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Modeling of a non maintained system with non constant deterioration rates

Saminu Iliyasu Bala

Department of Mathematical Sciences, Bayero University Kano, Nigeria

Abstract

In this paper we presents the reliability and mean time to failure estimation of a system where the deterioration rates follow the Weibull distribution. The system is two unit active parallel system where the units operates simultaneously and maintenance is not allowed. We are particularly interested in the effect of the shape parameter (from Weibull Distribution) on the system reliability. Explicit expression for Mean time to failure of the system is obtained analytically. Numerical study of the system using assumed data has shown that the choice of the shape parameter influences the time taken by the system to reach the failure state. Also, high values of the shape parameter decreases system reliability. The Model developed in this paper is important to engineers, maintenance managers, and plant management for good maintenance decision.

Keywords: *Mean Time to System Failure, Reliability, Shape Parameter, Weibul Distribution.*

1 Introduction

Equipment and technical systems are inevitably subjected to aging and wearing away through the course of their life time. Degradation has an implication on many important areas such as inventory management and control. Furthermore, it is a common knowledge that deterioration can reduce system performance and will ultimately lead to random failure. System failures are usually modelled by assuming that the process through failure can be explained through deterioration process. Modelling the degradation of a system can be done using a stochastic process. Many researches has been undertaking on reliability and mean time to system failure ($MTTF$) of deteriorating systems. For example, [2] study the $MTTF$ of a two-state complex with repairable system, the problem of dynamic preventive maintenance for deteriorating production system was studied in detail by [1]. In that paper, the author categorized the deterioration into consecutive states in which the deterioration of the system increase as it moves from one state to the next. [7] used semi-Markov process in modelling deteriorating systems. Other studies on deteriorating system can be found in [6, 5, 4, 3].

Modelling the system deterioration is important because it will assist in diagnosing the best time to carry out a preventive maintenance. This paper is a follow up on the work of [5] who studied reliability and mean time to failure estimation of two unit active parallel system. In that paper, the authors looked at the effect of time and deterioration rates on reliability and $MTTF$.
There are two reasons why we want to revisit that paper. First of all the authors considered the deteriorating rates as constants. Degradation is usually measured at several times and may be influenced by the system environment. Because deterioration rate is uncertain over time, it should ideally be represented as a stochastic and not a deterministic process. Here we consider the deteriorating rates as stochastic variables which follow the Weibull distribution. Secondly, there is an error in the the solution of the system of equations given in that paper. This error also affects the formula for $MTTF$ in that paper ([5]).

2 Model formulation and assumptions

To formulate the model we make certain underlying assumptions:

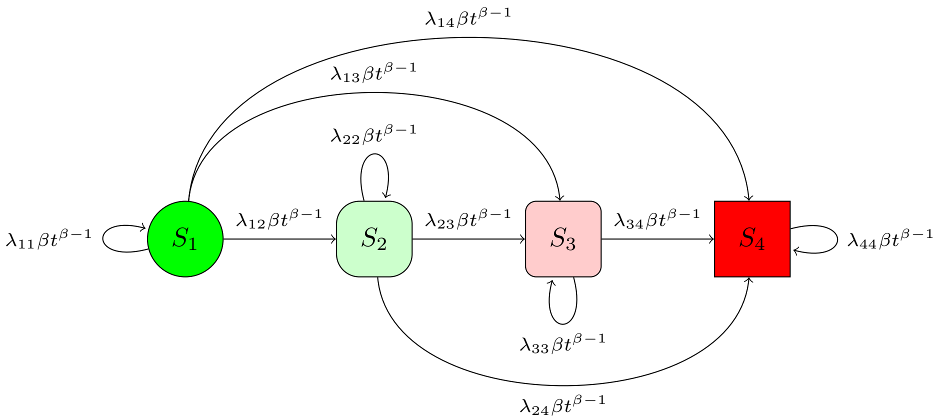

Fig. 1: Transition diagram for the non maintained system showing the various states and parameters

1. State of the system can be: Perfect (S_1), Minor deterioration (S_2), Major deterioration (S_3), Failure (S_4).
2. At any given time t, the system is either in the operating state, deteriorating state or in the failed state.
3. The units operate simultaneously.
4. State S_4 can be access from the previous state.
5. The state of the system changes as time progresses.
6. System/units work in S_1, S_2, S_3, and fail in S_4.
7. Deteriorating rates follow The Weibull distribution $\theta(t) = \lambda_{mn}\beta t^{\beta-1}$. β is the shape parameter.

2.1 Notations and nomenclature

The following notations are used throughout this paper.
$\lambda_{12}, \lambda_{13}, \lambda_{23}$: Deterioration rates
λ_{14} : Deterioration rate of the system while in S_1.
λ_{24}: Deterioration rate of the system while in S_2.
λ_{34}: Deterioration rate of the system while in S_3.
$R(t)$: Reliability of the system at time t.
MTTF: Mean time to failure of the system.

2.2 The model

Using the assumptions and notations presented in this section, we present the traction diagram of the model as in Fig1.

Fig 1 show the various states and parameters. From the Figure, we derived the following differential equations:

$$\frac{d}{dt}P_1(t) = -\beta t^{\beta-1}(\lambda_{12}+\lambda_{13}+\lambda_{14})P_1(t). \quad (1)$$
$$\frac{d}{dt}P_2(t) = \beta t^{\beta-1}(\lambda_{12}P_1(t)-(\lambda_{23}+\lambda_{24})P_2(t)). \quad (2)$$
$$\frac{d}{dt}P_3(t) = \beta t^{\beta-1}(\lambda_{13}P_1(t)+\lambda_{23}P_2(t)-\lambda_{34}P_3(t)). \quad (3)$$
$$\frac{d}{dt}P_4(t) = \beta t^{\beta-1}(\lambda_{14}P_1(t)+\lambda_{24}P_2(t)+\lambda_{34}P_3(t)). \quad (4)$$

We also imposed the following initial conditions

$$[P_1(0), P_2(0), P_3(0), P_4(0)] = [1, 0, 0, 0]. \quad (5)$$

3 Main results

We solved the differential equations 1-4 subject to initial condition 5 to obtained the following

$$P_1(t) = e^{-(\lambda_{12}+\lambda_{13}+\lambda_{14})t^\beta}, \tag{6}$$

$$P_2(t) = -\frac{\lambda_{12}e^{-(\lambda_{12}+\lambda_{13}+\lambda_{14})t^\beta}}{D_2} + \frac{\lambda_{12}e^{(-\lambda_{23}-\lambda_{24})t^\beta}}{D_2}, \tag{7}$$

$$\begin{aligned} P_3(t) &= \frac{R_1\left(e^{-(\lambda_{12}+\lambda_{13}+\lambda_{14})t^\beta} - e^{-t^\beta\lambda_{34}}\right)}{D_2\left(\lambda_{34}-\lambda_{12}-\lambda_{13}-\lambda_{14}\right)} \\ &+ \frac{\lambda_{23}\lambda_{12}\left(e^{-t^\beta(\lambda_{23}+\lambda_{24})} - e^{-t^\beta\lambda_{34}}\right)}{D_2\left(\lambda_{34}-\lambda_{23}-\lambda_{24}\right)}, \end{aligned} \tag{8}$$

where $R_1 = {\lambda_{13}}^2 + (-\lambda_{23}+\lambda_{12}+\lambda_{14}-\lambda_{24})\lambda_{13} - \lambda_{23}\lambda_{12}$, $D_1 = \Gamma(\beta)/\beta$ and $D_2 = -\lambda_{23}-\lambda_{24}+\lambda_{12}+\lambda_{13}+\lambda_{14}$. Equations 6-8 represent part of the contribution of this paper. This result is new because it expresses the probabilities in terms of general value of the shape-parameter (β). If $\beta = 1$, the current model should normally reduce to that of [5]. Under this condition it is useful to compare this model with that of [5]. It can be seen that equations 6 and 7 are the same as expressions for $P_1(t)$ and $P_2(t)$ in that paper. The expression for $P_3(t)$ from this paper (6) is different from that of [5]. In that paper, the term $\frac{-R_1e^{-t^\beta\lambda_{34}}}{D_2(\lambda_{34}-\lambda_{12}-\lambda_{13}-\lambda_{14})}$ is missing. In fact one of the objectives of paper is to highlight this correction. With a missing term in the expression for $P_3(t)$, one will expect a different expression for the $MTTF$ and consequently different numerical results.
Following [5], the mean time to system failure is defined by $MTTF = \int_0^\infty(P_1(t)+P_2(t)+P_3(t))dt$. In our formulation the $MTTF$ is given by

$$\begin{aligned} MTTF &= D_1\Bigg[\frac{\lambda_{12}\left(-\left(\lambda_{12}+\lambda_{13}+\lambda_{14}\right)^{-\beta^{-1}} + \left(\lambda_{23}+\lambda_{24}\right)^{-\beta^{-1}}\right)}{D_2} \\ &+ \left(\lambda_{12}+\lambda_{13}+\lambda_{14}\right)^{-\beta^{-1}} + \frac{R_1\left(\left(\lambda_{12}+\lambda_{13}+\lambda_{14}\right)^{-\beta^{-1}} - {\lambda_{34}}^{-\beta^{-1}}\right)}{D_2\left(\lambda_{34}-\lambda_{12}-\lambda_{13}-\lambda_{14}\right)} \\ &+ \frac{\lambda_{23}\lambda_{12}\left(\left(\lambda_{23}+\lambda_{24}\right)^{-\beta^{-1}} - {\lambda_{34}}^{-\beta^{-1}}\right)}{D_2\left(\lambda_{34}-\lambda_{23}-\lambda_{24}\right)}\Bigg]. \end{aligned} \tag{9}$$

This formula for the $MTTF$ is different from that of [5] see equation 5 of that paper. This also gives another contribution from this paper.

3.1 Numerical simulation of the model

In this section we investigate the behaviour of the model developed via numerical simulation. To do that, we assumed the following values for the deterioration rates shown on Table 1. The simulations were conducted with

Table 1: The Table shows the assumed values of the parameters used throughout the simulations shown on Figs 2 and 3

Parameter	Value
λ_{12}	0.1
λ_{13}	0.25
λ_{14}	0.9
λ_{23}	0.3
λ_{24}	0.4
λ_{43}	0.45

four values of $\beta = 1, 1.5, 2, 3$ for time t in the range [0 10].
Fig 2 depicts the plots of the transition probabilities for different values of $\beta(\beta = 1.5, 2, 2, 3)$. We assumed that the

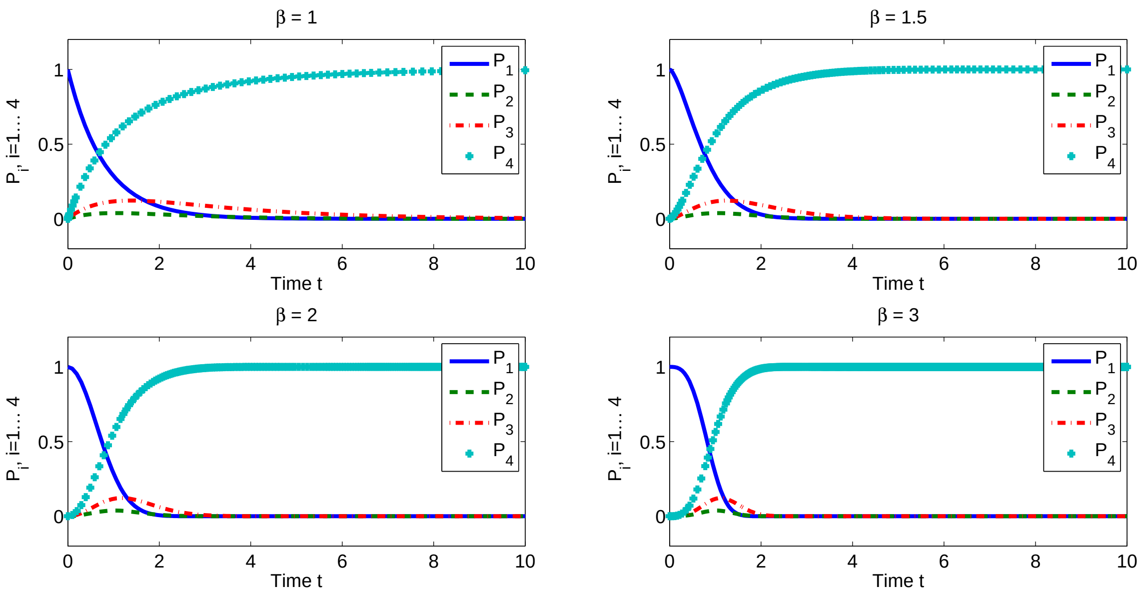

Fig. 2: Shows the various transition probabilities against time for four different values of β.

time taken for the system to be completely in the failed state to be equivalent to the time when $P_1(t), P_2(t)$ and $P_3(t) \leq 0.01$. Based on this assumption, we can see from the Figure that for $\beta = 1$, the system is completely in the failed state when $t \geq 8.4$. When $\beta = 1.5$, the system completely failed when $t \geq 4.14$. Thus, increasing β by 50% brings about a reduction of time taken to reach failure state by roughly $1/2$. Increasing β further to 2 and 3 gives corresponding failure time as $t \geq 2.9$ and $t \geq 2.05$ respectively. Based on these results, one can conclude that the choice of the shape parameter (β) influences the time taken for the system to reach the failure state. The higher the value of this parameter, the faster it takes the system to reach the failure state.

Fig 3 depicts the plots of the system reliability against time for 4 different values of the shape parameter β. From the Figure, similar observation can be made as we did for Fig 2. The higher, the value of β, the faster it takes the system to reach the failure state.

The results presented in this paper are based on the assumption that the system being considered has four states. Generally speaking, the model can be extended to any number of states (n). It is not difficult to see that equations 1 to 4 satisfy the general form given by equation 10.

$$\frac{dP_i}{dt} = -\sum_{j=1+i}^{n} \lambda_{ij} P_i + \sum_{j=1}^{n-1} \lambda_{ji} P_j, \quad i = 1 \ldots n, \lambda_{mn} = 0, \text{ if } m \geq n. \tag{10}$$

4 Conclusion

This paper is a follow up to an earlier paper as well as a correction to that paper. We first formulated a model of a non maintained system where the deterioration rates follow Weibull probability distribution. Then we obtained analytic expressions for the mean time to system failure (MTTF) for general shape parameter value. We used assumed data to study the various transition probabilities and the system reliability numerically. The results of the simulation indicates that the value of the shape parameter has an impact on the system reliability because high values of this parameter decreases system reliability and increases the probability of the system to be in failure state.

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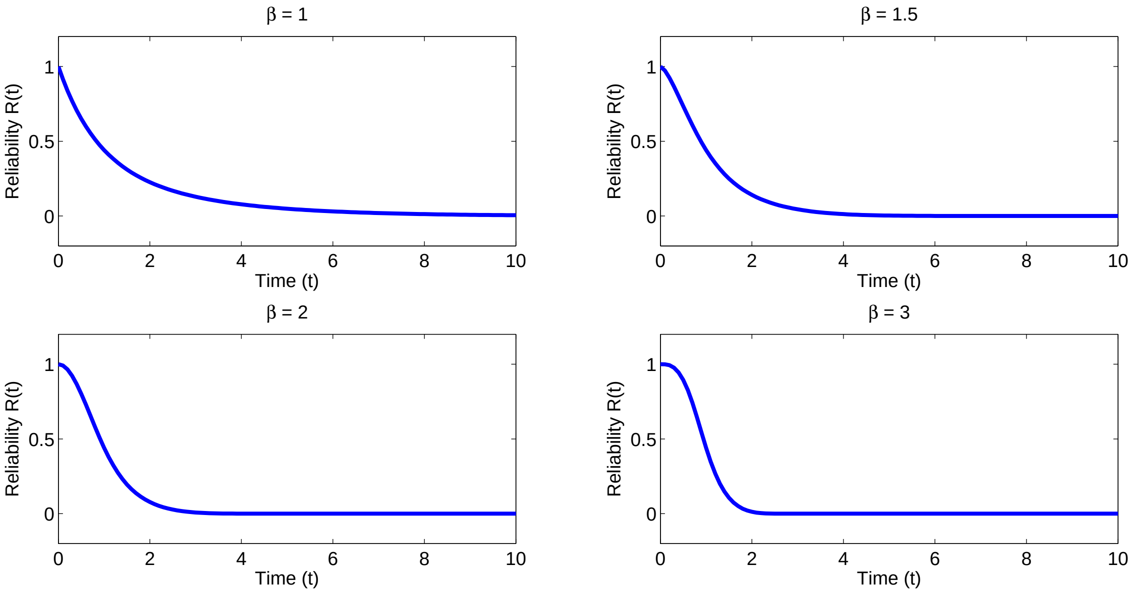

Fig. 3: The Figure shows the plots of the system reliability $R(t) = P_1(t) + P_2(t) + P_3(t)$ against time for various values of the shape parameter β.

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