***A Smarandache Completely Prime Ideal with Respect to an Element of Near Ring***

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**Abstact**

In this paper we introduce the notion of a smarandache completely prime ideal with respect to an element belated to a near field of a near ring N ( b-s-c.p.i ) of N . We study some properties of this new concept and link it with some there types of ideals of a near ring

**1- Introduction**

In 1905 , L.E.Drckson began the study of a near ring and later in 1930 , Wieland has investigated it { 1 } . in 1977 , G.Pilz , introduced the notion of a prime ideal of near ring { 1 } . in 1988 , N.G.Groenewald introduced of a completely prime ideal of a near ring { 5 } . in 2002 , W.B.Vasanth Kandasamy study samaradache near ring , (samaradache ideal , of a near ring { 7 } . in 2012 H.H. Abbass and M.A.Mohommed introduced the notion of a completely prime ideal with respect to an element of a near ring { 3 }

In this work We introduce a Samaradache completely prime ideal with respect to an element related to a near field of near ring as We mentioned in the abstract

**2- Preliminaries**

In this section, We review some basic concepts a bout a near ring, and some types of fields of a near rind that We need in our work

**Definition 2.1 [1]:-**

A left near ring is a set N together with two binary operations “+” and ”.” such that

(1) is a group (not necessarily abelian ),

(2) is a semi group ,

(3) .

Definition 2.2 [2]:-

The left near ring is called a zero symmetric if , for all .

Definition 2.3[7]:-

Left ­ be a near-ring. A normal subgroup of is called a left ideal of if

**Remark 2.4**

If N is a left near ring, then ,for all (from the left distributire law). Also, we will refer that all near rings and ideals in this work are left.

**Definition 2.5 [6]:-**

Let be an ideal of a near ring ,then is called a completely prime ideal of if for all , implies ,denoted by .

The a near ring in example is not

**Definition 2.6 [3]:-**

Let be a near ring, be an ideal of and let , then is called a completely ideal with respect to the element denoted by of , if for all , implies

Definition 2.7 [7]:-

A near ring is called an integral domain if has non\_zero divisors.

Definition 2.8 [7]:-

Let and be two near rings,

The mapping is called a near ring homomorphism if for all

**Definition 2.9 [7]:-**

Anon-empty set is said to be a near field if is defined by two binary operations “+” and ”.” such that

(1) is a group

(2) is a group

(3) .

**Definition 2.10 [7]:-**

The near ring is said to be a smarandache near ring denoted by (s-near ring) if it has aproper subset such that is a near field.

**Definition 2.11 [7]:-**

Let be s-near ring. A normal subgroup of is called a smarandache ideal (s-ideal) of related M if,

1. For all and for all ,,

Where M is the near field contained in .

 **Remark 2.12 [7]**

Let be a chain of s-ideals related to a near field M of a near ring N ,then

 is a s-ideals related to near field M

**Remark 2.13 [6]**

Let and be two s-near rings and let

 be an epimomorphism and has as near filed. Then is a near field of .

**Proposition 2.14 [4]:-**

Let and be two s-near rings and

 be an epimomorphism and let be a

s-ideals related to a near field M of a near ring N, then is s-ideals related to a near field .

 **Proposition 2.15 [4]:-**

Let be a s-near ring has a near filed , be a s-near ring, be an epimomorphism and let be s-ideals related to a near field of ,where of , then is a s-ideals related to a near field of .

**Definition 2.16 [7]:-**

Let N be a s-near ring. The s-ideals related to a near field M is called completely prime related to a near field M of N if ,for all implies .denoted by of .

**3. the main results**

In this section, we define the notion of smarandache completely ideal with respect to an element b

 and study some properties of this notion, we will discuss the image and pre image of under near rings epimomorphism and explain the relationships between it and of a near ring.

**Definition 3.1:-**

A s-ideals related to a near field M of a s-near ring is called a samarandache completely ideal with respect to an element b of , if implies for all .

***Example 3.2*:**

The left s-near ring with addition and multiplication defined by the following tables.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| + | 0 | a | b | c |
| 0 | 0 | a | b | c |
| a | a | 0 | c | b |
| b | b | c | 0 | a |
| c | c | b | a | 0 |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| . | 0 | a | b | c |
| 0 | 0 | 0 | 0 | 0 |
| a | 0 | a | a | 0 |
| b | 0 | a | b | c |
| c | 0 | 0 | c | c |

The s-ideal related to the near field is of since .

**Proposition 3.3:-**

Let be a s-ideal related to a near field M of a s-near ring ,then is a of if and only if is , where 1 is the multiplicative identity element of M.

Proof:

Suppose is a ideal of

and let such that .

then we have

 [since is a of ].

 of .

Conversely,

Let such that

 [since is of ].

**Remark (3.4)**

In general a S.C.P.I related to a near field M of a s-near ring N may not be b-S.C.P.I related to M of N as in the following example

**Example (3.5)**

Consider the s-near ring of integers mod 6 (z6, t6, .6); the s-ideal I= {0,2,4} is S.C.P.I related to the near field M = {0,3} , but it is not 2-S.C.P.I of N , since 3 M and 2.(3.3)=0  I but 3T .

**Proposition (3.6)**

Let I be a b-C.P.I related to a near field M of a s-near ring N. then I is a b-S.C.P.I of N.

Proof

Let x, y M , such that b. (x.y)  I

x,y  N [ since M is a proper subset of N]

xI or y I [since I is b-S.C.P.I of N]

I is a b-S.C.P.I of N.

**Remark (3.7)**

The conzerse of proposition (3.6) may not be true as in the following example .

**Example (3.8)**

Consider the s-near ring of integers mod 12 (Z12, t12, i12); s-ideal I = {0,2,4,6,8,10} if z-S.C.P.I related to the near field M= { 0,4,8} , but it is not 2-C.P.I , since 3,5 Z12 and 2.(3.5) = 6I, but 3 and 5 I .

**Proposition (3.9)**

Let N be a s-near ring and let I be a s-ideal related to a near field M of N. then I is a b-S.C.P.I of N if and only if M is a subset of I, for all bI >

Proof

Suppose I is a b-S.C.P.I , bI and XM.

Now,

X2 = x.xI , 0I and 0. x2 = 0. (x.x) =0I

xI [since I is o-S.C.P.I ],

M is a subset of I

Conversely,

Let bI and x ,y M such that b.(x.y) I

x or y  I [since MI ]

I is b-S.C.P.I of N.

**Proposition (3.10)**

Let N be a s-near integral domain . then I ={0} is b-S.C.P.I related to a near field M of N, for all nNl {0} .

Proof:

Let bNl {0} and x ,y M , such that b.(x.y) I

b. (x.y) =0

x.y =0 [since b0 and N is a near integral domain]

x=0 or y=0 xI or yI

xI or yI .

I is a b-S.C.P.I of N .

**Proposition (3.11)**

Let N be a zero symmetric s-near ring and let I={0}. Then I is not o-S.C.P.I of N related to all near fields of N .

Proof:

Suppose I is o-S.C.P.I related to a near field M of N .

Since M is a near field M {0}

? X M , such that x0 .

Now,

0x2 = 0.(x.x) = 0  I

xI x=0 and this contradiction [ since x0]

I is not 0- S.C.P.I related to M of N.

**Proposition (2.12)**

Let N be a s-near ring and let {Ii} iI be a chain of b-S.C.P.I related to a near field M of N , for all iI . then V iI Ii is a b-S.C.P.I related to M of N .

Proof

Since {Ii} iI is a chain a b-S.C.P.I related to M of N .

Ii is a s-ideal of N for all i I .

 V iI Ii is a s-deal of N [ By remark (2.12)]

Now,

Let x , y M , such that b.(x.y)  V iI Ii

there exists b-S.C.P.I related M Ik  {Ii} iI of N, such that b.(x.y)  Ik

xIk or y  Ik [ since Ik is a b-S.C.P.I of N]

x  V iI Ii or Y V iI Ii .  V iI Ii is a b-S.C.P.I of N .

**Remark (3.13)**

In general , if {Ii} iI is a family of b-S.C.P.I related to a near field M of as near ring N, then ????? and V iI Ii may not be b-S.C.P.I

Related to M of N , as in the following example

**Example (3.13):**

Consider the s-near ring of integers mod12. (Z12,t12, 12) , the s-ideals I={0,6} and J={0,4,8} are 3-S.C.P.I. related to the near field M= {0,4,8} of Z12, but the s-ideal ?????=
{0} is not 3-S.C.P.I related to M of Z12 , since 3.(3.8)=0 I, but and 8  I, Also , the subset I J= {0,4,6,8} is s-ideal of Z12 and this implies I J is not 3-S.C.P.I related to M of Z12 .

**Theorem : (3.15)**

Let (N1, \*, 0) and (N2, t, 0) be two s-near rings ,

F: N1N2 be an epimvor phism and let I be a b-S.C.P.I related to near field M of N, then f(I) is f(b) –S.C.P.I related to the near field f(M) of N2.

Proof

By remark (2.13), we have f(I) is a s-ideal related to a near field f(M)

Now Let f(m1) , f(m2) ∈ f(m) , such that

f(b) ! ( f( m1 ) ! f( m2) ∈ f(I)

⇒ f( b (m1 . m2 ) ) ∈ f ( I )

 ⇒ f( b (m1 . m2 ) ) ∈ f ( I )

⇒ either m1 ∈ I or m2 ∈ I or m2 [ since I is b- S.C .P. I related to M of N1 ]

⇒ f(m1) ∈ f ( I ) or (m2) ∈ f(I)

⇒ f(I) is a f ( b ) - S.C .P. I related to f( M) of N2

**Theorem ( 3.16) :**

 Let ( N1 , + , . ) be as – near ring has a near field M1 ,

( N2) be S- near ring , f: N1 → N2 be an epimomorphism , and Let J be a b- S.C .P.I related to the near field f(M) of N2 , then f-1 (I) is a - S.C .P.I related to a near field M of N1 , where b – f (a) .

Prof :

 By proposition ( 2.15) , we have f-1 (J) is a S – ideal related to M of N1 . Now , Let x,y ∈ M , such that a. (x.y ) ∈ f-1 (J)

⇒ f(x) , f(y) ∈ f(M) and f( a ! (x y) ∈ J

⇒ f(x) , f(y) ∈ f(M) and f( a ) ! f(x), f(y) ) ∈ J

⇒ either f(x) ∈ J or f(y) ∈ J [ since J is b- S.C .P. I related to f(M) of N2 ]

⇒ either x ∈ f-1 (J) or y f-1 (J) or y∈ f-1 (J)

⇒ f-1 (J) is a b- S.C .P. I related to f(M) of N2

**Corollary ( 3.17) :**

Let ( N1 ,+,0) be a S- near ring has a near field M, ( N2 , +' , ." ) be a S- near ring , f : N1 , → N2 be an e pimomorphism , and if { o1 } be a b- S.C .P. I related to the near field f(M) of N2 . . The ker(f) is b- S.C .P. I related to a near field M of N1 , where

Ker f = { x ∈ N1 : f(x) = 0 } and b=f(a)

Proof :

 Since f-1 ({ 01 )} l = ker (f) , then where Rer (f) is a - S.C .P. I related to M of N1

[ By theorem ( 3-16) ]

**Reference**

[1] G.Pilz , " Near Ring " , North Holland Publ . and Co., 1977 .

[2] H.A.S . Abujabal , M.A obaid and M.A.K han , " On structure and Commutatitiy of near Rings " , An to fagasta – chule , 2000 .

[3] H.H Abbass and M.A . who mmed , " On alompletely prime Ideal with respect to an element of a near ring " , J. of Kerbala university . Vol 10 , No 3 . 2012 .

[4] H.H Abbass and S.M . Ibrahem , " on Fuzzy completely semi orime Ideal with respect to an element of a near Ring " , M Sc. Thsis university of Kufa 2011 .

[5] N.J Groenewatd , " The completely prim radical near rings " , Acta Math . Hung , Vo 133 , 1988 .

[6] P. Dheena and G.Sathesh Kumar , " Completely z-prime Ideal in near ring ", India , 2007 .

[7] W.B Vasantha and A Samy , " Samarandache near ring ", U.S. America research press , 2002 .