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Anti-de-Sitter Schwarzschild black hole in fuzzy space

Mourad Nach[1], Abdellah EL Boukili[2], Moulay Brahim Sedra[1] and Abderrahman EL Boukili[1]*

[1]Ibn Tofail University, Faculty of Sciences, Physics Departement,
(LHESIR), Kénitra, Morocco.
[2]Mohamed V university, Faculty of Sciences Agdal,
(LECA), Rabat, Morocco.
**Corresponding author E-mail: aelboukili@gmail.com*

Abstract

In this paper we focus on the metric of an Anti-de-Sitter Schwarzschild black hole in fuzzy space with an exponential decay function as Mass density of the point particle. Given the metric and assuming that the fuzzy space parameter *h* is small, we study the horizon, the area spectrum and the Hawking temperature of the AdS-Schwarzschild black hole.

Keywords: *Anti-de-Sitter space, Schwarzschild black hole, fuzzy space, horizon, Hawking temperature*

1 Introduction

AdS Schwarzschild black hole is a black hole solution of general relativity or its extensions which represents an isolated massive object, but with a negative cosmological constant[1]. Such a solution asymptotically approaches anti de Sitter space at spatial infinity, and is a generalization of the Kerr vacuum solution, which asymptotically approaches Minkowski spacetime at spatial infinity[2]. The metric of the AdS-Schwarzschild black hole is given by[2]

$$ds^2=-\left(1-\frac{2M}{r}+\frac{r^2}{l^2}\right)dt^2+\frac{dr^2}{\left(1-\frac{2M}{r}+\frac{r^2}{l^2}\right)}+r^2\left(d\theta^2+\sin^2\theta d\varphi^2\right) \tag{1}$$

There is a horizon at[3]

$$r_{\mathrm{h}}=\sqrt[3]{l^2\left(M-\sqrt{M^2+\frac{1}{27}l^2}\right)}+\sqrt[3]{l^2\left(M+\sqrt{M^2+\frac{1}{27}l^2}\right)} \tag{2}$$

The horizon area A of the AdS-Schwarzschild black hole is given by[4]

$$\begin{aligned}A&=r_{\mathrm{h}}^2\int_0^{2\pi}d\varphi\int_0^{\pi}\sin\theta d\theta=4\pi r_{\mathrm{h}}^2\\&=4\pi\left(\sqrt[3]{l^2\left(M-\sqrt{M^2+\frac{1}{27}l^2}\right)}+\sqrt[3]{l^2\left(M+\sqrt{M^2+\frac{1}{27}l^2}\right)}\right)^2\end{aligned} \tag{3}$$

The Hawking temperature of the AdS-Schwarzschild black hole black hole is given by [5]

$$T_{\mathrm{H}}=\frac{1}{4\pi}\left|\frac{M}{r_{\mathrm{h}}^2}+\frac{2}{l^2}\right| \tag{4}$$

In this paper we focus on the metric of an AdS-Schwarzschild black hole in fuzzy space with the Mass density of the point particle is described by[6]

$$\rho_h(r)=\frac{1}{2\pi h^2}\exp\left(-\frac{r}{h}\right) \tag{5}$$

Given the metric and assuming that the noncommutative parameter h is small, we study the horizon, the area spectrum and the Hawking temperature of the AdS-Schwarzschild black hole. Since the fuzzy noncommutativity in space violates rotational symmetry, our horizon corrections have a preferred direction.
This paper is organized as follows. In section 2, we present the horizon, area spectrum and Hawking temperature of the AdS-Schwarzschild black hole in fuzzy spaces. In section 3 we present the thermodynamics properties of such black hole. Finally we discuss our results in Section4.

2 AdS-Schwarzschild black hole in fuzzy space

To find the solution of AdS-Schwarzschild 4D, we use the smeared source (5) following the method of Nicolini [7]. use the condition of the metric $g_{tt} = -g_{rr}^{-1}$ for AdS-Schwarzschild Noncommutative and conservation covariant tensor momentum energy, momentum energy tensor can be fixed to the form [8],

$$T_{\mu}^{\nu} = diag\left(-\rho_h, p_r, p_{\perp}, p_{\perp}\right) \tag{6}$$

in which

$$p_r = -\rho_h \quad , \; p_{\perp} = p_r - \frac{r}{2}\partial_r \rho_h \tag{7}$$

Thus, rather than a massive, structureless point, a source turns out to a self-gravitating, droplet of anisotropic fluid of density $\rho_h(r)$, radial pressure p_r and tangential pressure $p_{\perp}$.
The solution of Einstein equation, using (6) as the matter source, is the same as replacing the mass of Dirac-delta function source in standard Schwarzschild spacetime by the effective mass of smeared source[9]

$$M_h(r) = \int d^3 x \rho_h(r) = M\left[1 - \left(\frac{1}{2}\left(\frac{r}{h}\right)^2 + \frac{r}{h} + 1\right)\exp\left(-\frac{r}{h}\right)\right] \tag{8}$$

which approaches to M in the limit $h \to 0$ in the commutative space. The AdS-Schwarzschild metric in four dimensions reads

$$dS^2 = -f\left(r\right)dt^2 + \frac{1}{f\left(r\right)}dr^2 + r^2 d\varphi^2 \tag{9}$$

Where

$$f\left(r\right) = 1 - \frac{2M_h\left(r\right)}{r} + \frac{r^2}{l^2} \tag{10}$$

Note that the effective mass described in Moyal space with smeared source of (5) is $M_{\theta}(r) = \frac{M}{\sqrt{\pi}}\gamma\left(\frac{3}{2}, \frac{r^2}{4\theta}\right)$. It approaches to M in the limit $\theta \to 0$, the commutative space.
we see that the solution of condition $f\left(r\right) = 0,$ the horizon can be written in the following form

$$r_H = \sqrt[3]{l^2\left(M_h\left(r\right) - \sqrt{M_h^2\left(r\right) + \frac{1}{27}l^2}\right)} + \sqrt[3]{l^2\left(M_h\left(r\right) + \sqrt{M_h^2\left(r\right) + \frac{1}{27}l^2}\right)} \tag{11}$$

which reduces in the case commutative space to the following value

$$r_H = \sqrt[3]{l^2\left(M - \sqrt{M^2 + \frac{1}{27}l^2}\right)} + \sqrt[3]{l^2\left(M + \sqrt{M^2 + \frac{1}{27}l^2}\right)} \tag{12}$$

3 Thermodynamic properties of fuzzy AdS-Schwarzschild black hole

3.1 Hawking temperature

The Hawking temperature of the geometry of the fuzzy Ads-Schwarzschild black hole is

$$\begin{aligned} T_H &= \frac{1}{4\pi}\left|\frac{\partial f(r)}{\partial r}\right|_{r=r_+} \\ &= \frac{1}{4\pi r_+}\left\{\left[1+\frac{r_+^2}{l^2}\right]\left[1-\frac{1}{2}\left(\frac{r_+}{h}\right)^3 \frac{\exp\left(-\frac{r_+}{h}\right)}{1-\left(\frac{1}{2}\left(\frac{r_+}{h}\right)^2+\frac{r_+}{h}+1\right)\exp\left(-\frac{r_+}{h}\right)}\right]+\frac{2r_+^2}{l^2}\right\} \end{aligned} \tag{13}$$

Where

$$M_H = \frac{r_+ + \frac{r_+^3}{l^2}}{2\left(1-\left(\frac{1}{2}\left(\frac{r_+}{h}\right)^2+\frac{r_+}{h}+1\right)\exp\left(-\frac{r_+}{h}\right)\right)} \tag{14}$$

For large black holes, i.e. $\left(\frac{r_+}{h}\right) \succ\succ 0$, and one recovers the temperature of the Ads- Schwarzschild black hole[10]

$$T_H = \frac{1}{4\pi}\left[\frac{1}{r_+}+\frac{3r_+}{l^2}\right] \tag{15}$$

3.2 Entropy

According to a general form of entropy given by ref.[11]

$$S = \int_{r_0}^{r_+}\frac{1}{T_H}dM = \int_{r_0}^{r_+}\frac{1}{T_H}\frac{\partial M}{\partial r_+^{'}}dr_+^{'} \tag{16}$$

and using the expression of M given by the equation (14) we obtain the following integral expression of the entropy

$$S = \int_{r_0}^{r_+}\left(\frac{1}{T_H}\frac{\left(1+\frac{3r_+^{'2}}{l^2}\right)\left(1-\left(\frac{1}{2}\left(\frac{r_+^{'}}{h}\right)^2+\frac{r_+^{'}}{h}+1\right)\exp\left(-\frac{r_+^{'}}{h}\right)\right)}{\left(1-\left(\frac{1}{2}\left(\frac{r_+^{'}}{h}\right)^2+\frac{r_+^{'}}{h}+1\right)\exp\left(-\frac{r_+^{'}}{h}\right)\right)^2}\right)dr_+^{'} - \int_{r_0}^{r_+}\frac{1}{T_H}\frac{\left(r_+^{'}+\frac{3r_+^{'3}}{l^2}\right)\left(\frac{1}{2}\frac{r_+^{'2}}{h^3}\exp\left(-\frac{r_+^{'}}{h}\right)\right)}{\left(1-\left(\frac{1}{2}\left(\frac{r_+^{'}}{h}\right)^2+\frac{r_+^{'}}{h}+1\right)\exp\left(-\frac{r_+^{'}}{h}\right)\right)^2}dr_+^{'} \tag{17}$$

4 Conclusion

We have focused in this paper on the metric of an Anti-de-Sitter Schwarzschild black hole in fuzzy space with an exponential decay function as Mass density of the point particle. we have study the horizon, the area spectrum and the Hawking temperature of the AdS-Schwarzschild black hole. we have shown that the fuzzy noncommutativity in space violates the symmetry.

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