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Research Paper

A reliable iterative method for solving the epidemic model and the prey and predator problems

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Abstract

In the present article, we implement the new iterative method proposed by Daftardar-Gejji and Jafari (NIM) [V. Daftardar-Gejji, H. Jafari, An iterative method for solving nonlinear functional equations, J. Math. Anal. Appl. 316 (2006) 753-763] to solve two problems; the first one is the problem of spread of a non-fatal disease in a population which is assumed to have constant size over the period of the epidemic, and the other one is the problem of the prey and predator. The results demonstrate that the method has many merits such as being derivative-free, overcome the difficulty arising in calculating Adomian polynomials to handle the nonlinear terms in Adomian Decomposition Method (ADM), does not require to calculate Lagrange multiplier as in Variational Iteration Method (VIM) and no needs to construct a homotopy as in Homotopy Perturbation Method (HPM). The results obtained are compared with the results by existing methods and prove that the presented method is very effective, simple and does not require any restrictive assumptions for nonlinear terms. The software used for the numerical calculations in this study was MATHEMATICA ® 8.0.

Keywords: *New iterative method, Prey and predator problem, The epidemic model*

1. Introduction

Mathematical modeling has become important tools in analyzing the spread and control of infectious diseases. These models help us to understand different factors like the transmission and recovery rates and predict how the diseases will spread over a period of time [1].

To understand the dynamical interaction of epidemic in population usually ordinary differential equations (ODEs) system namely an SIR (susceptible, infectious, recovered) model is used [1] and references therein. In the SIR epidemic model the disease incubation is negligible such that once infected, each susceptible individual becomes infectious instantaneously and later recovers with a temporary acquired immunity.

Recently, many attempts have been made to develop analytic and approximate methods to solve the epidemic model, see for examples [1, 2, 3]. Although such methods have been successfully applied but some difficulties have appeared, for examples, construct a homotopy as in HPM and solve the corresponding the algebraic equations, in calculating Adomian polynomials to handle the nonlinear terms in ADM and calculate Lagrange multiplier as in VIM, respectively.

On the other hand, the problem in which some rabbits and foxes are considered living together, where foxes eat the rabbits and rabbits eat clover, and there is an increase and decrease in the number of foxes and rabbits appears to be interesting for the researchers to solve it and find the exact solution and approximate solutions to this problem, see for examples [3, 4, 5, 6, 7].

Recently Daftardar-Gejji and Jafari [8] have proposed a new technique for solving linear/nonlinear functional equations namely new iterative method (NIM) or (DJ method). The DJ method has been extensively used by many researchers for the treatment of linear and nonlinear ordinary and partial differential equations of integer and fractional order [9, 10, 11, 12]. The method converges to the exact solution if it exists through successive approximations. For concrete problems, a few number of approximations can be used for numerical purposes with high degree of accuracy. The DJM is simple to understand and easy to implement using computer packages and yields better results and does not require any restrictive assumptions for nonlinear terms as required by some existing techniques [9].

In this paper, the applications of the DJ method for both the epidemic model and the problem of the prey and predator will be presented to find the approximate solutions. The main attractive features of the current method are being derivative-free, overcome the difficulty some existing techniques, simple to understand and easy to implement using computer packages.

The paper is organized as follows. Section 2 is devoted to the basic idea of DJ method and the convergence of DJM is presented. In section 3 the mathematical model of the epidemic model and the problem of the prey and predator is illustrated. In section 4 the two problems are solved by DJ method and the numerical results for the approximate solutions are discussed with comparison with some existing techniques and finally in section 5 the conclusion is presented.

2. The basic idea of DJ method

Consider the following general functional equation:

$$u = N(u) + f, \tag{1}$$

where N is a nonlinear operator from a Banach space $B \to B$ and f is a known function[8]. We are looking for a solution u of Eq.(1) having the series form:

$$u = \sum_{i=0}^{\infty} u_i. \tag{2}$$

The nonlinear operator N can be decomposed as

$$N(\sum_{i=0}^{\infty} u_i) = N(u_0) + \sum_{i=1}^{\infty}\{N(\sum_{j=0}^{i} u_j) - N(\sum_{j=0}^{i-1} u_j)\}. \tag{3}$$

From Eqs.(2) and (3), Eq.(1) is equivalent to

$$\sum_{i=0}^{\infty} u_i = f + N(u_0) + \sum_{i=1}^{\infty}\{N(\sum_{j=0}^{i} u_j) - N(\sum_{j=0}^{i-1} u_j)\}. \tag{4}$$

We define the recurrence relation:

$$\begin{array}{l} G_0 = u_0 = f, \\ G_1 = u_1 = N(u_0), \\ G_m = u_{m+1} = N(u_0 + \cdots + u_m) - N(u_0 + \cdots + u_{m-1}), \;\; m = 1, 2, ... \end{array} \tag{5}$$

Then

$$(u_1 + ... + u_{m+1}) = N(u_1 + ... + u_m), \;\; m = 1, 2, ..., \tag{6}$$

and

$$u(x) = f + \sum_{i=1}^{\infty} u_i. \tag{7}$$

The m-term approximate solution of Eq.(2) is given by $u = u_0 + u_1 + ... + u_{m-1}$.

2.1. Convergence of the DJM

We present below the condition for convergence of the series $\sum u_i$. For more details we refer the reader to [13].

Theorem 2.1.1 :[13]

If N is $C^{(\infty)}$ in a neighbourhood of u_0 and $\| N^{(n)}(u_0) \| \leq L$, for any n and for some real $L > 0$ and $\| u_i \| \leq M < \frac{1}{e}$, $i = 1, 2, ...$, then the series $\sum_{n=0}^{\infty} G_n$ is absolutely convergent and moreover, $\| G_n \| \leq LM^n e^{n-1}(e-1), n = 1, 2, ...$

Theorem 2.1.2 : [13]

If N is $C^{(\infty)}$ and $\|N^{(n)}(u_0)\| \leq M \leq e^{-1}$, $\forall n$, then the series $\sum_{n=0}^{\infty} G_n$ is absolutely convergent.

3. The epidemic model and the problem of the prey and predator

3.1. The epidemic model

The problem of spreading of a non-fatal disease in a population which is assumed to have constant size over the period of the epidemic is considered in [1, 2, 14, 15, 16]. At time t suppose the population consist of:

$x(t)$ susceptible population: those so far uninfected and therefore liable to infection;
$y(t)$ infective population: those who have the disease and are still at large;
$z(t)$ isolated population, or who have recovered and are therefore immune.

Then the following system determines the progress of the disease, for more details on the mathematical modeling leading to the following system of nonlinear equations, governing the problem we refer the readers to [1, 2, 14, 15, 16].

$$\begin{aligned} \tfrac{dx}{dt} &= -\beta x(t)y(t), \\ \tfrac{dy}{dt} &= \beta x(t)y(t) - \gamma y(t), \\ \tfrac{dz}{dt} &= \gamma y(t) \end{aligned} \tag{8}$$

with initial conditions:

$$x(0) = N_1 \quad y(0) = N_2 \quad z(0) = N_3.$$

Biazar [2] used the Adomian decomposition method (ADM) to solve this problem. Islam et al. [1] used homotopy perturbation method to the Eq.(8). Moreover, Rafei et al. [3, 6] used both the variational iteration method to solve this problem.

3.2. The prey and predator problem

There are some rabbits and foxes living together. Foxes eat the rabbits and rabbits eat clover, and there is an increase and decrease in the number of foxes and rabbits [3, 4, 5, 6, 7]. Another example of the prey and predator problem is given in [15] in a lake there are two species of fish: A, which lives on plants of which there is a plentiful supply, and B (the predator) which subsists by eating A (the prey). For more details on the mathematical modeling leading to the following system of nonlinear equations, governing the problem; the readers can be found more details in [3, 4, 5, 6, 7, 14, 15, 16].

$$\begin{aligned} \tfrac{dx}{dt} &= x(t)(a - by(t)), \\ \tfrac{dy}{dt} &= -y(t)(c - dx(t)), \end{aligned} \tag{9}$$

where $x(t)$ and $y(t)$ are respectively the populations of rabbits and the foxes at time t and a, b, c, d are constants.

Rafei et al. [3] used the variational iteration method to solve this problem. Also, Biazar and Montazeri [4] used the Adomian decomposition method (ADM) to solve this problem. Moreover, Biazar et al. [5] have successfully implemented the power series method to compute an approximation to the solution of the this problem. Moreover, the homotopy perturbation method is employed to find the approximate solution for this problem in [6]. In addition, the classical ADM is converted into a hybrid numeric-analytic method and applied to solve the prey and predator problem.

4. Solution of the epidemic model and the problem of the prey and predator by DJ method

4.1. Solution of the epidemic model by DJ method

In order to apply the DJ method to solve the system of differential equations in Eq. (8) it should convert it to the corresponding system of integral equations, this can be done by applying inverse of the operator $\frac{d(\cdot)}{dt}$, which is integration operator $\int\limits_0^t (\cdot)dt$ to each equations in the system (8) we derive

$$\begin{aligned} x(t) &= x_0 - \beta \int\limits_0^t x(x)y(t)dt, \\ y(t) &= y_0 + \int\limits_0^t \left[\beta x(x)y(t) - \gamma y(t)\right]dt, \\ z(t) &= z_0 + \gamma \int\limits_0^t y(t)dt. \end{aligned} \tag{10}$$

where $x_0 = x(t = 0) = N_1, \quad y_0 = y(t = 0) = N_2, \quad z_0 = z(t = 0) = N_3$, are the initial conditions.
A few first terms being calculated by DJ method:

$$\begin{aligned} x_1 &= I_1(x_0, y_0, z_0) = -\beta \int\limits_0^t x_0 y_0 dt = -N_1 N_2 t\beta, \\ y_1 &= I_2(x_0, y_0, z_0) = \int\limits_0^t [\beta x_0 y_0 - \gamma y_0]dt = t(N_1 N_2 \beta - N_2 \gamma), \\ z_1 &= I_3(x_0, y_0, z_0) = \gamma \int\limits_0^t y0 dt = N_2 t\gamma, \end{aligned} \tag{11}$$

$$\begin{aligned} &x_2 = I_1(x_0 + x_1, y_0 + y_1, z_0 + z_1) = -\beta \int\limits_0^t [(x_0 + x_1)(y_0 + y_1)]dt - x_1 = \\ &= \tfrac{1}{6} N_1 N_2 t^2 \beta(N_1 \beta(-3 + 2N_2 t\beta) + 3\gamma + \mathrm{N}_2 \beta(3 - 2t\gamma)), \\ &y_2 = I_2(x_0 + x_1, y_0 + y_1, z_0 + z_1) = \int\limits_0^t [\beta(x_0 + x_1)(y_0 + y_1) - \gamma(y_0 + y_1)]dt - y1 = \\ &= -\tfrac{1}{6} N_2 t^2 ({N_1}^2 \beta^2 (-3 + 2N_2 t\beta) - 3\gamma^2 + N_1 \beta(6\gamma + N_2 \beta(3 - 2t\gamma))), \\ &z_2 = I_3(x_0 + x_1, y_0 + y_1, z_0 + z_1) = \gamma \int\limits_0^t (y_0 + y_1)dt - z_1 = \tfrac{1}{2} N_2 t^2 (N_1 \beta - \gamma)\gamma, \\ &x_3 = I_1(x_0 + x_1 + x_2, y_0 + y_1 + y_2, z_0 + z_1 + z_2) = -\beta \int\limits_0^t [(x_0 + x_1 + x_2)(y_0 + y_1 + y_2)]dt + \\ &+\beta \int\limits_0^t [(x_0 + x_1)(y_0 + y_1)]dt = \tfrac{1}{2520} N_1 N_2 t^3 \beta(2{N_1}^3 N_2 t^2 \beta^4 (63 - 70N_2 t\beta + 20{N_2}^2 t^2 \beta^2) - \\ &2{N_1}^2 \beta^2 (210 + 10{N_2}^3 t^3 \beta^3 (-7 + 4t\gamma) + 21N_2 t\beta(-20 + 9t\gamma) - 7{N_2}^2 t^2 \beta^2 (-42 + 25t\gamma)) + \\ &2N_1 \beta(420\gamma + 21N_2 \beta(20 - 35t\gamma + 9t^2\gamma^2) + {N_2}^3 t^2 \beta^3 (63 - 70t\gamma + 20t^2\gamma^2) - 7{N_2}^2 t\beta^2 (60 - 63t\gamma + 20t^2\gamma^2)) + \\ &7(-60\gamma^2 - 6N_2 \beta\gamma(10 - 15t\gamma + 3t^2\gamma^2) + {N_2}^2 \beta^2 (-60 + 75t\gamma - 42t^2\gamma^2 + 10t^3\gamma^3))), \end{aligned} \tag{12}$$

$$y_3 = I_2(x_0+x_1+x_2, y_0+y_1+y_2, z_0+z_1+z_2) = \int_0^t [\beta(x_0+x_1+x_2)(y_0+y_1+y_2)$$
$$-\gamma(y_0+y_1+y_2)]dt - \int_0^t [\beta(x_0+x_1)(y_0+y_1) - \gamma(y_0+y_1)]dt = -\frac{1}{2520}N_2$$
$$t^3(2{N_1}^4N_2t^2\beta^5(63-70N_2t\beta+20{N_2}^2t^2\beta^2)+420\gamma^3-2{N_1}^3\beta^3(210+10{N_2}^3t^3\beta^3(-7+4t\gamma)+$$
$$21N_2t\beta(-20+9t\gamma)-7{N_2}^2t^2\beta^2(-42+25t\gamma))+2{N_1}^2\beta^2(630\gamma+21N_2\beta(20-40t\gamma+9t^2\gamma^2)+ \quad (13)$$
$${N_2}^3t^2\beta^3(63-70t\gamma+20t^2\gamma^2)-7{N_2}^2t\beta^2(60-63t\gamma+20t^2\gamma^2))+7N_1\beta(-180\gamma^2-6N_2\beta\gamma(20-20t\gamma+3t^2\gamma^2)+$$
$${N_2}^2\beta^2(-60+75t\gamma-42t^2\gamma^2+10t^3\gamma^3))),$$
$$z_3 = I_3(x_0+x_1+x_2, y_0+y_1+y_2, z_0+z_1+z_2) = \gamma\int_0^t (y_0+y_1+y_2)dt =$$
$$\frac{1}{12}N_2t^3\gamma({N_1}^2\beta^2(2-N_2t\beta)+2\gamma^2+N_1\beta(-4\gamma+N_2\beta(-2+t\gamma)))$$

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Continuing in this manner, we can find the rest of components $x_4, x_5; y_4, y_5; z_4, z_5$ were also determined and will be used, but for brevity not listed.

4.1.1. Numerical results and comparison with the ADM and HPM

For comparison with the results done by Biazar [2] using ADM and the results done by Rafei et al. [3], the following values, for parameters, are considered:

$N_1 = 20$ Initial population of $x(t)$, who are susceptible

$N_2 = 15$ Initial population of $y(t)$, who are infective

$N_3 = 10$ Initial population of $z(t)$, who are immune

$\beta = 0.01$ Rate of change of susceptible population to infective population

$\gamma = 0.02$ Rate of change of infective population to immune population

A few first approximations for $x(t)$, $y(t)$ and $z(t)$, are calculated and presented below after substituting the above values in Eqs. (11), (12) and (13):

$$\begin{aligned} x_1(t) &= 20-3t, \\ y_1(t) &= 15+2.7t, \\ z_1(t) &= 10+0.3t, \end{aligned} \quad (14)$$

$$\begin{aligned} x_2(t) &= 20-3t-0.045t^2+0.027t^3, \\ y_2(t) &= 15+2.7t+0.018t^2-0.027t^3, \\ z_2(t) &= 10+0.3t+0.027t^2, \end{aligned} \quad (15)$$

$$\begin{aligned} x_3(t) &= 20-3t-0.045t^2+0.02805t^3+0.00077625t^4-0.00030618t^5-2.835*10^{-6}t^6+ \\ &1.04143*10^{-6}t^7, \\ y_3(t) &= 15+2.7t+0.018t^2-0.02817t^3-0.00064125t^4+0.00030618t^5+2.835*10^{-6}t^6- \\ &1.04143*10^{-6}t^7, \\ z_3(t) &= 10+0.3t+0.027t^2+0.00012t^3-0.000135t^4, \end{aligned} \quad (16)$$

$$\begin{aligned}
&x_4(t) = 20 - 3t - 0.045t^2 + 0.02805t^3 + 0.000795375t^4 - 0.000316507t^5 - \\
&0.0000122051t^6 + 3.54056 * 10^{-6}t^7 + 1.00638 * 10^{-7}t^8 - 2.49702 * 10^{-8}t^9 - \\
&6.59004 * 10^{-10}t^{10} + 1.34797 * 10^{-10}t^{11} + 2.67689 * 10^{-12}t^{12} - \\
&4.84378 * 10^{-13}t^{13} - 4.21779 * 10^{-15}t^{14} + 7.23049 * 10^{-16}t^{15}, \\
&y_4(t) = 15 + 2.7t + 0.018t^2 - 0.02817t^3 - 0.000654525t^4 + \\
&0.000319072t^5 + 0.0000111845t^6 - 3.54866 * 10^{-6}t^7 - \\
&9.80339 * 10^{-8}t^8 + 2.49702 * 10^{-8}t^9 + 6.59004 * 10^{-10}t^{10} - \\
&1.34797 * 10^{-10}t^{11} - 2.67689 * 10^{-12}t^{12} + 4.84378 * 10^{-13}t^{13} + \\
&4.21779 * 10^{-15}t^{14} - 7.23049 * 10^{-16}t^{15}, \\
&z_4(t) = 10 + 0.3\text{t} + 0.027t^2 + 0.00012t^3 - 0.00014085t^4 - 2.565 * 10^{-6}t^5 + \\
&1.0206 * 10^{-6}t^6 + 8.1 * 10^{-9}t^7 - 2.60357 * 10^{-9}t^8,
\end{aligned} \tag{17}$$

These results are plotted in Figs. 1-4. As the plots show while the number of susceptible population increases the population of who are infective decreases in the period of the epidemic, meanwhile the number of immune population increases. But the size of the population over the period of the epidemic is constant.

A comparison between the results obtained by the DJ method and the ADM and the VIM in [2, 3], shows that the results of four terms approximations of the of both DJ method and VIM is the same as the results of six terms approximations of the ADM. In fact, the results obtained by DJ method is completely the same with VIM but it is appear the results obtained by ADM in [2] are not accurate enough.

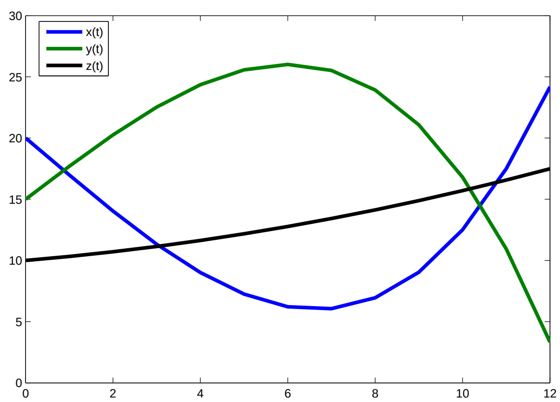

Figure 1: Three terms approximations

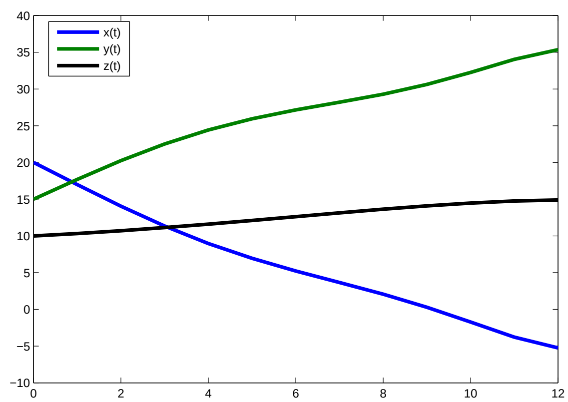

Figure 2: Four terms approximations

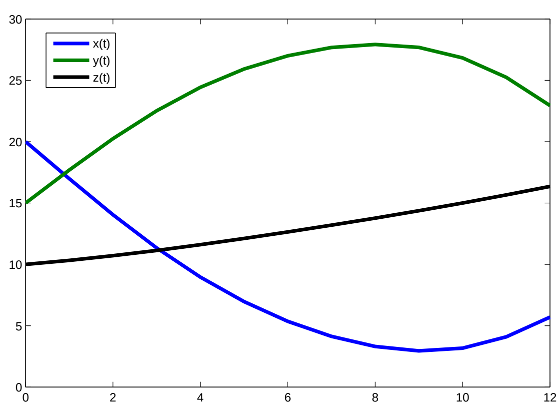

Figure 3: Five terms approximations

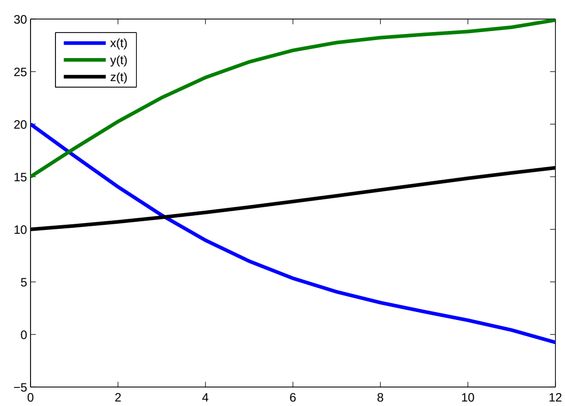

Figure 4: Six terms approximations

4.2. Solution of the the prey and predator problem by DJ method

By following the same procedure as for the epidemic model. The application of DJ method for the system of differential equations in Eq. (9) can be done by converting it to the corresponding system of integral equations, this can be done by applying inverse of the operator $\frac{d(\cdot)}{dt}$, which is integration operator $\int\limits_0^t (\cdot)dt$ to each equations in the system (9) we derive

$$\begin{aligned} x(t) &= x_0 + \int\limits_0^t x(t)(a - by(t))dt, \\ y(t) &= y_0 - \int\limits_0^t y(t)(c - dx(t))dt. \end{aligned} \tag{18}$$

By starting with initial approximations $x_0(t) = x_0,\ y_0(t) = y_0$. A few first terms can be calculated by DJ method:

$$\begin{aligned} x_1 &= L_1(x_0, y_0) = \int\limits_0^t x_0(a - by_0)dt = (x_0 a - x_0 y_0 b)t, \\ y_1 &= L_2(x_0, y_0) = -\int\limits_0^t y_0(c - dx_0) = -(dx_0 y_0 d - cy_0)t, \end{aligned} \tag{19}$$

$$\begin{aligned} & x_2 = L_1(x_0 + x_1, y_0 + y_1) = \int\limits_0^t [(x_0 + x_1)(a - b(y_0 + y_1))]dt - x_1 = \\ & = -\tfrac{1}{6}t^2 x_0(-3a^2 + 2ab(3 - ct + dtx_0)y_0 + by_0(-3by_0 + dx_0(3 - 2bty_0) + c(-3 + 2bty_0))), \\ & y_2 = L_2(x_0 + x_1, y_0 + y_1) = -\int\limits_0^t [(y_0 + y_1)(c - d(x_0 + x_1))]dt - y1 = \\ & \tfrac{1}{6}t^2 y_0(3c^2 - 2cdx_0(3 + at - bty_0) + dx_0(a(3 + 2dtx_0) - 3by_0 + dx_0(3 - 2bty_0))). \end{aligned} \tag{20}$$

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Continuing in this manner, we can find the rest of components $x_3, x_4, x_5; y_3, y_4, y_5$; were also determined and will be used, but for brevity not listed.

4.2.1. Numerical results and comparison with the ADM and HPM

For comparison with the results done by Biazar [2] using ADM and the results done by Rafei et al. [3], the following values, for parameters, are considered:

Case	x_0	y_0	a	b	c	d
1	14	18	0.1	1	1	1
2	16	10	1	1	0.1	1

For comparison with the results done in [2, 3] a four terms approximation and according to the values introduced in the table, the following solutions are derived for the two cases by using Eqs.(19) and (20):

Case 1:

$$x(t) = 14 - 250.6t + 604.87t^2 + 19364.9t^3 - 100453.2t^4 - 1.80563 * 10^6 t^5 + 4.36307 * 10^6 t^6 + 5.45825 * 10^7 t^7$$

$$y(t) = 18 + 234t - 734.4t^2 - 19099.98t^3 + 105828.5t^4 + 1.80563 * 10^6 t^5 - 4.36307 * 10^6 t^6 - 5.45825 * 10^7 t^7$$

Case 2:

$$x(t) = 16 - 144t - 624t^2 + 6602.4t^3 + 57745.8t^4 - 394602t^5 - 1.48576 * 10^6 t^6 + 8.32106 * 10^6 t^7$$

$$y(t) = 10 + 159t + 544.05t^2 - 6828.54t^3 - 55647t^4 + 394602t^5 + 1.48576 * 10^6 t^6 - 8.32106 * 10^6 t^7$$

Figs. 5-12 show the relations between the number of foxes and the rabbits versus time for the two cases 1 and 2. A comparison between the results derived by the DJ method with those obtained by the Adomian decomposition [2] and VIM in [3], shows that the results of three terms approximations of the DJ method is approximately the same with VIM and both the same as the results of four terms approximations of the Adomian decomposition and the homotopy perturbation method [6].

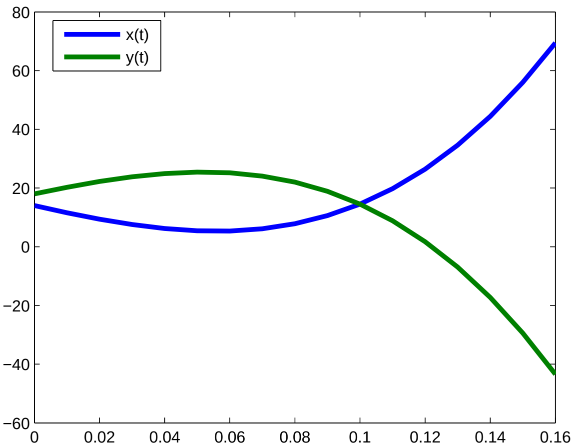

Figure 5: Three terms approximations for case 1

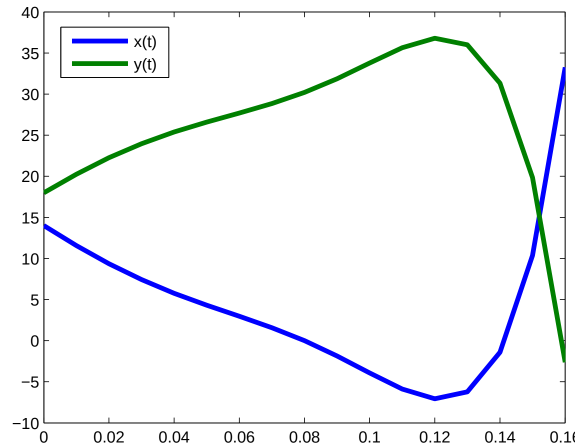

Figure 6: Four terms approximations for case 1

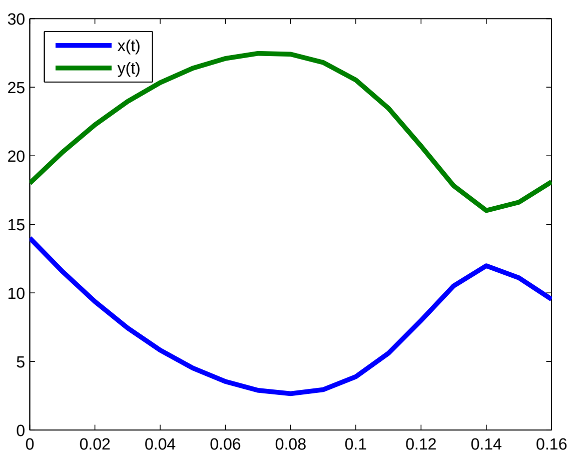

Figure 7: Five terms approximations for case 1

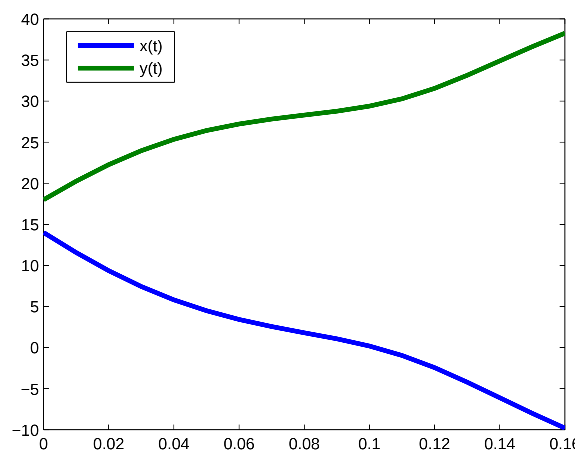

Figure 8: Six terms approximations for case 1

5. Conclusion

In this paper, the nonlinear systems of differential equations, governing the epidemic model and prey and predator problem are successfully solved using new iterative method or DJ method [8]. The main advantages of proposed method are being derivative-free, overcome the difficulty some existing techniques, simple to understand and easy to implement and can be easily comprehended with only a basic knowledge of Calculus. There is less computation needed and it is economical in terms of computer power/memory in comparison with the Adomian decomposition method, the homotopy perturbation method and variational iteration method. For computations the MATHEMATICA® 8.0 is used and for plots the MATLAB R2011a is used.

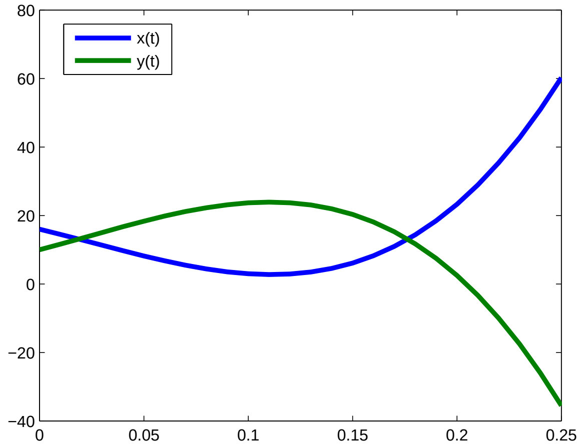

Figure 9: Three terms approximations for case 2

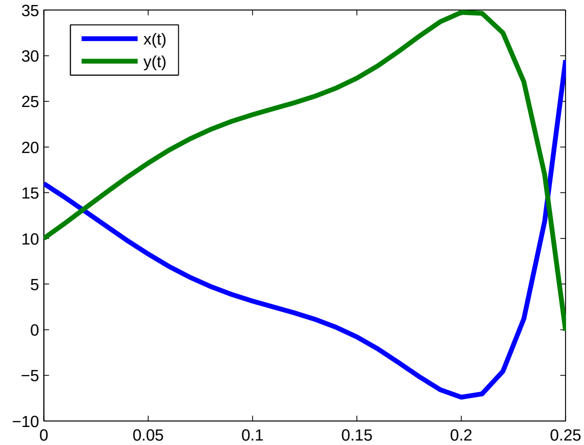

Figure 10: Four terms approximations for case 2

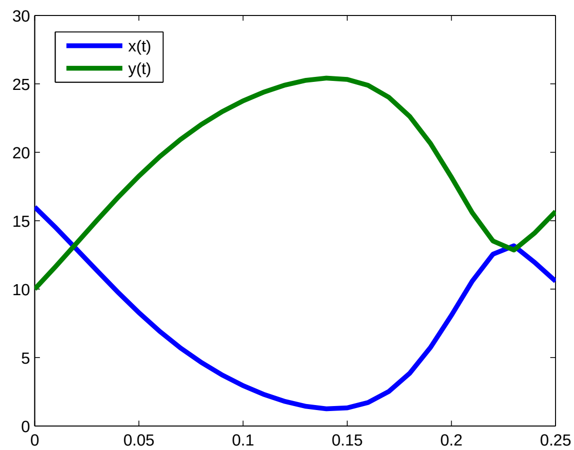

Figure 11: Five terms approximations for case 2

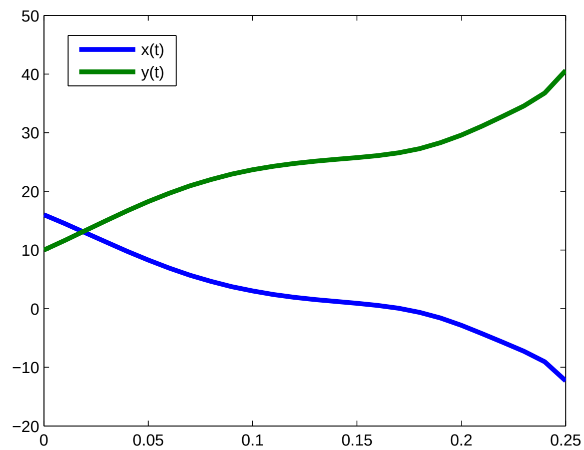

Figure 12: Six terms approximations for case 2

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