

# An integration of fuzzy TOPSIS and fuzzy logic for multi-criteria decision making problems

Ratih Fitria Jumarni\*, Nurnadiah Zamri

Faculty of Informatics and Computing, Universiti Sultan Zainal Abidin, Tembilala Campus, 22200 Besut, Terengganu, Malaysia

\*Corresponding author E-mail: [ratihfitriajumarni@gmail.com](mailto:ratihfitriajumarni@gmail.com)

## Abstract

Multi-Criteria Decision Making (MCDM) is a decision-making methods, which it is able to find a unique agreement from number of experts by evaluating the uncertain judgment among them. Several fuzzy logic based approaches have been employed in MCDM to handle the linguistic uncertainties and hesitancy. However, there is still a need to handle high level of uncertainties that exists in decision making problems. Thus, the purpose of this paper is to introduce the new concept namely fuzzy TOPSIS and fuzzy logic based MCDM. The proposed concepts aims to handle the high levels of uncertainties which exists due to the varying experts' judgments and the vagueness of the appraisal. The proposed method utilized fuzzy logic rule-base in determining the alternatives and criteria for decision matrix. Then, in the aggregation phase, the min operator is used to compute the firing strength for each rule. The feasibility and applicability of the proposed methods are illustrated with an example. This new concept is seen to be able to handle intangibles and less cumbersome in mathematical calculations.

**Keywords:** Fuzzy logic; Fuzzy set; Fuzzy TOPSIS; Multi-criteria decision-making;

## 1. Introduction

Hesitation and conflicting decisions/views/opinions in experts evaluation happen when experts appraise their preferences among the criteria and alternatives [1]. Conflict in decision making can lead to worries, arguments, confrontations, litigation and separation [2]. The hesitations, conflicts and misperception exist internally and externally. The internally conflicts such as self-esteem and confidence level effect the DMs judgment during the assessment. The external circumstances such as the political situation, the global circumstances prevailing at that time and the environmental conditions effects also on DM's opinion. In a decision making system, for the DMs to provide fair, neutral and unbiased decisions these uncertainties are impossible to control in order for the DMs to provide fair, neutral and unbiased decisions. With this said, hesitations and conflicting thoughts and ideas regarding a particular decision making process can also be fruitful and productive [3].

In other words, hesitancy and conflicting views and ideas, provoke controversy and thus generate new insights which can lead to an expansion of knowledge about the role conflict plays in groups and organizations and towards theorizing [4]. The Multiple Criteria Decision Making (MCDM) method is one of the most widely used approaches for evaluating multiple conflicting criteria [5]. This approach often requires the experts to provide qualitative and quantitative measurements for determining the performance of each alternative with respect to the criteria and the relative importance of evaluation criteria with respect to overall judgments [6].

Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) is a conventional means of solving MCDM problems [7-9]. The main advantage of the TOPSIS is its inherent ability to handle intangibles and less cumbersome mathematical calculations.

Then, Fuzzy TOPSIS (FTOPSIS) based fuzzy set " $\tilde{A} = \{(x, \mu_A(x)) | x \in X\}$ " where  $X$  be a universe of discourse,

characterized by a membership function  $\mu_A(x)$  which associates with each element  $x$  in  $X$  a real number in the interval  $[0,1]$ " was introduced to handle uncertainty in linguistic judgment. Initial research on FTOPSIS was conducted by [10], who discussed the use of linguistic variables in Fuzzy Set (FS).

Researches have been investigating techniques to handle the faced uncertainties in many decision making applications. Fuzzy logic; The IF-THEN statement "IF  $u$  is  $A$ , THEN  $v$  is  $B$ ," is regarded as an appropriate methodology for decision making systems which is able to simultaneously handle numerical data and linguistic knowledge. Research in fuzzy decision making has grown rapidly in the utilization of extended fuzzy set theories (i.e., Intuitionistic Fuzzy Sets (IFSs) [8], Hesitant Fuzzy Sets [9], Vague Sets [10], Interval-valued Fuzzy Sets [11]). The work in [12] developed an interactive decision support system for sustainable energy management and the application of fuzzy methods to tackle uncertainties in the data. The work presented in [13] studied the supplier selection which involved several conflicting criteria where the decision maker's knowledge is usually vague and imprecise.

Thus, the purpose of this paper is to introduce the new linguistic scales to the decision environment of interval type-2 fuzzy context for solving IT2FTOPSIS problems. This new linguistic scales react to the subjective judgments from the experts where the lowest of the scale and the highest of the scale are equally strong. It is due to the negative association rule: "birds can fly is a well-known fact, but penguins cannot fly although they are birds" [14]. The negative data in this decision making study does not mean the data is wrong or corrupt. The positive and negative is relative.

## 2. The proposed method

This section proposed a new concept that combines fuzzy TOPSIS with fuzzy logic. A step-by-step flow of the proposed method is briefly explained on Figure 1.

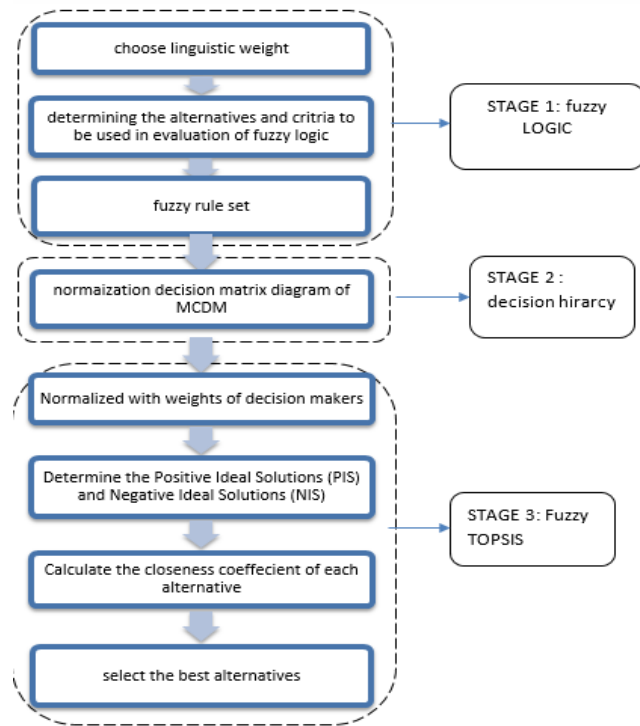


Fig.1: Schematic of the new proposed method

STEP 1. Construct the weighting matrix of the criteria of the experts and construct the  $p$ th average weighting matrix respectively using Table 1.

Table 1: Linguistic variables and their fuzzy number values of criteria weights

Linguistic Variable	Fuzzy Value
Very Low (VL)	(0, 0, 0.1)
Low (L)	(0, 0.1, 0.3)
Medium low (ML)	(0.1, 0.3, 0.5)
Medium (M)	(0.3, 0.5, 0.7)
Medium High (MH)	(0.5, 0.7, 0.9)
High (H)	(0.7, 0.9, 1)
Very High (VH)	(0.9,1,1)

STEP 2. Establish a decision matrix for ranking. A MCDM problem can be concisely expressed in matrix format. In MCDM problems, responses from experts are mainly focused on the opinion of the experts regarding rating of the attributes of the problems based on the identified criteria. The experts were asked to specify rating using ten of the new linguistic scales varying from ‘Very Poor’ to ‘Very Good’ over the factors associated with FMCDM problems. The linguistic scale of FTOPSIS is used to define the expert measurements of each criteria and alternatives of the MCDM problems. This linguistic scale is shown in Table 2.

Table 2: Linguistic variables for the rating of alternatives

Linguistic Variable	Fuzzy Value
Very Poor (VP)	(0, 0, 1)
Poor (P)	(0, 1, 3)
Medium poor (MP)	(1, 3, 5)
Fair (F)	(3,5,7)
Medium good (mg)	(5,7,9)
Good (G)	(7,9,10)
Very Good (VG)	(9,10,10)

STEP 3. This leads to the construction of the fuzzy decision matrices. Based on the experts  $z_k \in D$ , we can construct reciprocal decision matrices. Following rule according to the experts’ opinions:

$$\text{IF } x_i \text{ is } \tilde{F}_i \text{ and } x_j \text{ is } \tilde{F}_j \text{ THEN } \tilde{I}_{ij} \quad (1)$$

Then, we use the minimum implication operator set theoretic operation by [15] to compute the firing strength for each rule.

$$A \cap B \Rightarrow \mu_{A \cap B}(x) = \min[\mu_A(x), \mu_B(x)] \quad (2)$$

STEP 4. Normalization of decision matrix and weighted normalized decision matrix use the fuzzy arithmetic averaging operation to aggregate all  $x_{ij}^{(r,k)}$  over the  $k$  expert as follows:

$$x_{ij}^r = \frac{1}{k} \sum_{k=1}^m x_{ij}^{(r,k)} \quad (3)$$

STEP 5. Normalized with weights of decision makers’ preference decision matrix as below calculated fuzzy number is in [0, 1] interval

$$V_{ij} = r_{ij} \times w_{ij} \quad (4)$$

STEP 6. Determine the positive ideal solutions and negative ideal solutions respectively

$$A^* = \{v_1^*, \dots, v_n^*\} = \{(\max_j v_{ij} \mid i \in I^+), (\min_j v_{ij} \mid i \in I^-)\},$$

$$A^- = \{v_1^-, \dots, v_n^-\} = \{(\min_j v_{ij} \mid i \in I^+), (\max_j v_{ij} \mid i \in I^-)\},$$

where  $I^+$  is associated with the positive criteria and  $I^-$  is associated with the negative criteria.

STEP 7. Calculate the separation measures using the n-dimensional Euclidean distance. The separation of each alternative from the ideal solution is given as

$$D_j^* = \sqrt{\sum_{i=1}^n (v_{ij} - v_i^*)^2}, j = 1, \dots, J. \quad (5)$$

Similarly, the separation from the negative-ideal solution is given as

$$D_j^- = \sqrt{\sum_{i=1}^n (v_{ij} - v_i^-)^2}, j = 1, \dots, J. \quad (6)$$

STEP 8. Calculate the relative closeness to the ideal solution. The relative closeness of the alternative  $a_j$  with respect to  $A^*$  is defined as

$$C_j^* = \frac{D_j^-}{D_j^* + D_j^-}, j = 1, \dots, J. \quad (7)$$

Rank the preference order. A large value of closeness coefficient  $C_j^*$  indicates a good performance of the alternative  $A_j$ . The best alternative is the one with the greatest relative closeness to the ideal solution.

### 3. Numerical example

We modified the example that was taken from [8]. The FMCDM problem in evaluating three different cars towards the four attributes; safety, price, appearance and performance. The computations of this example using the proposed fuzzy TOPSIS with fuzzy logic are executed in the eight steps as follows:

STEP 1. The data that considers fuzzy TOPSIS and fuzzy logic and its conversion (see Table 1 and Table 2) are referred in order to construct matrix of attributes.

**Table 3:** The relative importance weights of the five criteria by three DMs

Criterion	DMs			Aggregated Fuzzy Number
	D1	D2	D3	
C1	H	VH	MH	(0.70, 0.87, 0.97)
C2	VH	VH	VH	(0.90, 1.0, 1.0)
C3	VH	H	H	(0.7, 0.93, 1.0)
C4	VH	VH	VH	(0.9, 1.0, 1.0)
C5	M	MH	MH	(0.43, 0.63, 0.83)

STEP 2. The data that considers fuzzy sets (see Table 2) are referred in order to construct matrix of attributes. Then, the fuzzy decision matrix is shown as Table 4.

**Table 4:** The ratings of the three candidates by decision makers under all criteria

No. of Rules	Input Variable					Output Variable
	C1	C2	C3	C4	C5	
1	MG	G	F	VG	F	A1
2	G	MG	G	G	F	A1
3	MG	F	G	VG	F	A1
4	G	VG	VG	VG	VG	A2
5	G	VG	VG	VG	MG	A2
6	MG	VG	G	VG	G	A2
7	VG	MG	G	G	G	A3
8	G	G	MG	VG	G	A3
9	F	VG	VG	MG	MG	A3

An example from Table 5, rule no 1 for the ‘A1’ alternative which is according to

DM1: if the *emotional steadiness* is ‘medium good’, *oral communication skill* is ‘Good’, *personality* is ‘Fair’, *past experience* is ‘Very good, and *self-confidence* is ‘Fair’ THEN future analyze engineer is A1.

STEP 3. Fuzzy rule set. Constructed the reciprocal decision matrices based on fuzzy rule set collected from all decision makers

according to three DMs evaluated, their opinions were based on three output variable/alternatives (A1, A2 and A3), we construct nine rules for component and output variable (as shown in Table 4). These three output variables will be tested but using the same rules (according Table 4), we have three different set of output. Example for rule no 1 according to Table 5,

**Table 5:** Fuzzy rule no. 1

	C1	C2	C3	C4	C5
	C1	-	MG and G	MG and F	MG and VG
C2	G and MG	-	G and F	G and VG	G and F
C3	F and MG	F and G	-	F and VG	F and F
C4	VG and MG	VG and G	VG and F	-	VG and F
C5	F and MG	F and G	F and F	F and VG	-

The following is an example of how the expert interpret the rule no.1 based on the example shown above:

Rule no 1, for output (in decision making we refer as an alternatives) A1:

- X<sub>12</sub> – IF C1 is MG and C2 is G THEN A1
- X<sub>13</sub> – IF C1 is MG and C2 is F THEN A1
- X<sub>14</sub> – IF C1 is MG and C2 is VG THEN A1
- X<sub>15</sub> – IF C1 is MG and C2 is F THEN A1
- X<sub>23</sub> – IF C1 is G and C2 is F THEN A1
- X<sub>24</sub> – IF C1 is G and C2 is VG THEN A1
- X<sub>25</sub> – IF C1 is G and C2 is F THEN A1
- X<sub>34</sub> – IF C1 is F and C2 is VG THEN A1
- X<sub>35</sub> – IF C1 is F and C2 is F THEN A1

Then, we use the min operator to compute the firing strength for each rule. This leads to the construction of the fuzzy decision matrices. Based on the DMS / expert, we can construct reciprocal decision matrices. The decision use the linguistic rating variable (show in Table 2) and we use minimum implication set theoretic operation by [15]

$$A \cup B \Rightarrow \mu_{A \cup B}(x) = \max[\mu_A(x), \mu_B(x)]$$

$$A \cap B \Rightarrow \mu_{A \cap B}(x) = \min[\mu_A(x), \mu_B(x)]$$

$$\mu_{\bar{A}}(x) = 1 - \mu_A(x)$$

For example, X<sub>12</sub> – IF C1 is MG and C2 is G THEN A1

$$C_1 \cap C_2 \Rightarrow \mu_{C_1 \cap C_2}(x) = \min[\mu_{C_1}(5,7,9), \mu_{C_2}(7,9,1)]$$

**Table 6:** Fuzzy number for linguistic variable

A1	DM1 =	C1	C2	C3	C4	C5	Average
	C1	0,0,0	5,7,9	3,5,7	5,7,9	3,5,7	3.2, 4.8, 6.4
	C2	5,7,9	0,0,0	3,5,7	5,7,9	3,5,7	3.2, 4.8, 6.4
	C3	3,5,7,	3,5,7	0,0,0,	3,5,7	3,5,7	2.4, 4, 5.6
	C4	5,7,9	7,9,10	3,5,7	0,0,0	3,5,7	3.2, 5.2, 6.6
	C5	3,5,7	3,5,7	3,5,7	3,5,7	0,0,0	2.4, 4, 5.6
	DM2 =	C1	C2	C3	C4	C5	Average
	C1	0,0,0	5,7,9	7,9,10	7,9,10	3,5,7	4.4, 6, 7.2
	C2	5,7,9	0,0,0	5,7,9	5,7,9	3,5,7	3.6, 5.2, 6.8
	C3	3,5,7	3,5,7	0,0,0,	3,5,7	3,5,7	4.4, 6, 7.2
	C4	7,9,10	7,9,10	7,9,10	0,0,0	3,5,7	4.8, 6.4, 7.4
	C5	3,5,7	3,5,7	3,5,7	3,5,7	0,0,0	2.4, 4, 5.6
	DM3 =	C1	C2	C3	C4	C5	Average
	C1	0,0,0	3,5,7	5,7,9	5,7,9	3,5,7	3.2, 4.8, 6.4
	C2	3,5,7	0,0,0	3,5,7	3,5,7	3,5,7	2.4, 4, 5.6
	C3	5,7,9	3,5,7	0,0,0,	7,9,10	3,5,7	3.6, 5.2, 6.6
	C4	5,7,9	3,5,7	7,9,10	0,0,0	3,5,7	3.6, 5.2, 6.6
	C5	3,5,7	3,5,7	3,5,7	3,5,7	0,0,0	2.4, 4, 5.6

STEP 4: Normalization of decision matrix and weighted normalized decision matrix are created as in Table 7 according to this information. Use the fuzzy arithmetic averaging operation to aggregate all  $x_{ij}^{(r,k)}$  over the k expert as follows:

$$x_{ij}^r = \frac{1}{k} \sum_{k=1}^m x_{ij}^{(r,k)}$$

**Table 7:** The fuzzy decision matrix and fuzzy weights of three alternatives

		C14	C2	C3	C4	C5
A1	D1	(3.2, 4.8, 6.4)	(3.2, 4.8, 6.4)	(2.4, 4, 5.6)	(3.6, 5.2, 6.6)	(2.4, 4, 5.6)
	D2	(4.4, 6, 7.2)	(3.6, 5.2, 6.8)	(4.4, 6, 7.2)	(4.8, 6.4, 7.4)	(2.4, 4, 5.6)
	D3	(3.2, 4.8, 6.4)	(2.4, 4, 5.6)	(3.6, 5.2, 6.6)	(3.6, 5.2, 6.6)	(2.4, 4, 5.6)
A2	D1	(3.6, 5.2, 6.67)	(3.07, 4.67, 6.27)	(3.47, 5.07, 6.43)	(4, 5.6, 6.9)	(2.4, 4, 5.6)
	D2	(5.6, 7.2, 8)	(6.4, 7.6, 8)	(6.8, 7.8, 8)	(6.8, 7.8, 8)	(6.8, 7.8, 8)
	D3	(5.2, 6.8, 7.8)	(6, 7.2, 7.8)	(6, 7.2, 7.8)	(6, 7.2, 7.8)	(4.8, 6.2, 7.4)
A3	D1	(4, 5.6, 7.2)	(5.2, 6.6, 7.6)	(4.8, 6.2, 7.6)	(5.2, 6.6, 7.6)	(5.2, 6.8, 7.8)
	D2	(4.93, 6.53, 7.67)	(5.87, 7.13, 7.8)	(5.87, 7.13, 7.8)	(6, 7.2, 7.8)	(5.6, 6.93, 7.73)
	D3	(5.2, 7.2, 7.8)	(4, 5.6, 7.2)	(5.2, 6.8, 7.8)	(5.2, 6.8, 7.8)	(5.2, 6.8, 7.8)
	weight	(5.2, 7.2, 7.8)	(6, 7.2, 7.8)	(6, 7.2, 7.8)	(6, 7.2, 7.8)	(4, 5.6, 7.2)
		(4, 5.6, 7.2)	(5.6, 7.2, 7.8)	(5.2, 6.8, 7.8)	(6, 7.2, 7.8)	(5.2, 6.8, 7.8)
		(4.8, 6.4, 7.6)	(5.2, 6.6, 7.6)	(5.47, 6.93, 7.8)	(5.73, 7.07, 7.8)	(4.8, 6.4, 7.6)
		(0.7, 0.87, 0.97)	(0.9, 1, 1)	(0.77, 0.93, 1)	(0.9, 1, 1)	(0.43, 0.63, 0.83)

Then, we constructing the normalized fuzzy decision matrix as Table 8.

**Table 8:** The fuzzy normalized decision matrix

	C14	C2	C3	C4	C5
A1	(0.36, 0.52, 0.67)	(0.31, 0.47, 0.63)	(0.35, 0.51, 0.65)	(0.4, 0.56, 0.69)	(0.24, 0.4, 0.56)
A2	(0.49, 0.65, 0.77)	(0.59, 0.71, 0.78)	(0.59, 0.71, 0.78)	(0.6, 0.72, 0.78)	(0.56, 0.69, 0.77)
A3	(0.48, 0.64, 0.76)	(0.52, 0.66, 0.76)	(0.55, 0.69, 0.78)	(0.57, 0.71, 0.78)	(0.48, 0.64, 0.76)
weight	(0.7, 0.87, 0.97)	(0.9, 1, 1)	(0.77, 0.93, 1)	(0.9, 1, 1)	(0.43, 0.63, 0.83)

STEP 5. Normalized with weights of decision makers' preference decision matrix as below in Table 9, calculated fuzzy number is in [0, 1] interval

**Table 9:** The fuzzy normalized decision matrix

	C14	C2	C3	C4	C5
A1	(0.25, 0.45, 0.64)	(0.28, 0.47, 0.63)	(0.27, 0.47, 0.65)	(0.36, 0.56, 0.69)	(0.10, 0.25, 0.47)
A2	(0.35, 0.57, 0.74)	(0.53, 0.71, 0.78)	(0.45, 0.67, 0.78)	(0.54, 0.72, 0.78)	(0.24, 0.44, 0.64)
A3	(0.34, 0.55, 0.73)	(0.47, 0.66, 0.76)	(0.42, 0.65, 0.78)	(0.52, 0.71, 0.78)	(0.21, 0.41, 0.63)

STEP 6. Determine the Positive Ideal Solutions (PIS) and Negative Ideal Solutions (NIS) as:  $A^+ = [(1,1,1), (1,1,1), (1,1,1), (1,1,1), (1,1,1)]$ ,  $A^- = [(0,0,0), (0,0,0), (0,0,0), (0,0,0), (0,0,0)]$ .

**Table 10:** Positive and negative ideal solutions

	C1	C2	C3	C4	C5
$A_1^+$	0.99	0.97	0.97	0.84	1.28
$A_1^-$	0.83	0.83	0.84	0.96	0.54
$A_2^+$	0.83	0.59	0.68	0.58	1.01
$A_2^-$	0.99	1.18	1.12	1.19	0.82
$A_3^+$	0.84	0.68	0.71	0.61	1.06
$A_3^-$	0.98	1.11	1.10	1.17	0.78

STEP 7. Calculate the distance of PIS and NIS. Calculate the distance of each candidate from FPIS and FNIS, respectively as Table 11.

**Table 11:** The distance measurement

	D*	D-
A1	5.06	4.00
A2	3.69	5.30
A3	3.90	5.14

STEP 8. Calculate the relative closeness  
Next, the relative closeness is calculated with respect to the ideal solution and shown as Table 12 as follows:

**Table 12:** The distance the relative closeness coefficient

	Closeness Coefficient (Cc)
A1	0.441
A2	0.590
A3	0.569

In conclusion, the best alternative selection is  $A_2$  and the ranking order of the alternative is given by  $A_2 > A_3 > A_1$ .

### 4. Comparative analysis

Since the proposed model introduced a new concept of fuzzy logic and fuzzy TOPSIS, it is important to compare it with the existing approach. A different case is used to employ this comparison. The proposed method is compared with the method from the [8]. Thus, the results of the comparative analysis were obtained as given in Table 13.

**Table 13:** Comparative analysis

Method	Ranking Order According to Closeness Coefficient
Proposed method	$A_2 > A_3 > A_1$
Proposed method with [8]	$A_2 > A_3 > A_1$

In light of the result from Table 13, it can be concluded that the proposed Fuzzy TOPSIS with fuzzy logic method gives beautifully consistent with the other method.

### 5. Conclusion

Time and inter-dependent conditions in network are parts of the problems for solving cloud solutions problem to manage big data projects. In this paper, affinity set has been applied as an assessment model in proposing weights of criteria of big data projects. Linguistic assessments provided by experts were used as input weight data to the FTOPSIS. The results suggest that Google is the best cloud solutions out of four alternatives. The FTOPSIS

with affinity weight is found to be practically feasible and compatible in solving big data projects due to the considerations of time. A ranking comparison using the same example under two other methods was made to validate the proposed method. From the results, this study has shown the ability of the FTOPSIS with affinity weights in finding a way to close the gaps between big data applications. This is just the beginning of FTOPSIS with affinity weights in big data areas, thus for our future work, we aim to apply affinity sets to the whole FTOPSIS method.

## Acknowledgement

This research is supported by *Dana Penyelidikan Universiti UNISZA/2016/DPU/10*, Universiti Sultan Zainal Abidin. This support is gratefully acknowledged.

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