+u_1\big) & ,\tau_1\le t<\tau_2\\ \vdots & \\ F_k\big((t-\tau_{k-1})+u_{k-1}\big) & ,\tau_{k-1}\le t<T\end{cases} \tag{3}$$

Where $F_j\left((t_{ij}-\tau_{j-1})+u_{j-1}\right)=\left(e^{\lambda exp\left(-cS_j^{p}\left((t_{ij}-\tau_{j-1})+u_{j-1}\right)^{\alpha}\right)}-e^{\lambda}\right)\left(1-e^{\lambda}\right)^{-1}$ the cumulative distribution function of the failure at stresses $V_j, j=1,2,\ldots,k$, $u\_(j-1)$ is the solution of the equation $F_j\left(u_{j-1};S_j\right)=F_{j-1}\big(\tau_{j-1}-\tau_{j-2}+$ $uj-2;Sj-1$. Therefore the general form solution is as follows:

$$u_j = \left(\frac{s_j}{s_{j+1}}\right)^{1/\alpha} \left(\tau_j - \tau_{j-1} + u_{j-1}\right). \quad (4)$$

Note that $u_0 = 0$ and $\tau_0 = 0$ where τ_j is the time of changing stress.
Thus the associated pdf will be as follows:

$$g(t) = \begin{cases} f_1(t) & , \ 0 \le t < \tau_1 \\ f_2\big((t-\tau_1)+u_1\big) & , \ \tau_1 \le t < \tau_2 \\ \vdots & \\ f_k\big((t-\tau_{k-1})+u_{k-1}\big) & , \ \tau_{k-1} \le t < T \end{cases} \quad (5)$$

3. Point and interval maximum likelihood estimations

In this Section, the point and interval estimators of the model parameters are introduced using the maximum likelihood method.

3.1. Point estimation

According to type-I censoring, the test applied to N identical sample units which will be terminated when all units fail or censoring time T is reached. Let $t_{ij}, i = 1,2,\ldots,n_j, j = 1,2,\ldots,k$ be independent and identically distributed Weibull Poisson random variable, the likelihood function for multiple SSALT with type I censored data is considered to have the following form:

$$l = \frac{N!}{B!}\prod_{j=1}^{k}\prod_{i=1}^{n_j} f_j\left(t_{ij} - \tau_{j-1} + u_{j-1}\right)\left(1 - F_k(T - \tau_{k-1} + u_{k-1})\right)^B, \quad (6)$$

where $B = N - \sum_{j=1}^{k} n_j$ is the number of surviving units? Then the likelihood function for three parameter WP distribution for k-step stress ALT with type I censored is as follows:

$$l = \frac{N!}{B!}\prod_{j=1}^{k}\prod_{i=1}^{n_j} \frac{\alpha C S_j^p \lambda (t^*)^{\alpha-1} e^{-\lambda - C S_j^p (t^*)^\alpha + \lambda exp\left(-C S_j^p (t^*)^\alpha\right)}}{1 - e^{-\lambda}} \left[1 - \left(e^{\lambda exp\left(-C S_k^p D^\alpha\right)} - e^{\lambda}\right)\left(1 - e^{\lambda}\right)^{-1}\right]^B \quad (7)$$

$$l = \frac{N!}{B!}\prod_{j=1}^{k}\prod_{i=1}^{n_j} \frac{\alpha C S_j^p \lambda (t^*)^{\alpha-1} e^{-\lambda - C S_j^p (t^*)^\alpha + \lambda exp\left(-C S_j^p (t^*)^\alpha\right)}}{1 - e^{-\lambda}} \left[1 - \frac{e^{\lambda exp\left(-C S_k^p D^\alpha\right) - \lambda} - 1}{e^{-\lambda} - 1}\right]^B \quad (8)$$

where $t^* = t_{ij} - \tau_{j-1} + u_{j-1}$, $D = T - \tau_{k-1} + u_{k-1}$ and $u_{j=k-1} = \left(\frac{\beta_{k-1}}{\beta_k}\right)^{1/\alpha} . (\tau_{k-1} - \tau_{k-2} + u_{k-2})$.

The MLEs of the unknown parameters are obtained by maximizing the logarithm of the likelihood function expressed in the following form:

$$ln\, l = ln\frac{N!}{B!} + \eta\, ln(\alpha) + \eta\, ln(\lambda) + \eta\, ln(C) + p\sum_{j=1}^{k} n_j\, ln(S_j) + (\alpha - 1)\sum_{j=1}^{k}\sum_{i=1}^{n_j} ln(t^*)$$
$$+\lambda \sum_{j=1}^{k}\sum_{i=1}^{n_j} e^{-C S_j^p (t^*)^\alpha} - C\sum_{j=1}^{k}\sum_{i=1}^{n_j} S_j^p (t^*)^\alpha - \eta\, ln\left(e^{\lambda} - 1\right) + B\, ln\left[\frac{1 - e^{\lambda . e^{\left(-C S_k^p D^\alpha\right)}}}{1 - e^{\lambda}}\right] \quad (9)$$

The first partial derivatives of the likelihood equation with respect to the parameters C, p, λ and α respectively, will be as the follows:

$$\frac{\partial\, ln\, l}{\partial C} = \frac{\eta}{C} - \sum_{j=1}^{k}\sum_{i=1}^{n_j}(\lambda H G + H) + \left[\frac{B}{M} . \lambda S_k^p D^\alpha L\right], \quad (10)$$

$$\frac{\partial\, ln\, l}{\partial p} = \sum_{j=1}^{k} n_j ln(S_j) - \left[\sum_{j=1}^{k}\sum_{i=1}^{n_j}\left(\lambda C H G\, ln(S_j) + C H\, ln(S_j)\right)\right] + \left[\frac{B . \lambda C S_k^p D^\alpha L\, ln(S_k)}{M}\right], \quad (11)$$

$$\frac{\partial\, ln\, l}{\partial \lambda} = \frac{\eta}{\lambda} + \sum_{j=1}^{k}\sum_{i=1}^{n_j} G - \frac{\eta e^{\lambda}}{e^{\lambda} - 1} + \left[\frac{B}{M}\left(-L + \frac{1 - e^{\lambda . e^{\left(-C S_k^p D^\alpha\right)}}}{e^{-\lambda} - 1}\right)\right] \quad (12)$$

and

$$\frac{\partial\, ln\, l}{\partial \alpha} = \frac{\eta}{\alpha} + \sum_{j=1}^{k}\sum_{i=1}^{n_j}\left[ln(t^*) - \lambda C G\, H ln(t^*) - C H\, ln(t^*)\right] + \left[\frac{B . \lambda C S_k^p D^\alpha L\, ln\, D}{M}\right] \quad (13)$$

where $G = e^{-C S_j^p (t^*)^\alpha}$, $H = S_j^p (t^*)^\alpha$, $M = 1 - e^{\lambda . e^{\left(-C S_k^p D^\alpha\right)}}$ and $L = e^{\lambda e^{\left(-C S_k^p D^\alpha\right)} - C S_k^p D^\alpha}$.

After equating the Equations (10) to (13) with zero and solving it together, the likelihood estimates $\hat{C}, \hat{p}, \hat{\lambda}$ and $\hat{\alpha}$ will be obtained. Notice that the first derivatives of Equations (10) to (13) be nonlinear equations and their solutions will be obtained numerically. In addition, estimates values $\hat{\beta}_j$ for each stress will be obtained by substituting the estimates values of $\hat{C}$ and $\hat{p}$ in the inverse power law relationship $\beta_j = CS_j^p$.

3.2. Interval estimation

It can be said that the MLEs have an asymptotic variance-covariance matrix defined by the inverse of F. The approximate confidence intervals (CI) of the parameters are derived based on the asymptotic distribution of the MLEs for the unknown parameters. The asymptotic distribution of $\frac{\hat{\theta}-E(\hat{\theta})}{\sqrt{var(\hat{\theta})}}$ can be approximated by a standard normal distribution, where $\mathrm{var}(\hat{\theta})$ is the asymptotic variance. Therefore, the two-sided approximate $\alpha\,100$ percent confidence limits for θ (lower bound (LB), upper bound (UB)) can be obtained, such that

$$LB\,(\theta) = \hat{\theta} - Z_{\alpha/2}\sqrt{var(\hat{\theta})},\ UB\,(\theta) = \hat{\theta} + Z_{\alpha/2}\sqrt{var(\hat{\theta})} \tag{14}$$

were $Z_{\alpha/2}$ is the $100(\alpha/2)\%$ standard normal percentile and $\hat{\theta} \equiv (\hat{\lambda}, \hat{\alpha}, \hat{C}, \hat{p})$.

In relation to the asymptotic variance-covariance matrix of the MLE of the parameters, it can be approximated by numerically inverting the observed Fisher-information matrix. The observed Fisher-information matrix is composed of the negative second derivatives of the natural logarithm of the likelihood function evaluated at the MLEs. It can be given by the following matrix:

$$F = -\begin{bmatrix} \frac{\partial^2 \ln l}{\partial C^2} & \frac{\partial^2 \ln l}{\partial C \partial p} & \frac{\partial^2 \ln l}{\partial C \partial \lambda} & \frac{\partial^2 \ln l}{\partial C \partial \alpha} \\ \frac{\partial^2 \ln l}{\partial C \partial p} & \frac{\partial^2 \ln l}{\partial p^2} & \frac{\partial^2 \ln l}{\partial p \partial \lambda} & \frac{\partial^2 \ln l}{\partial p \partial \alpha} \\ \frac{\partial^2 \ln l}{\partial C \partial \lambda} & \frac{\partial^2 \ln l}{\partial p \partial \lambda} & \frac{\partial^2 \ln l}{\partial \lambda^2} & \frac{\partial^2 \ln l}{\partial \lambda \partial \alpha} \\ \frac{\partial^2 \ln l}{\partial C \partial \alpha} & \frac{\partial^2 \ln l}{\partial p \partial \alpha} & \frac{\partial^2 \ln l}{\partial \lambda \partial \alpha} & \frac{\partial^2 \ln l}{\partial \alpha^2} \end{bmatrix} \downarrow (\hat{C}, \hat{p}, \hat{\lambda}, \hat{\alpha}) \tag{15}$$

The elements of trace of the matrix F can be expressed by the following equations:

$$\frac{\partial^2 \ln l}{\partial C^2} = \frac{-\eta}{C^2} + \lambda \sum_{j=1}^{k} \sum_{i=1}^{n_j} G(H)^2 - \frac{B.\left(\lambda S_k^p D^\alpha L\right)^2}{M^2} - \frac{B\lambda S_k^p D^\alpha L}{M}\left\{\lambda e^{\left(-CS_k^p D^\alpha\right)}.\left(S_k^p D^\alpha\right) + S_k^p D^\alpha\right\}, \tag{16}$$

$$\frac{\partial^2 \ln l}{\partial p^2} = \sum_{j=1}^{k} \sum_{i=1}^{n_j} \left(\lambda G\left(CH \ln(S_j)\right)^2 - \lambda HCG\left(\ln(S_j)\right)^2 - CH\left(\ln(S_j)\right)^2\right) - \frac{B\left(L\lambda CS_k^p D^\alpha \ln(S_k)\right)^2}{M^2}$$
$$- \frac{B\lambda CS_k^p D^\alpha L \ln(S_k)}{M}\left(CS_k^p D^\alpha \lambda \ln(S_k)\, e^{\left(-CS_k^p D^\alpha\right)} + CS_k^p D^\alpha \ln(S_k)\right) + \frac{BL\lambda CS_k^p D^\alpha (\ln(S_k))^2}{M}, \tag{17}$$

$$\frac{\partial^2 \ln l}{\partial \lambda^2} = \frac{-\eta}{\lambda^2} - \frac{\left(\eta e^\lambda\left(e^\lambda - 1\right)\right) - \eta\left(e^\lambda\right)^2}{(e^\lambda - 1)^2} + \frac{B.L}{M^2}\left(-L + \frac{M}{e^{-\lambda} - 1}\right) + \frac{B}{M}\left[\left(-Le^{\left(-CS_k^p D^\alpha\right)}\right) - \frac{L\left(e^{-\lambda} - 1\right) - Me^{-\lambda}}{(e^{-\lambda} - 1)^2}\right] \tag{18}$$

and

$$\frac{\partial^2 \ln l}{\partial \alpha^2} = \frac{-\eta}{\alpha^2} + \lambda\left[\sum_{j=1}^{k} \sum_{i=1}^{n_j} G(CH \ln t^*)^2 - \sum_{j=1}^{k} \sum_{i=1}^{n_j} CGH(\ln t^*)^2\right] - C \sum_{j=1}^{k} \sum_{i=1}^{n_j} H(\ln t^*)^2 - \frac{B\left(\lambda CS_k^p D^\alpha L \ln D\right)^2}{M^2}$$
$$+ \frac{BL\lambda CS_k^p D^\alpha (\ln D)^2}{M} - \frac{B.CS_k^p D^\alpha \lambda L \ln D\left\{CS_k^p D^\alpha \lambda \ln D\ e^{\left(-CS_k^p D^\alpha\right)} + CS_k^p D^\alpha \ln D\right\}}{M}. \tag{19}$$

4. Optimum criterion

The purpose of this Section is to explore the problem of determining the optimal time of changing stress level for multiple SSALT under type I censoring data assuming WP as a lifetime model. Furthermore, the optimal censoring time which leads to the most accurate estimate is determined.

4.1. Optimum time of changing stress level

An optimal test plan allows maximum possible information to be obtained from the tests, using the same number of test items. So, it provides certain objective, such as providing the most precise estimates to improve the quality of the statistical inference. In addition, this test reduces the total required time on test. A useful criterion for optimum plans is A-optimality criterion which was mentioned in Aly [3]. A-optimality criterion is based on minimizing the trace of the variance-covariance matrix of the MLEs for the unknown parameters, and then the optimal time of changing stress levels τ_{j-1} can be obtained by solving the following equation:

$$\frac{\partial tr\ F}{\partial \tau_{j-1}} = 0 \qquad , j = 2, \dots, k \tag{20}$$

where

$$tr\ F = \frac{\partial^2 \ln l}{\partial C^2} + \frac{\partial^2 \ln l}{\partial p^2} + \frac{\partial^2 \ln l}{\partial \lambda^2} + \frac{\partial^2 \ln l}{\partial \alpha^2} \tag{21}$$

The solution of (20) is not in a closed form. It requires an iterative method.

4.2. Optimum censoring time

Based on A-optimality criterion that minimizes the trace of the variance-covariance matrix of the MLEs of the unknown parameters, the optimal censored time of T can be obtained by solving the following equation:

$$\frac{\partial tr\ F}{\partial T} = 0 \tag{22}$$

where

$$tr\ F = \frac{\partial^2 \ln l}{\partial C^2} + \frac{\partial^2 \ln l}{\partial p^2} + \frac{\partial^2 \ln l}{\partial \lambda^2} + \frac{\partial^2 \ln l}{\partial \alpha^2} \tag{23}$$

In general, the solution of (22) is not in a closed form and therefore requires a numerical method to obtain the optimal censored time T which minimizes $tr\ F$.

5. Simulation study

In order to obtain the MLEs of the unknown parameters and evaluate the performance of the MLEs, several sample sizes $N = (30, 50, 70, 100)$ are generated from three parameter WP distribution of multiple SSALT data. The mean square errors (MSEs) and the biases for the MLEs are calculated. In addition, optimum change time for each stress level and optimum censoring time are calculated. The simulation procedures are described through the following steps:

i. For a given values of the parameters $C = 0.5, \lambda = 1,\ p = (1.25, 1.5, 2)$ and $\alpha = (1.25, 1.5, 2)$ and selected values of stresses $V_1 = 0.75, V_2 = 1$ and $V_3 = 2$ calculate $\beta_j = CS_j^p$ for each stress level where $(k = 3)$.
ii. Generate random samples of size $N = (30, 50, 70, 100)$ from uniform (0,1) distribution and obtain the order statistics $(U_{1:N} \dots, U_{N:N})$.
iii. For a given value of the first stress change time $\tau_1 = 0.5$, find n_1 such that
$$U_{n_1:N} \le \left(e^{\lambda exp^{(-\beta_1 \tau_1{}^{\alpha})}} - e^{\lambda}\right)\left(1 - e^{\lambda}\right)^{-1} < U_{n_1+1:N}.$$
iv. For a given value of the second stress change time $\tau_2 = 1$, find n_2 such that
$$U_{n_2:N-n_1} \le \left(e^{\lambda exp^{\left(-CS_2^p\left(\tau_2 - \tau_1 + \left(\frac{-CS_1^p}{-CS_2^p}\right)^{1/\alpha} . \tau_1\right)^{\alpha}\right)}} - e^{\lambda} \right)\left(1 - e^{\lambda}\right)^{-1} < U_{n_2+1:N-n_1}.$$
v. For a given value of the censoring time $T = 5$, find n_3 such that
$$U_{n_3:N-n_1-n_2} \le \left(e^{\lambda exp^{\left(-CS_3^p\left((T-\tau_2) + \left(\frac{-CS_2^p}{-CS_3^p}\right)^{1/\alpha} . \left(\tau_2 - \tau_1 + \left(\frac{-CS_1^p}{-CS_2^p}\right)^{1/\alpha} . \tau_1\right)\right)^{\alpha}\right)}} - e^{\lambda} \right)\left(1 - e^{\lambda}\right)^{-1} < U_{n_3+1:N-n_1-n_2}$$
vi. Then the order observations $t_{1:N} \le \dots \le t_{n_1:N} \le t_{n_1+1:N} \le \dots \le T$ are calculated from (2).
vii. Based on $n_1, n_2, n_3, \tau_1, \tau_2, T$ and the observed observations, MLEs for the Parameters C, p, λ and α can be obtained by solving the nonlinear equations from (10) to (13) numerically. Also, both the LB and the UB for each parameter are obtained.
viii. Compute the biases and MSEs for the model parameters over 100 samples are obtained.
ix. The optimum time of changing stress level and the optimum censoring time are obtained by solving Equations (20) and (22) respectively, numerically through iteration.

Simulation results based on k-SSALT for Weibull Poisson distribution under type I censoring with $k = 3$ are summarized in Tables from 1 to 7. Tables 1-4 present the biases and the MSEs for the unknown parameters. Table 5 shows the CI for the MLEs. Tables 6-7 give the number of failures at each stress level, the optimum time of changing stress levels and the optimum censoring time. From these tables, the following conclusions can be observed:

1. For fixed values for time of changing stress levels τ_j and censoring time T as N increases, the MSEs and the biases of model parameters decrease.
2. The MSEs for the MLEs $\hat{\alpha}$ and $\hat{p}$ are smaller as the value of the parameters α and p increase.

3. The MSEs and the biases are the smallest for $\hat{\lambda}, \hat{\beta}_3$ and $\widehat{C}$.
4. The estimate values of $\hat{\lambda}$ has the shortest confidence interval.
5. As the sample size increases the interval of the estimator's decreases.
6. As the stress increases, it is evident that the MSE of the estimated scale parameter β_j tends to decrease
7. For fixed values for time of changing stress levels τ_j and censoring time T, the number of failures n_1 before time τ_1 increase and number of failures n_2, n_3 decrease.
8. For different values of the parameters, as sample size increases, the censoring time T tends to increase.

Table 1: The Biases and MSEs for the MLEs of the Unknown Parameters (C, p, λ, α) For $N = 30$

$N = 30, C = 0.5, p = 2, \lambda = 1, \alpha = 2$							
Parameter	C	p	λ	α	β_1	β_2	β_3
Bias	-0.053	0.144	-0.028	-0.027	0.192	0.253	0.072
MSE	0.007	0.442	0.002	0.013	0.040	0.066	0.006
$N = 30, C = 0.5, p = 1.5, \lambda = 1, \alpha = 2$							
Parameter	C	p	λ	α	β_1	β_2	β_3
Bias	-0.124	0.507	-0.0160	-0.203	0.586	0.406	-0.004
MSE	0.033	0.484	0.014	0.163	0.340	0.166	0.005
$N = 30, C = 0.5, p = 2, \lambda = 1, \alpha = 1.25$							
Parameter	C	p	λ	α	β_1	β_2	β_3
Bias	-0.069	-0.151	-0.002	0.237	0.475	0.467	0.162
MSE	0.020	0.040	0.045	0.032	0.247	0.227	0.027
$N = 30, C = 0.5, p = 2, \lambda = 1, \alpha = 1.5$							
Parameter	C	p	λ	α	β_1	β_2	β_3
Bias	-0.051	0.103	-0.002	-0.161	0.567	0.537	0.194
MSE	0.002	0.012	0.0008	0.028	0.351	0.300	0.039
$N = 30, C = 0.5, p = 1.25, \lambda = 1, \alpha = 2$							
Parameter	C	p	λ	α	β_1	β_2	β_3
Bias	-0.041	0.067	-0.004	-0.267	0.513	0.380	0.108
MSE	0.001	0.004	0.0001	0.080	0.271	0.148	0.012
$N = 30, C = 0.5, p = 1.5, \lambda = 1, \alpha = 1.25$							
Parameter	C	p	λ	α	β_1	β_2	β_3
Bias	0.106	0.082	-0.049	-0.109	0.472	0.431	-0.028
MSE	0.044	0.236	0.379	0.079	0.595	0.225	0.004
$N = 30, C = 0.5, p = 1.5, \lambda = 1, \alpha = 1.5$							
Parameter	C	p	λ	α	β_1	β_2	β_3
Bias	-0.078	-0.234	-0.047	-0.239	0.476	0.321	0.054
MSE	0.008	0.089	0.006	0.064	0.259	0.109	0.003
$N = 30, C = 0.5, p = 1.25, \lambda = 1, \alpha = 1.5$							
Parameter	C	p	λ	α	β_1	β_2	β_3
Bias	-0.041	-0.096	-0.007	-0.217	0.577	0.364	0.057
MSE	0.001	0.011	0.0006	0.053	0.351	0.140	0.004
$N = 30, C = 0.5, p = 1.25, \lambda = 1, \alpha = 1.25$							
Parameter	C	p	λ	α	β_1	β_2	β_3
Bias	0.108	0.377	-0.073	-0.172	0.462	0.450	-0.018
MSE	0.054	0.580	0.021	0.068	0.349	0.233	0.003

Table 2: The Biases and MSEs for the MLEs of the Unknown Parameters (C, p, λ, α) For $N = 50$

$N = 50, C = 0.5, p = 2, \lambda = 1, \alpha = 2$							
Parameter	C	p	λ	α	β_1	β_2	β_3
Bias	-0.006	0.021	-0.001	-0.043	0.267	0.171	0.092
MSE	0.0008	0.387	0.00002	0.003	0.034	0.073	0.008
$N = 50, C = 0.5, p = 1.5, \lambda = 1, \alpha = 2$							
Parameter	C	p	λ	α	β_1	β_2	β_3
Bias	-0.078	0.097	-0.0002	-0.368	0.432	0.347	0.221
MSE	0.006	0.009	0.00008	0.223	0.320	0.300	0.049
$N = 50, C = 0.5, p = 2, \lambda = 1, \alpha = 1.25$							
Parameter	C	p	λ	α	β_1	β_2	β_3
Bias	-0.060	-0.130	-0.002	-0.204	0.334	0.311	0.156
MSE	0.003	0.018	0.006	0.044	0.181	0.192	0.025
$N = 50, C = 0.5, p = 2, \lambda = 1, \alpha = 1.5$							
Parameter	C	p	λ	α	β_1	β_2	β_3
Bias	-0.034	0.070	-0.001	-0.110	0.441	0.430	0.199
MSE	0.001	0.006	0.0004	0.015	0.295	0.282	0.040
$N = 50, C = 0.5, p = 1.25, \lambda = 1, \alpha = 2$							
Parameter	C	p	λ	α	β_1	β_2	β_3
Bias	-0.042	0.068	-0.004	-0.267	0.404	0.375	0.106
MSE	0.001	0.005	0.00002	0.080	0.256	0.142	0.012

$N = 50, C = 0.5, p = 1.5, \lambda = 1, \alpha = 1.25$							
Parameter	C	p	λ	α	β_1	β_2	β_3
Bias	-0.020	0.157	-0.014	-0.149	0.338	0.316	-0.064
MSE	0.0004	0.025	0.02	0.023	0.291	0.102	0.004
$N = 50, C = 0.5, p = 1.5, \lambda = 1, \alpha = 1.5$							
Parameter	C	p	λ	α	β_1	β_2	β_3
Bias	-0.047	0.110	-0.006	-0.260	0.450	0.337	0.079
MSE	0.002	0.012	0.0004	0.068	0.207	0.115	0.006
$N = 50, C = 0.5, p = 1.25, \lambda = 1, \alpha = 1.5$							
Parameter	C	p	λ	α	β_1	β_2	β_3
Bias	-0.031	-0.071	-0.005	-0.164	0.485	0.375	0.065
MSE	0.0009	0.005	0.0003	0.028	0.346	0.142	0.004
$N = 50, C = 0.5, p = 1.25, \lambda = 1, \alpha = 1.25$							
Parameter	C	p	λ	α	β_1	β_2	β_3
Bias	0.013	0.371	-0.0006	-0.090	0.343	0.328	-0.084
MSE	0.001	0.484	0.0006	0.021	0.307	0.189	0.008

Table 3: The Biases and MSEs for the MLEs of the Unknown Parameters (C, p, λ, α) For $N = 70$

$N = 70, C = 0.5, p = 2, \lambda = 1, \alpha = 2$							
Parameter	C	p	λ	α	β_1	β_2	β_3
Bias	-0.010	0.368	-0.005	0.067	0.306	0.291	0.027
MSE	0.0002	0.0510	0.0002	0.095	0.323	0.088	0.007
$N = 70, C = 0.5, p = 1.5, \lambda = 1, \alpha = 2$							
Parameter	C	p	λ	α	β_1	β_2	β_3
Bias	-0.053	0.099	-0.002	-0.259	0.500	0.384	0.161
MSE	0.003	0.010	0.00004	0.072	0.565	0.359	0.026
$N = 70, C = 0.5, p = 2, \lambda = 1, \alpha = 1.25$							
Parameter	C	p	λ	α	β_1	β_2	β_3
Bias	-0.025	-0.080	-0.105	-0.396	0.462	0.436	0.063
MSE	0.004	0.024	0.05	0.058	0.559	0.193	0.008
$N = 70, C = 0.5, p = 2, \lambda = 1, \alpha = 1.5$							
Parameter	C	p	λ	α	β_1	β_2	β_3
Bias	-0.036	-0.074	-0.002	-0.115	0.386	0.271	0.184
MSE	0.001	0.005	0.00004	0.014	0.233	0.240	0.034
$N = 70, C = 0.5, p = 1.25, \lambda = 1, \alpha = 2$							
Parameter	C	p	λ	α	β_1	β_2	β_3
Bias	-0.034	0.056	-0.004	-0.213	0.475	0.358	0.103
MSE	0.001	0.003	0.00002	0.054	0.228	0.130	0.011
$N = 70, C = 0.5, p = 1.5, \lambda = 1, \alpha = 1.25$							
Parameter	C	p	λ	α	β_1	β_2	β_3
Bias	0.090	0.038	-0.278	-0.184	0.390	0.371	-0.044
MSE	0.044	0.040	0.085	0.039	0.595	0.150	0.003
$N = 70, C = 0.5, p = 1.5, \lambda = 1, \alpha = 1.5$							
Parameter	C	p	λ	α	β_1	β_2	β_3
Bias	-0.045	0.105	-0.007	-0.241	0.403	0.309	0.071
MSE	0.002	0.011	0.00005	0.059	0.167	0.097	0.005
$N = 70, C = 0.5, p = 1.25, \lambda = 1, \alpha = 1.5$							
Parameter	C	p	λ	α	β_1	β_2	β_3
Bias	-0.029	0.066	-0.005	-0.151	0.450	0.355	-0.059
MSE	0.0008	0.004	0.00003	0.024	0.306	0.127	0.003
$N = 70, C = 0.5, p = 1.25, \lambda = 1, \alpha = 1.25$							
Parameter	C	p	λ	α	β_1	β_2	β_3
Bias	0.115	0.337	0.028	-0.049	0.499	0.361	-0.027
MSE	0.045	0.327	0.002	0.012	0.347	0.196	0.003

Table 4: The Biases and MSEs for the MLEs of the Unknown Parameters (C, p, λ, α) For $N = 100$

$N = 100, C = 0.5, p = 2, \lambda = 1, \alpha = 2$							
Parameter	C	p	λ	α	β_1	β_2	β_3
Bias	0.012	-0.393	-0.012	0.17	0.554	0.287	-0.033
MSE	0.001	0.0007	0.0005	0.153	0.635	0.085	0.010
$N = 100, C = 0.5, p = 1.5, \lambda = 1, \alpha = 2$							
Parameter	C	p	λ	α	β_1	β_2	β_3
Bias	-0.055	0.103	-0.002	-0.266	0.540	0.384	0.154
MSE	0.003	0.011	0.000005	0.074	0.413	0.335	0.024
$N = 100, C = 0.5, p = 2, \lambda = 1, \alpha = 1.25$							
Parameter	C	p	λ	α	β_1	β_2	β_3
Bias	0.019	0.027	-0.108	-0.158	0.571	0.407	0.096
MSE	0.009	0.015	0.0044	0.0166	0.466	0.169	0.017
$N = 100, C = 0.5, p = 2, \lambda = 1, \alpha = 1.5$							
Parameter	C	p	λ	α	β_1	β_2	β_3
Bias	-0.036	0.073	-0.002	-0.113	0.436	0.464	0.177
MSE	0.001	0.005	0.00004	0.013	0.202	0.220	0.032

$N = 100, C = 0.5, p = 1.25, \lambda = 1, \alpha = 2$							
Parameter	C	p	λ	α	β_1	β_2	β_3
Bias	-0.036	0.059	-0.004	-0.219	0.451	0.342	0.096
MSE	0.001	0.003	0.00002	0.056	0.207	0.118	0.009
$N = 100, C = 0.5, p = 1.5, \lambda = 1, \alpha = 1.25$							
Parameter	C	p	λ	α	β_1	β_2	β_3
Bias	0.188	0.017	-0.050	-0.197	0.496	0.443	-0.033
MSE	0.097	0.0110	0.009	0.040	0.496	0.228	0.002
$N = 100, C = 0.5, p = 1.5, \lambda = 1, \alpha = 1.5$							
Parameter	C	p	λ	α	β_1	β_2	β_3
Bias	0.027	0.009	-0.011	0.079	0.575	0.360	-0.051
MSE	0.004	0.008	0.0003	0.048	0.304	0.140	0.006
$N = 100, C = 0.5, p = 1.25, \lambda = 1, \alpha = 1.5$							
Parameter	C	p	λ	α	β_1	β_2	β_3
Bias	0.016	0.045	-0.0008	0.062	0.331	0.499	-0.043
MSE	0.001	0.0005	0.00001	0.022	0.533	0.249	0.005
$N = 100, C = 0.5, p = 1.25, \lambda = 1, \alpha = 1.25$							
Parameter	C	p	λ	α	β_1	β_2	β_3
Bias	0.109	0.041	-0.074	-0.043	0.567	0.523	-0.062
MSE	0.058	0.39	0.021	0.054	0.589	0.32	0.008

Table 5: The MLEs and the Confidence Intervals for the MLEs for the Unknown Parameters (C, p, λ, α) For $N = (30,50,70,100)$

$C = 0.5, p = 2, \lambda = 1, \alpha = 2$												
N		C			p			λ			α	
	$\hat{C}$	L.B	U.B	$\hat{p}$	L.B	U.B	$\hat{\lambda}$	L.B	U.B	$\hat{\alpha}$	L.B	U.B
30	0.547	0.416	0.677	1.944	1.604	2.283	0.672	0.644	0.699	1.273	1.058	1.487
50	0.593	0.226	0.959	1.821	0.586	3.055	0.699	0.542	0.855	1.257	0.521	1.992
70	0.590	0.571	0.608	2.568	1.832	3.303	0.705	0.685	0.724	1.467	0.879	2.055
100	0.612	0.583	0.640	2.593	1.189	3.996	0.688	0.654	0.721	1.47	0.779	2.160
$C = 0.5, p = 1.5, \lambda = 1, \alpha = 2$												
N		C			p			λ			α	
	$\hat{C}$	L.B	U.B	$\hat{p}$	L.B	U.B	$\hat{\lambda}$	L.B	U.B	$\hat{\alpha}$	L.B	U.B
30	0.576	0.313	0.838	2.507	0.968	4.045	0.540	0.344	0.736	1.597	0.915	2.278
50	0.822	0.808	0.835	1.397	1.377	1.416	0.400	0.399	0.412	1.232	1.108	1.355
70	0.847	0.827	0.866	1.699	1.655	1.742	0.698	0.697	0.699	1.241	1.117	1.364
100	0.845	0.825	0.864	1.703	1.669	1.736	0.698	0.697	0.699	1.234	1.126	1.341
$C = 0.5, p = 2, \lambda = 1, \alpha = 1.25$												
N		C			p			λ			α	
	$\hat{C}$	L.B	U.B	$\hat{p}$	L.B	U.B	$\hat{\lambda}$	L.B	U.B	$\hat{\alpha}$	L.B	U.B
30	0.831	0.811	0.850	1.751	1.699	1.802	0.698	0.696	0.699	0.763	0.697	0.828
50	0.840	0.820	0.859	1.730	1.674	1.785	0.697	0.696	0.699	0.796	0.708	0.883
70	0.875	0.699	1.050	2.386	1.190	3.581	0.495	0.138	0.851	0.904	0.718	1.089
100	0.919	0.899	0.938	2.127	2.093	2.160	0.592	0.591	0.620	0.842	0.734	0.949
$C = 0.5, p = 2, \lambda = 1, \alpha = 1.5$												
N		C			p			λ			α	
	$\hat{C}$	L.B	U.B	$\hat{p}$	L.B	U.B	$\hat{\lambda}$	L.B	U.B	$\hat{\alpha}$	L.B	U.B
30	0.849	0.821	0.876	1.703	1.637	1.768	0.698	0.694	0.699	0.839	0.737	0.940
50	0.866	0.838	0.893	1.670	1.608	1.731	0.688	0.686	0.689	0.890	0.802	0.977
70	0.864	0.848	0.879	1.674	1.640	1.707	0.698	0.696	0.699	0.885	0.836	0.933
100	0.864	0.586	1.141	1.673	0.340	3.005	0.698	0.336	1.059	0.887	0.882	1.248
$C = 0.5, p = 1.25, \lambda = 1, \alpha = 2$												
N		C			p			λ			α	
	$\hat{C}$	L.B	U.B	$\hat{p}$	L.B	U.B	$\hat{\lambda}$	L.B	U.B	$\hat{\alpha}$	L.B	U.B
30	0.659	0.639	0.678	1.467	1.427	1.475	0.796	0.794	0.797	1.333	1.157	1.508
50	0.658	0.630	0.685	1.468	1.436	1.499	0.795	0.793	0.796	1.333	1.245	1.420
C = 0.5, p = 1.25, λ = 1, α = 2												
N		C			p			λ			α	
	$\hat{C}$	L.B	U.B	$\hat{p}$	L.B	U.B	$\hat{\lambda}$	L.B	U.B	$\hat{\alpha}$	L.B	U.B
70	0.666	0.646	0.685	1.456	1.412	1.499	0.796	0.794	0.797	1.387	1.211	1.562
100	0.664	0.650	0.677	1.459	1.431	1.480	0.795	0.794	0.796	1.381	1.337	1.424
$C = 0.5, p = 1.5, \lambda = 1, \alpha = 1.25$												
N		C			p			λ			α	
	$\hat{C}$	L.B	U.B	$\hat{p}$	L.B	U.B	$\hat{\lambda}$	L.B	U.B	$\hat{\alpha}$	L.B	U.B
30	0.706	0.349	1.062	2.782	1.790	3.773	0.151	0.012	0.314	1.141	0.685	1.596
50	0.580	0.552	0.607	2.857	2.813	2.900	1.186	1.183	1.188	1.151	0.975	1.326
70	0.690	0.318	1.061	2.738	2.350	3.125	0.922	0.028	1.815	1.116	0.977	1.254
100	0.788	0.768	0.807	2.717	2.677	2.756	0.670	0.668	0.672	1.103	0.939	1.266
$C = 0.5, p = 1.5, \lambda = 1, \alpha = 1.5$												
N		C			p			λ			α	
	$\hat{C}$	L.B	U.B	$\hat{p}$	L.B	U.B	$\hat{\lambda}$	L.B	U.B	$\hat{\alpha}$	L.B	U.B
30	0.622	0.560	0.683	1.834	1.472	2.195	0.853	0.729	0.976	1.061	0.909	1.212
50	0.653	0.651	0.654	1.710	1.661	1.758	0.893	0.884	0.901	1.040	1.012	1.067
70	0.655	0.648	0.661	1.705	1.685	1.724	0.893	0.891	0.894	1.059	1.057	1.060
100	0.627	0.138	1.115	2.909	2.258	3.559	0.789	0.083	1.661	1.379	1.317	1.440

$C = 0.5, p = 1.25, \lambda = 1, \alpha = 1.5$												
N		C			p			λ			α	
	$\hat{C}$	L.B	U.B	$\hat{p}$	L.B	U.B	$\hat{\lambda}$	L.B	U.B	$\hat{\alpha}$	L.B	U.B
30	0.659	0.631	0.686	1.696	1.634	1.757	0.893	0.886	0.899	1.083	0.944	1.221
50	0.669	0.666	0.671	1.671	1.662	1.679	0.894	0.891	0.896	1.136	1.128	1.144
70	0.864	0.848	0.879	1.666	1.646	1.685	0.894	0.696	0.699	1.149	1.142	1.204
100	0.864	0.586	1.141	2.645	2.421	2.868	0.599	0.571	0.626	1.362	0.960	1.763
$C = 0.5, p = 1.25, \lambda = 1, \alpha = 1.25$												
N		C			p			λ			α	
	$\hat{C}$	L.B	U.B	$\hat{p}$	L.B	U.B	$\hat{\lambda}$	L.B	U.B	$\hat{\alpha}$	L.B	U.B
30	0.708	0.306	1.109	2.348	1.976	2.719	0.227	0.020	0.474	1.128	0.745	1.510
50	0.613	0.453	1.679	2.937	1.863	4.003	0.299	0.294	0.303	1.290	1.066	1.513
70	0.715	0.369	1.060	2.371	1.518	3.223	0.328	0.317	0.336	1.151	0.805	1.496
100	0.809	0.388	1.229	2.941	1.605	4.276	0.226	0.022	0.473	1.257	0.810	1.703

Table 6: The Values of the Optimum Time (τ_1) For $N = (30,50,70,100)$

$C = 0.5, p = 2, \lambda = 1, \alpha = 2$					$C = 0.5, p = 1.25, \lambda = 1, \alpha = 1.25$					$C = 0.5, p = 1.25, \lambda = 1, \alpha = 1.5$				
N	n_1	n_2	n_3	τ_1	N	n_1	n_2	n_3	τ_1	N	n_1	n_2	n_3	τ_1
30	10	9	10	0.913	30	11	6	9	1.519	30	10	7	11	1.022
50	17	16	16	0.959	50	19	10	16	1.176	50	17	12	18	0.994
70	23	22	23	0.901	70	24	17	26	1.439	70	28	14	22	0.944
100	34	31	34	0.913	100	39	20	32	1.35	100	34	24	37	0.986
$C = 0.5, p = 1.5, \lambda = 1, \alpha = 2$					$C = 0.5, p = 1.5, \lambda = 1, \alpha = 1.5$					$C = 0.5, p = 1.5, \lambda = 1, \alpha = 1.25$				
N	n_1	n_2	n_3	τ_1	N	n_1	n_2	n_3	τ_1	N	n_1	n_2	n_3	τ_1
30	8	9	11	0.824	30	11	7	10	1.137	30	10	5	11	0.752
50	14	16	18	1.092	50	19	12	16	1.118	50	21	9	14	0.937
70	20	22	26	1.016	70	26	16	24	1.087	70	29	13	20	1.272
100	29	31	38	1.009	100	37	23	34	0.968	100	42	19	29	1.39
$C = 0.5, p = 2, \lambda = 1, \alpha = 1.25$					$C = 0.5, p = 1.25, \lambda = 1, \alpha = 2$					$C = 0.5, p = 2, \lambda = 1, \alpha = 1.5$				
N	n_1	n_2	n_3	τ_1	N	n_1	n_2	n_3	τ_1	N	n_1	n_2	n_3	τ_1
30	14	5	6	1.870	30	8	9	11	1.784	30	13	6	8	1.743
50	24	8	11	1.862	50	13	16	19	1.126	50	22	11	14	1.533
70	34	12	15	1.223	70	19	22	28	1.709	70	30	15	20	1.947
100	48	17	22	1.088	100	27	32	40	0.958	100	43	22	28	1.008

Table 7: The Values of the Optimum Censoring Time (T) For $N = (30,50,70,100)$

$C = 0.5, p = 2, \lambda = 1, \alpha = 2$		$C = 0.5, p = 1.25, \lambda = 1, \alpha = 1.25$		$C = 0.5, p = 1.25, \lambda = 1, \alpha = 1.5$	
N	T	N	T	N	T
30	2.481	30	2.14	30	3.62
50	3.249	50	2.562	50	3.283
70	3.044	70	2.47	70	3.156
100	4.002	100	4.266	100	3.339
$C = 0.5, p = 1.5, \lambda = 1, \alpha = 2$		$C = 0.5, p = 1.5, \lambda = 1, \alpha = 1.5$		$C = 0.5, p = 1.5, \lambda = 1, \alpha = 1.25$	
N	T	N	T	N	T
30	2.934	30	3.087	30	3.05
50	3.037	50	3.381	50	5.491
70	3.042	70	3.359	70	2.533
100	4.04	100	4.624	100	2.381
$C = 0.5, p = 2, \lambda = 1, \alpha = 1.25$		$C = 0.5, p = 1.25, \lambda = 1, \alpha = 2$		$C = 0.5, p = 2, \lambda = 1, \alpha = 1.5$	
N	T	N	T	N	T
30	3.796	30	3.457	30	4.398
50	2.302	50	3.561	50	3.041
70	3.458	70	4.213	70	3.602
100	4.923	100	4.041	100	3.516

6. Conclusion

In this paper, the maximum likelihood method for estimating the unknown parameters with type-I censoring are obtained. The data failure times for multiple SSALT are assumed to follow the three parameter WP distribution at each stress level with scale parameter which is an inverse power law function of the stress. The performance of the estimated parameters is evaluated using the biases and the mean square error criteria. MLEs and approximate confidence intervals are obtained using different values of the parameters.

In addition, the corresponding optimum time of changing stress levels and optimum censoring time are obtained numerically by using A-optimality. This optimum design is an important decision problem at the designing stage of the test as it gives guide to experimenters about when the stress change should be carried out during the test and when the test should be terminated.

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