



Repairable Queue with Non-exponential Interarrival Time and Variable Breakdown Rates

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Abstract

This paper considers a single server queue in which the service time is exponentially distributed and the service station may breakdown according to a Poisson process with the rates γ and γ' in busy period and idle period respectively. Repair will be performed immediately following a breakdown. The repair time is assumed to have an exponential distribution. Let $g(t)$ and $G(t)$ be the probability density function and the cumulative distribution function of the interarrival time respectively. When t tends to infinity, the rate of $g(t)/[1 - G(t)]$ will tend to a constant. A set of equations will be derived for the probabilities of the queue length and the states of the arrival, repair and service processes when the queue is in a stationary state. By solving these equations, numerical results for the stationary queue length distribution can be obtained.

Keywords: Interarrival Time, Constant Asymptotic Rate, Stationary Queue Length Distribution, Repairable Queue.

1. Introduction

In reality, most of the queueing systems may subject to failure or breakdown due to various causes. White & Christie [10] were the first to study an M/M/1 queueing system with the server subjects to exponentially distributed interruption. Thenceforth, numerous studies related to server breakdown or interruptions were published [1, 2, 6, 9, 11]. Most of these papers assumed that the breakdown rate is constant during different operation periods. However, in reality an unreliable server may experience distinct breakdown rates in different operation periods. Gray et al. [3] considered a model where the server takes vacation and is subjected to breakdown during system idle state and operation periods respectively. Sheng-li et al. [7] studied an M/M/1 queue in which the server may breakdown according to a Poisson process and they assume that the breakdown rates could be different during idle period and busy period. Sheng-li and Jing-bo [8] extended the model studied in Sheng-li et al. [7] to multi-state queueing system. Koh et al. [4] studied a model similar to that in Sheng-li et al. [7] but non-exponential service time distribution is considered.

In this paper, we change the interarrival time distribution in the model studied by Sheng-li et al. [7] to the one where the rate $g(t)/[1 - G(t)]$ tends to a constant as the time t tends to infinity. This distribution is known as a distribution with constant asymptotic rate (CAR). In practice, many distributions such as Erlang, exponential, gamma, hyperexponential, etc. are distributions with constant asymptotic rate. By using the method similar to that of Koh [5], stationary queue length distribution can be found for the model studied in Sheng-li et al. [7] where non-exponential interarrival time distribution can be taken into consideration.

Sections of this paper will be organized as follows: Section 2 briefly describes the model considered. In Section 3, the set of equations for the stationary probabilities will be derived and the method used to solve for this set of equations will be discussed in Section 4. Some numerical examples will be shown in Section 5 and a concluding remark is presented in section 6.

2. Model Description

Single server queue similar to that in Sheng-li et al. [7] is considered in which the server may breakdown according to a Poisson process and the breakdown rates could be different in idle period and busy period. We denote the breakdown rates in idle period and busy period as γ and γ' respectively. The repair time is assumed to have an exponential distribution with the rate δ and the repair will be performed immediately once the breakdown occurs. The service time is assumed to be exponentially distributed with rate μ while the interarrival time is changed to the one where the distribution has a constant asymptotic rate. Proposed numerical method in Koh [5] will be used to solve for the queueing problem in which the interarrival time is non-exponentially distributed.

3. Derivation of a Set of Equations for Stationary Probabilities

In this section, a set of equations is derived to find the stationary probabilities of the queue length and the states of the arrival, repair and service processes. The time axis is first segmented into equal length of interval Δt . We then let $g(t)$ be the probability

density function (pdf) of the interarrival time and τ_k be the interval $((k-1)\Delta t, k\Delta t)$ for $k = 1, 2, 3, \dots$. Let

$$\lambda_k = \frac{g(k\Delta t)}{\int_{k\Delta t}^{\infty} g(u)du}, \quad 1 \leq k \leq I \quad (1)$$

where I is large enough such that

$$\lambda_I = \lim_{k \rightarrow \infty} \lambda_k \quad (2)$$

Suppose that an arrival begins at time $t = 0$. Then the probability that there is an arrival in the interval τ_1 is approximately $\lambda_1 \Delta t$ and given that there is no arrival in interval $\tau_1, \tau_2, \dots, \tau_{k-1}$, the probability that there is an arrival in interval τ_k is approximately $\lambda_k \Delta t$, where $k = 2, 3, 4, \dots$ and $\lambda_k = \lambda_I$ for $k \geq I$.

Let the interval before τ_1 as τ_0 and denote the state number of the arrival process at the end of interval τ_k as ψ_k . Given that there is an arrival in τ_0 , we define ψ_k as

$$\psi_k = \begin{cases} 0, & \text{if} \\ & - k = 0; \text{ or} \\ & - \text{the next customer arrive in } \tau_k, \\ & k \geq 1. \\ \min(k, I), & \text{if the next customer does not arrive} \\ & \text{in } \tau_k, k \geq 1. \end{cases}$$

Next, we define the state number φ_k of the repair process at the end of interval τ_k as

$$\varphi_k = \begin{cases} 0, & \text{if the server is functioning in } \tau_k, k \geq 1. \\ 1, & \text{if the server is broken down and there is a completion of repair in } \tau_k, k \geq 2. \\ 2, & \text{if the server is broken down and there is no completion of repair in } \tau_k, k \geq 2. \end{cases}$$

The state number of the service process at the end of interval τ_k is denoted as ξ_k and is defined as follow:

$$\xi_k = \begin{cases} 0, & \text{If} \\ & - \text{no completion of service in } \tau_k, k \geq 1; \text{ or} \\ & - \text{there is no customer in the system.} \\ 1, & \text{if there is a completion of service in } \tau_k, k \geq 1. \end{cases}$$

The queue length at the end of the interval τ_k is denoted as n_k and a vector consisted of the state number for queue length, arrival process, repair process and service process at the end of the interval τ_k is defined as $\mathbf{h}_k = (n_k, \psi_k, \varphi_k, \xi_k)$.

Let the probability at the end of interval τ_k be $P_{nirj}^{(k)}$, where n is the number of customers in the system, i is the state of the arrival process, r is the state of repair process and j is the state of the service process. Assume that

$$P_{nirj} = \lim_{k \rightarrow \infty} P_{nirj}^{(k)} \quad (3)$$

In order to find the P_{nirj} , we first make the following observations. Suppose at the end of the interval τ_{k-1} , the queue is not empty where $n_{k-1} = n \geq 1$, the arrival process is in state $i-1$. There was a completion of service in τ_{k-1} and the state of the service process at the end of the interval τ_{k-1} is 1. The repair process is in state 0. The vector characteristic at the end of the interval τ_{k-1} is hence

$\mathbf{h}_{k-1} = (n, i-1, 0, 1)$. Let $i^* = \min(i, I)$, only one of the following events can occur in the next interval τ_k :

1. An arrival with the arrival rate λ_{i^*} and the vector of queueing characteristics at the end of interval τ_k becomes $\mathbf{h}_k = (n+1, 0, 0, 0)$.
2. A completion of service with the rate μ and $\mathbf{h}_k = (n-1, i^*, 0, 1)$.
3. A breakdown occurs with the rate γ and $\mathbf{h}_k = (n-1, i^*, 2, 0)$.
4. No arrival, no completion of service and no breakdown occurs, leading to $\mathbf{h}_k = (n, i^*, 0, 0)$.

If the system is empty at the end of interval τ_{k-1} with $n_{k-1} = n = 0$, the arrival process and service process are $i-1$ and 0 respectively. There was a completion of repair in the interval τ_{k-1} and at the end of the interval τ_{k-1} , the state of repair process is 1. We denote the vector of characteristics as $\mathbf{h}_{k-1} = (0, i-1, 1, 0)$ and one of the following events can occur in the next interval τ_k :

1. An arrival with the arrival rate λ_{i^*} , $\mathbf{h}_k = (1, 0, 0, 0)$.
2. A breakdown occurs with the rate γ , $\mathbf{h}_k = (0, i^*, 2, 0)$.
3. No arrival and no breakdown occurs, $\mathbf{h}_k = (0, i^*, 0, 0)$.

If the system is non-empty and under repair at the end of the interval τ_{k-1} , then $n_{k-1} = n \geq 1$ and the state of the repair process is 2. The arrival process and service process is in state $i-1$ and 0 respectively. The vector of characteristics at the end of the interval τ_{k-1} is $\mathbf{h}_{k-1} = (n, i-1, 2, 0)$. Subsequently, one of the following events can occur in the interval τ_k :

1. An arrival with the arrival rate λ_{i^*} , $\mathbf{h}_k = (n+1, 0, 2, 0)$.
2. A completion of repair with the repair rate δ , $\mathbf{h}_k = (n, i^*, 1, 0)$.
3. No arrival and no completion of repair, $\mathbf{h}_k = (n, i^*, 2, 0)$.

Suppose at the end of the interval τ_{k-1} , the vector of characteristics is $\mathbf{h}_{k-1} = (0, 1, 0, 1)$ and there is a breakdown occurs in the interval τ_k , we will obtain:

$$P_{0220}^{(k)} = P_{0101}^{(k)} (1 - \lambda_2 \Delta t) (\gamma \Delta t) \quad (4)$$

When k tends to infinity, we will obtain the following equation:

$$P_{0220} = P_{0101} (1 - \lambda_2 \Delta t) (\gamma \Delta t) \quad (5)$$

Similarly, by combining \mathbf{h}_{k-1} and the events that could occur in the interval τ_k , we can obtain the vector characteristics \mathbf{h}_k at the end of the interval τ_k . The set of equations to find the probability P_{nirj} is then obtained.

$$\text{When } n = 0, \\ P_{n200} \cong P_{n101} (1 - \lambda_2 \Delta t) (1 - \gamma \Delta t) \quad (6)$$

$$P_{n300} \cong (P_{n200} + P_{n201}) (1 - \lambda_3 \Delta t) (1 - \gamma \Delta t) \quad (7)$$

$$P_{ni00} \cong \left(P_{n(i-1)00} + P_{0(i-1)01} + P_{0(i-1)10} \right) \times (1 - \lambda_i \Delta t) (1 - \gamma \Delta t) \quad \text{for } 3 < i < I \quad (8)$$

$$P_{nt00} \equiv \left(\begin{array}{l} P_{n(t-1)00} + P_{n(t-1)01} + P_{n(t-1)10} \\ + P_{nt00} + P_{nt01} + P_{nt10} \end{array} \right) \times (1 - \lambda_t \Delta t)(1 - \gamma \Delta t) \quad (9)$$

$$P_{ni10} \equiv P_{n(i-1)20} (1 - \lambda_t \Delta t)(\delta \Delta t) \quad \text{for } 2 < i < I \quad (10)$$

$$P_{ni10} \equiv \left(P_{n(t-1)20} + P_{ni20} \right) (1 - \lambda_t \Delta t)(\delta \Delta t) \quad (11)$$

$$P_{n220} \equiv P_{n101} (1 - \lambda_2 \Delta t)(\gamma \Delta t) \quad (12)$$

$$P_{n320} \equiv \left(P_{n200} + P_{n201} \right) (1 - \lambda_3 \Delta t)(\gamma \Delta t) + P_{n220} (1 - \lambda_3 \Delta t)(1 - \delta \Delta t) \quad (13)$$

$$P_{ni20} \equiv \left(\begin{array}{l} P_{n(i-1)00} + P_{n(i-1)01} + P_{n(i-1)10} \\ \times (1 - \lambda_t \Delta t)(\gamma \Delta t) \\ + P_{n(i-1)20} (1 - \lambda_t \Delta t)(1 - \delta \Delta t) \end{array} \right) \quad \text{for } 3 < i < I \quad (14)$$

$$P_{nt20} \equiv \left(\begin{array}{l} P_{n(t-1)00} + P_{n(t-1)01} + P_{n(t-1)10} \\ + P_{nt00} + P_{nt01} + P_{nt10} \end{array} \right) \times (1 - \lambda_t \Delta t)(\gamma \Delta t) + \left(P_{n(t-1)20} + P_{nt20} \right) (1 - \lambda_t \Delta t)(1 - \delta \Delta t) \quad (15)$$

For $n \geq 0$,

$$P_{n101} \equiv P_{(n+1)000} (1 - \lambda_t \Delta t)(1 - \gamma \Delta t)(\mu \Delta t) \quad (16)$$

$$P_{ni01} \equiv \left(\begin{array}{l} P_{(n+1)(i-1)00} + P_{(n+1)(i-1)01} \\ + P_{(n+1)(i-1)10} \end{array} \right) \times (1 - \lambda_t \Delta t)(1 - \gamma \Delta t)(\mu \Delta t) \quad \text{for } 1 < i < I \quad (17)$$

$$P_{nt01} \equiv \left(\begin{array}{l} P_{(n+1)(t-1)00} + P_{(n+1)(t-1)01} + P_{(n+1)(t-1)10} \\ + P_{(n+1)t00} + P_{(n+1)t01} + P_{(n+1)t10} \end{array} \right) \times (1 - \lambda_t \Delta t)(1 - \gamma \Delta t)(\mu \Delta t) \quad (18)$$

When $n = 1$,

$$P_{n000} \equiv \sum_{i=1}^{I-1} P_{(n-1)i01} (\lambda_{i+1} \Delta t) + P_{(n-1)t01} (\lambda_t \Delta t) + \sum_{i=2}^{I-1} P_{(n-1)i00} (\lambda_{i+1} \Delta t) + P_{(n-1)t00} (\lambda_t \Delta t) + \sum_{i=3}^{I-1} P_{(n-1)i10} (\lambda_{i+1} \Delta t) + P_{(n-1)t10} (\lambda_t \Delta t) \quad (19)$$

$$P_{n020} \equiv \sum_{i=2}^{I-1} P_{(n-1)i20} (\lambda_{i+1} \Delta t) + P_{(n-1)t20} (\lambda_t \Delta t) \quad (20)$$

For $n \geq 1$,

$$P_{n100} \equiv P_{n000} (1 - \lambda_1 \Delta t)(1 - \gamma \Delta t)(1 - \mu \Delta t) \quad (21)$$

$$P_{ni00} \equiv \left(\begin{array}{l} P_{n(i-1)00} + P_{0(i-1)01} + P_{0(i-1)10} \\ \times (1 - \lambda_t \Delta t)(1 - \gamma \Delta t)(1 - \mu \Delta t) \end{array} \right) \quad \text{for } 1 < i < I \quad (22)$$

$$P_{nt00} \equiv \left(\begin{array}{l} P_{n(t-1)00} + P_{n(t-1)01} + P_{n(t-1)10} \\ + P_{nt00} + P_{nt01} + P_{nt10} \end{array} \right) \times (1 - \lambda_t \Delta t)(1 - \gamma \Delta t)(1 - \mu \Delta t) \quad (23)$$

$$P_{ni10} \equiv P_{n(i-1)20} (1 - \lambda_t \Delta t)(\delta \Delta t) \quad \text{for } 0 < i < I \quad (24)$$

$$P_{ni10} \equiv \left(P_{n(t-1)20} + P_{ni20} \right) (1 - \lambda_t \Delta t)(\delta \Delta t) \quad (25)$$

$$P_{n120} \equiv P_{n000} (1 - \lambda_1 \Delta t)(\gamma \Delta t) + P_{n020} (1 - \lambda_1 \Delta t)(1 - \delta \Delta t) \quad (26)$$

$$P_{ni20} \equiv \left(\begin{array}{l} P_{n(i-1)00} + P_{n(i-1)01} + P_{n(i-1)10} \\ \times (1 - \lambda_t \Delta t)(\gamma \Delta t) \\ + P_{n(i-1)20} (1 - \lambda_t \Delta t)(1 - \delta \Delta t) \end{array} \right) \quad \text{for } 1 < i < I \quad (27)$$

$$P_{nt20} \equiv \left(\begin{array}{l} P_{n(t-1)00} + P_{n(t-1)01} + P_{n(t-1)10} \\ + P_{nt00} + P_{nt01} + P_{nt10} \end{array} \right) \times (1 - \lambda_t \Delta t)(\gamma \Delta t) + \left(P_{n(t-1)20} + P_{nt20} \right) (1 - \lambda_t \Delta t)(1 - \delta \Delta t) \quad (28)$$

For $n \geq 2$,

$$P_{n000} \equiv \sum_{i=0}^{I-1} P_{(n-1)i00} (\lambda_{i+1} \Delta t) + P_{(n-1)t00} (\lambda_t \Delta t) + \sum_{i=1}^{I-1} P_{(n-1)i01} (\lambda_{i+1} \Delta t) + P_{(n-1)t01} (\lambda_t \Delta t) + \sum_{i=1}^{I-1} P_{(n-1)i10} (\lambda_{i+1} \Delta t) + P_{(n-1)t10} (\lambda_t \Delta t) \quad (29)$$

$$P_{n020} \equiv \sum_{i=0}^{I-1} P_{(n-1)i20} (\lambda_{i+1} \Delta t) + P_{(n-1)t20} (\lambda_t \Delta t) \quad (30)$$

4. Stationary Queue Length Distribution

We will apply the method in Koh¹¹ to solve equations (6) to (30) to find the stationary queue length distribution. Firstly, let b_{irj} , c_{irj} , d_{irj} , e_{irj} and f_{irj} be constants and the following notations are introduced:

- $P_{n***} = \{P_{mirj} : 0 \leq i \leq I, r = 0, 1, 2 \text{ and } j = 0, 1\}$;
- $(P_{m***}, P_{(m+1)***}, P_{(m+2)***})$ denotes the set of equations of the form:
$$\sum_{i=0}^I \sum_{r=0}^2 \sum_{j=0}^1 b_{irj} P_{mirj} + \sum_{i=0}^I \sum_{r=0}^2 \sum_{j=0}^1 c_{irj} P_{(m+1)irj} + \sum_{i=0}^I \sum_{r=0}^2 \sum_{j=0}^1 d_{irj} P_{(m+2)irj} \equiv 0$$
- $(P_{mirj} | P_{0***}, P_{(m+1)***})$ denotes the equation of the form:
$$P_{mirj} \equiv \sum_{i=1}^I \sum_{r=0}^2 \sum_{j=0}^1 e_{irj} P_{0irj} + \sum_{i=0}^I \sum_{r=0}^2 \sum_{j=0}^1 f_{irj} P_{(m+1)irj}$$

Using the above notations, (16) to (28) in the case when $n = 1$ can be represented as

$$(P_{0***}, P_{1***}, P_{2***}), \quad (31)$$

and (16) to (18) together with (21) to (30) in the case when $n = 2$ may be represented as

$$(P_{1***}, P_{2***}, P_{3***}). \quad (32)$$

In general, for $n > 2$ can be represented as

$$(P_{(n-1)***}, P_{n***}, P_{(n+1)***}). \quad (33)$$

By rearranging the sets of equation given by (31), we can obtain the following equation:

$$(P_{1irj} | P_{0***}, P_{2***}). \quad (34)$$

By substituting (34) into (32) and rearrange the equations, we can get the expression of P_{2irj} in the form of:

$$(P_{2irj} | P_{0***}, P_{3***}). \quad (35)$$

By substituting (35) into (33) for $n = 3$, we can get the expression of P_{3irj} in the form $(P_{3irj} | P_{0***}, P_{4***})$. For $n \geq 4$, by repeating the process of substituting the expression of the $P_{(n-1)irj}$ given by

$$(P_{(n-1)irj} | P_{0***}, P_{n***}) \quad (36)$$

into (33) and solving for the P_{mirj} to get the following expression:

$$(P_{mirj} | P_{0***}, P_{(n+1)***}). \quad (37)$$

When $n = N$ is large enough, we may set all the $P_{(n+1)irj}$ in (37) to be zero and obtain:

$$(P_{Nirj} | P_{0***}). \tag{38}$$

Next, we may substitute the expression of $(P_{Nirj} | P_{0***})$ into (36) and repeat the substitution process for $n = N - 1, N - 2, \dots, 1$ to obtain the following equation:

$$(P_{nirj} | P_{0***}). \tag{39}$$

When $n = 1$, (39) yields $(P_{1irj} | P_{0***})$, together with the equation of (6) to (18), we can get the following system of equations:

$$(P_{0irj} | P_{0***}). \tag{40}$$

An inspection of (40) reveals that there are two equations that are linearly dependent. Hence, we need to add in another linearly independent equation so that the resulting system of equations has a unique solution. By equating the sum of the left sides of the equation given by (39) to the sum of the right sides of (39), we can obtain an equation with the following form,

$$\sum_{n=1}^N \sum_i \sum_r \sum_j P_{nirj} = \sum_i \sum_r \sum_j k_{irj} P_{0irj} \tag{41}$$

where $i \geq 0, r = 0, 1, 2, j = 0, 1$ and the k_{irj} are constants. As

$$\sum_{n=1}^N \sum_i \sum_r \sum_j P_{nirj} \cong 1 \tag{42}$$

substituting one of the equations in (40) with (42) and solve for the system of equations, we will get the values for P_{0irj} . By substituting these values of P_{0irj} into (39), all the numerical values of P_{nirj} will be obtained and the probability that the queue length is n is obtain by the summation $P_n = \sum_i \sum_r \sum_j P_{nirj}$.

5. Numerical Result

Table 1 shows the numerical results for the steady-state mean system size when the interarrival time is exponentially distributed. We then relax the assumption of exponentially distributed interarrival time and the numerical results for the stationary queue length distribution are obtained and showed in Tables 2 and 3. Simulation is carried out to verify all the results.

Table 1. Comparison of results computed using the proposed numerical method and those given in Sheng-li et al. [7] and simulation procedure [$\Delta t = 10^{-7}$ for mean system size, $\lambda_i = \lambda = 1, I = 4$]

μ	γ	γ'	δ	Mean system size		
				Numerical Method	Sheng-li et al. [7]	Simulation
1.3	0	0.1	1.2	5.3205	5.3205	5.2524
1.3	0.1	0.1	1.4	4.9583	4.9583	4.9749
1.3	0.1	0.2	2.2	5.4348	5.4348	5.4311
1.9	0	0.1	1.2	1.4116	1.4116	1.4103
1.9	0.1	0.1	1.4	1.4023	1.4023	1.3990
1.9	0.1	0.2	2.2	1.4192	1.4192	1.4235

Table 2. Comparison of results obtained by the proposed numerical method, and simulation procedure [$(\kappa, \theta) = (1.2, 0.5), \mu = 4, \delta = 1.9, \gamma = 0.5, \gamma' = 0.3$] [$\Delta t = 0.025$ for Gamma interarrival time distribution, $I = 200, N = 50$]

Queue Size, n	P(Queue Size = n)	
	Numerical method	Simulation
0	0.438033	0.438058
1	0.249506	0.249521
2	0.133823	0.133816
3	0.074847	0.074833

4	0.04294	0.042941
5	0.024998	0.024995
6	0.014672	0.014666
...
49	1.74E-12	0
Mean System Size	1.305486	1.305242

Table 3. Comparison of results obtained by the proposed numerical method, and simulation procedure [$(\kappa, \theta) = (1.6, 0.6), \mu = 1.8, \delta = 1.1, \gamma = 0.2, \gamma' = 0.1$] [$\Delta t = 0.0125$ for Gamma interarrival time distribution, $I = 400, N = 60$]

Queue Size, n	P(Queue Size = n)	
	Numerical method	Simulation
0	0.288541	0.288702
1	0.227341	0.227499
2	0.146811	0.146907
3	0.099640	0.099625
4	0.069337	0.069321
5	0.048826	0.048769
6	0.034572	0.034485
...
59	2.63E-10	0
Mean System Size	2.352468	2.349814

Table 1 shows that the results obtained by the proposed method are close to those given in Sheng-li et al. [7]. In Tables 2 and 3, the interarrival time is assumed to have gamma distribution and the numerical results found by the proposed method are closed to those obtained by the simulation procedure.

6. Conclusion

In this paper, the model given in Sheng-li et al. [7] is studied. The assumption of exponential interarrival time distribution is relaxed and the stationary queue length distribution is obtained by the proposed numerical method in Koh [5]. The results found in this paper and in Koh et al. [4] show that the numerical method is successfully used for finding the stationary queue length distribution for a queuing system with a more general interarrival time or service time distribution.

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