International Journal of Basic and Applied Sciences, 1 (4) (2012) 490-497

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New Exact Solutions of (2 + 1)-Dimensional Bogoyavlenskii Equation by the Sine-Cosine Method

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Abstract

By using the sine-cosine method proposed recently, we give the exact periodic and soliton solutions of the (2 + 1)-dimensional Bogoyavlenskii equation in this paper. Many new families of exact traveling wave solutions of the (2 + 1)-dimensional Bogoyavlenskii equation are successfully obtained. The computation for the method appears to be easier and faster by general mathematical software.

Keywords: *Sine-cosine method, Bogoyavlenskii equation, Periodic solution, Soliton solution.*

1 Introduction

Many important phenomena and dynamic processes in physics, mechanics, chemistry and biology can be represented by nonlinear partial differential equations. The study of exact solutions of nonlinear evolution equations plays an important role in soliton theory and explicit formulas of nonlinear partial differential equations play an essential role in the nonlinear science. Also, the explicit formulas may provide physical information and help us to understand the mechanism of related physical models. In recent years, many kinds of powerful methods have been proposed to find solutions of nonlinear partial differential equations, numerically and/or analytically, e.g., the tanh-method [1, 2, 3], the extended tanh method [4, 5, 6], the sine-cosine method [7, 8, 9], the homogeneous balance method [10], homotopy analysis method [11, 12, 13, 14], the $F-$expansion method [15], three-wave method [16, 17, 18], extended homoclinic test approach [19, 20], the $(\frac{G'}{G})-$expansion method [21] and the exp-function method [22, 23, 24].

In this paper, by means of the Sine-cosine method, we will obtain some exact and new solutions for the (2 + 1)-dimensional Bogoyavlenskii equation. In the following section we have a brief review on the Sine-cosine method and in Section 3, we apply the Sine-cosine method to obtain analytic solutions of the (2 + 1)-dimensional Bogoyavlenskii equation. Finally, the paper is concluded in Section 4.

2 The Sine-Cosine Method

1. We introduce the wave variable $\xi = x - ct$ into the PDE

$$P(u, u_t, u_x, u_{tt}, u_{xx}, u_{tx}, \ldots) = 0, \tag{1}$$

where $u(x,t)$ is traveling wave solution. This enables us to use the following changes:

$$\frac{\partial}{\partial t} = -c\frac{\partial}{\partial \xi}, \frac{\partial^2}{\partial t^2} = c^2\frac{\partial^2}{\partial \xi^2}, \frac{\partial}{\partial x} = \frac{\partial}{\partial \xi}, \frac{\partial^2}{\partial x^2} = \frac{\partial^2}{\partial \xi^2}, \ldots. \tag{2}$$

One can immediately reduce the nonlinear PDE (1) into a nonlinear ODE

$$Q(u, u_\xi, u_{\xi\xi}, u_{\xi\xi\xi}, \ldots) = 0. \tag{3}$$

The ordinary differential equation (3) is then integrated as long as all terms contain derivatives, where we neglect integration constants.

2. The solutions of many nonlinear equations can be expressed in the form

$$u(x,t) = \begin{cases} \lambda\, sin^\beta(\mu\,\xi), & |\xi| \le \dfrac{\pi}{\mu}, \\ 0 & \text{otherwise}, \end{cases} \tag{4}$$

or in the form

$$u(x,t) = \begin{cases} \lambda\, cos^\beta(\mu\,\xi), & |\xi| \le \dfrac{\pi}{2\mu}, \\ 0 & \text{otherwise}, \end{cases} \tag{5}$$

where λ, μ and$\beta \neq 0$ are parameters that will be determined, μ and c are the wave number and the wave speed respectively. We use

$$\begin{aligned}
&u(\xi) = \lambda\, sin^{\beta}(\mu\, \xi),\\
&u^n(\xi) = \lambda^n\, sin^{n\,\beta}(\mu\, \xi),\\
&(u^n)_\xi = n\mu\beta\lambda^n \cos(\mu\xi)\sin^{n\beta-1}(\mu\xi),\\
&(u^n)_{\xi\xi} = -n^2\mu^2\beta^2\lambda^n \sin^{n\beta}(\mu\xi) + n\mu^2\lambda^n\beta(n\beta-1)\sin^{n\beta-2}(\mu\xi),
\end{aligned} \tag{6}$$

and the derivatives of 5 becoms

$$\begin{aligned}
&u(\xi) = \lambda\, cos^{\beta}(\mu\, \xi),\\
&u^n(\xi) = \lambda^n\, cos^{n\,\beta}(\mu\, \xi),\\
&(u^n)_\xi = -n\mu\beta\lambda^n \sin(\mu\xi)\cos^{n\beta-1}(\mu\xi),\\
&(u^n)_{\xi\xi} = -n^2\mu^2\beta^2\lambda^n \cos^{n\beta}(\mu\xi) + n\mu^2\lambda^n\beta(n\beta-1)\cos^{n\beta-2}(\mu\xi),
\end{aligned} \tag{7}$$

and so on for other derivatives.

3.We substitute (6) or (7) into the reduced equation obtained above in (3), balance the terms of the cosine functions when (7) is used, or balance the terms of the sine functions when (6) is used, and solving the resulting system of algebraic equations by using the computerized symbolic calculations. We next collect all terms whit same power in $\cos^k(\mu\xi)$ or $\sin^k(\mu\xi)$ and set to zero their coefficients to get a system of algebraic equations among the unknowns μ, β and λ. We obtained all possible value of the parameters μ, β and λ.

3 The (2+1)-Dimensional Bogoyavlenskii Equation:

The dimensionless form of the (2+1)-dimensional Bogoyavlenskii equation, that is going to be studied in this paper is given by [25]

$$\begin{aligned}
&4\, u_t + u_{xxy} - 4\, u^2\, u_y - 4\, u_x\, v = 0\\
&u\, u_y = v_x\, .
\end{aligned} \tag{8}$$

To solve (8), we use the transformation

$$u(x, y, t) = u(\xi), \quad v(x, y, t) = v(\xi), \quad \xi = x + y - ct. \tag{9}$$

Substituting (9) into (8) and integrating once the second equation of (8) and for simplicity, equating the integration constant equal to zero, we have

$$
\begin{aligned}
&4\,c\,u^{'} + u^{'''} - 4\,u^{2}\,u^{'} - 4\,u^{'}\,v = 0 \\
&\frac{u^{2}}{2} = v\,.
\end{aligned}
\tag{10}
$$

The substitution of the second equation of (10) into (8), after integrating once the resultant, yields

$$
u^{''} - 2\,u^{3} + 4\,c\,u = 0 \tag{11}
$$

Substituting (4) into (11) gives

$$
\begin{aligned}
&-\mu\,k^{2}\beta^{2}\lambda\sin^{\beta}(\mu\xi) + \mu^{2}k^{2}\lambda\beta(\beta-1)\sin^{\beta-2}(\mu\xi) \\
&-2\,\lambda^{3}\sin^{3\beta}(\mu\xi) + 4\,c\lambda\sin^{\beta}(\mu\xi) = 0.
\end{aligned}
\tag{12}
$$

Equating the exponents and the coefficients of each pair of the sine functions we find the following system of algebraic equations:

$$
\begin{aligned}
&(\beta-1) \neq 0, \\
&\beta - 2 = 3\,\beta, \\
&-\mu^{2}k^{2}\,\beta^{2}\,\lambda + 4\,c\,\lambda = 0, \\
&\mu^{2}k^{2}\,\beta(\beta-1)\,\lambda - 2\,\lambda^{3} = 0,
\end{aligned}
\tag{13}
$$

Solving the system (13) yields

$$
\beta = -1 \quad , \quad \mu = \pm\frac{2\sqrt{c}}{k} \quad , \quad \lambda = \pm 2\sqrt{c}, \tag{14}
$$

where c and k are free parameters. The results (14) give for $c > 0$, the following periodic solutions:

$$
u_{11}(\xi) = 2\sqrt{c}\,\csc\left(\frac{2\sqrt{c}}{k}(\xi)\right), \; v_{11}(\xi) = 2\,c\,\csc^{2}\left(\frac{2\sqrt{c}}{k}(\xi)\right)
$$

where $0 < \pm 2\sqrt{c}\,(\xi) < \pi$, and

$$
u_{12}(\xi) = 2\sqrt{c}\,\sec\left(\frac{2\sqrt{c}}{k}(\xi)\right), \; v_{12}(\xi) = 2\,c\,\sec^{2}\left(\frac{2\sqrt{c}}{k}(\xi)\right)
$$

and

$$
u_{13}(\xi) = -2\sqrt{c}\,\sec\left(\frac{2\sqrt{c}}{k}(\xi)\right), \; v_{13}(\xi) = 2\,c\,\sec^{2}\left(\frac{2\sqrt{c}}{k}(\xi)\right)
$$

where $|2\sqrt{c}\,(\xi)| < \dfrac{\pi}{2}$.
However, for $c < 0$, we obtain the soliton solutions

$$u_{21}(\xi) = 2\sqrt{c}\,csch\left(\frac{2\sqrt{c}}{k}\,(\xi)\right),\ v_{21}(\xi) = 2\,c\,csch^2\left(\frac{2\sqrt{c}}{k}\,(\xi)\right)$$

where $0 < \pm 2\sqrt{c}\,(\xi) < \pi$, and

$$u_{22}(\xi) = 2\sqrt{c}\,sech\left(\frac{2\sqrt{c}}{k}\,(\xi)\right),\ v_{22}(\xi) = 2\,c\,sech^2\left(\frac{2\sqrt{c}}{k}\,(\xi)\right)$$

and

$$u_{23}(\xi) = -2\sqrt{c}\,sech\left(\frac{2\sqrt{c}}{k}\,(\xi)\right),\ v_{23}(\xi) = 2\,c\,sech^2\left(\frac{2\sqrt{c}}{k}\,(\xi)\right)$$

where $|2\sqrt{c}\,(\xi)| < \dfrac{\pi}{2}$.
As we know, various other types of exact solutions for the (2+1)-dimensional Bogoyavlenskii equation, such as rational solutions, polynomial solutions and the traveling wave solutions have been obtained by many authors under different approaches [25, 26, 27]. However, the periodic and soliton solutions are firstly given out to our knowledge.

4 Conclusion

In this paper, We successfully obtained exact and explicit analytic solutions to the (2+1)-dimensional Bogoyavlenskii equation via the sinecosine approach. Some of these results are in agreement with the results reported by others in the literature, and new results are formally developed in this work. It is shown that the algorithm can be also applied to other NLPDEs in mathematical physics.

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