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Research Paper

Rainfall frequency analysis using LH-moments approach: A case of Kemaman Station, Malaysia

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Abstract

Statistical analysis of extreme events is often carried out to obtain the probability distribution of floods data and then predict the occurrence of floods for a significant return period. L-moments approach is known as the most popular approach in frequency analysis. This paper discusses comparison of the L-moments method with higher order moments (LH-moments) method. LH-moment, a generalization of L-moment, which is proposed based on the linear combinations of higher-order statistics has been used to characterize the upper part of distributions and larger events in flood data. It is observed from a comparative study that the results of the analysis of observed data and the diagram based on the K3D-II distribution using LH-moments method is more efficient and reasonable than the L-moments method for estimating data of the upper part of the distribution events.

Keywords: *Frequency analysis; Higher-order statistic; K3D distribution; LH moments; Linear combination.*

1. Introduction

There is an increase in the number of natural hazards, which have been observed in recent times where there is almost no area on the planet not prone to natural disasters that may have negative consequences in some cases. There are areas that may be known to be in the circle of natural disaster risk, but this does not apply to all regions of the globe. It is clear that floods which the resulting by rainfall are one of the most important types of natural disasters because of the wide geographical area on which they are spread, as well as the amount of damage they produce. Therefore, the issue of natural hazards must be addressed from the perspective of preparedness and forecasting but a precautionary action must be taken to avoid any catastrophic consequences in the event of any natural hazard, so as to protect humans and nature.
Frequency analysis is an estimation of the frequency of a given event [6]. Compiling the previous observations and fitting them on the probability distributions helps to predict future events. Usually, the data analysis requires estimating the parameters of probability distributions. The performance of a particular model depends on the method of estimating parameters. Estimating probability distributions parameters is very important in data analysis. This study focusing on L-moments and LH-moments methods.
The concept of L-moment as parameter estimation method for several probability distributions can be regarded as a linear combination of probability weighted moments (PWMs) [5]. The PWMs is defined for a non-negative integer r. Many researchers used L-moment to analysis the frequency of flooding and precipitation. Probability distributions could be used to relate the magnitude of extreme events to their frequency occurrence [23]. Analysis of precipitation data and other phenomena related to them is very important in order to obtain the appropriate probability distribution, which in turn helps to prediction and identify the characteristics of the data. Many of researches like as [10-12, 18] used L-moments method based regional flood frequency to find flood frequency association for both gauged and ungauged catchments of a various area. In [13] applied the L-moment estimators for generalized Rayleigh distribution. In [9] used the method of L-moment estimators to estimate the parameters of polynomial quantile mixture. Some of literatures used L-moments such as [1, 7, 16, 21].
In [24] suggested LH-moments method that considered a generalization of L-moments based on linear combinations of higher-order statistics for describing the larger events in data and the upper part of distributions and has lately been used in hydrology by [14]. LH-moment was used in the regional flood frequency analysis by [15] for Karkhe watershed that located in Western Iran. In [22] compared between L-moments diagram and LH-moments diagram to examine the appropriateness of the General Extreme Value (GEV) distribution using annual maximum flow data from 31 streams in Malaysia. In the study of [2], a comparative study has been made between L-moments and LH-moments for regional flood frequency analysis of North-Bank of the river Brahmaputra, India. In [17], they have formulated LH-moments for five distributions useful in hydrology such as Mielke-Johnson's three parameter kappa distribution(K3D), three parameter kappa type-II distribution (K3D-II), beta-k distribution, beta-p distribution and a generalized Gumbel distribution (GGD).
This study concentrates on the three parameters kappa type-II distribution for rainfall frequency analysis using LH-moments method.

2. Methods of moments

2.1. L-moments

Hosking developed the L- moments theory based on order statistic and defined the L-moments as the linear combinations of probability weighted moments (PWM) [5]. In [3] defined and summarized the theory of PWM as:

$$\beta_r = \int_0^1 x(F)F^r dF \quad (1)$$

where β_r is the r^{th} order PWM , $x(F)$ is called the quantile function of X and F being non-exceedance probability [3]. The first four L-moment are defined as:

$$\lambda_1 = \beta_0$$
$$\lambda_2 = 2\beta_1 - \beta_0 \quad (2)$$
$$\lambda_3 = 6\beta_2 - 6\beta_1 + \beta_0$$
$$\lambda_4 = 20\beta_3 - 30\beta_2 + 12\beta_1 - \beta_0$$

The L-moment ratios defined and calculated by [5] as:
$\tau_2 = \frac{\lambda_2}{\lambda_1}$, $\tau_3 = \frac{\lambda_3}{\lambda_2}$, $\tau_4 = \frac{\lambda_4}{\lambda_2}$ with $\tau_r = \frac{\lambda_r}{\lambda_2}$, $r \geq 3$
where λ_r *is* a linear function of the expected order statistics is, λ_1 is a measure of location, τ_2 is L-coefficient variance (LCv), τ_3 is L-skewness (LCs) and τ_4 is L-kurtosis (LCk).
Let *x* be a non-degenerate random variable with finite mean. Hosking showed that r is greater than or equal to three($r \geq 3$), often the higher moments λ_r , $r \geq 3$ are standardized so that they are independent of the units of measurement of x, the absolute value of τ_r is less than one($|\tau_r| < 1$). If in addition x ≥ 0 almost surely, then τ the L-Cv of x satisfies $0 < \tau < 1$. These bounds are advantageous since they restrict the skewness to be within the interval (-1, 1) where it is zero for symmetric distributions [5].
Unbiased sample estimators of the first four L-moments by PWM sample estimators and unbiased sample estimators of PWMs as (b_i) obtained and defined by [6]. To compute unbiased sample estimate of the PWM for any distribution can be using

$$b_r = \frac{1}{n}\sum_{i=r+1}^{n}\frac{(i-1)(i-2)....(i-r)}{(n-1)(n-2)....(n-r)}x_i \quad (3)$$

where x_i is an ordered set of observations $x_1 \leq x_2 \leq \cdots \leq x_n$. We can compute the first four L-moment for any distribution from PWM using:

$$l_1 = b_0 ,$$
$$l_2 = 2b_1 - b_0 ,$$
$$l_3 = 6b_2 - 6b_1 + b_0 , \quad (4)$$
$$l_4 = 20b_3 - 30b_2 + 12b_1 - b_0$$

Estimates the sample L-moment ratios are given by:
$\hat{\tau}_2 = \frac{l_2}{l_1}$, $\hat{\tau}_3 = \frac{l_3}{l_2}$, $\hat{\tau}_4 = \frac{l_4}{l_2}$
In general the sample L-moment ratios of the r^{th} order are given by:
$\hat{\tau}_r = \frac{l_r}{l_2}$, $r \geq 3$.

2.2. Higher order moments (LH-moments)

LH-moments considered a generalization of L-moments which are suggested by [24] based on linear combinations of higher-order statistics to describing the larger events in data and the upper part of distributions. The expectation of an order statistic for a given of size (m) drawn from a distribution $F(x) = \Pr(X \leq x)$ can be written as

$$E[X_{j:m}] = \frac{m!}{(j-1)!(m-j)!}\int_0^1 X(F)F^{j-1}(1-F)^{m-j}\,dF \quad (5)$$

The first four LH-moments are defined by [24] for $\eta = 0,1,2,3, ...$

$$\lambda_1^\eta = E[X_{(\eta+1):(\eta+1)}],$$
$$\lambda_2^\eta = \frac{1}{2}E[X_{(\eta+2):(\eta+2)} - X_{(\eta+1):(\eta+2)}], \quad (6)$$
$$\lambda_3^\eta = \frac{1}{3}E[X_{(\eta+3):(\eta+3)} - 2X_{(\eta+2):(\eta+3)} + X_{(\eta+1):(\eta+3)}]$$
$$\lambda_4^\eta = \frac{1}{4}E[X_{(\eta+4):(\eta+4)} - 3X_{(\eta+3):(\eta+4)} + 3X_{(\eta+2):(\eta+4)} - X_{(\eta+1):(\eta+4)}]$$

The LH-moments ratios written as
$\tau_2^\eta = \frac{\lambda_2^\eta}{\lambda_1^\eta}$, $\tau_3^\eta = \frac{\lambda_3^\eta}{\lambda_2^\eta}$ and $\tau_4^\eta = \frac{\lambda_4^\eta}{\lambda_2^\eta}$.

When $\eta = 0$, LH-moments are equal to L-moments. For $\eta = 0,1,2, ..., n$, the LH-moment are called L_η as η increases, LH-moments reflects more and more characteristics of the upper part of the distribution [24].

3. L-moments and LH-moments of K3D-II

3.1. K3D-II distribution

The three parameter kappa type-II distribution (K3D-II) was proposed by [20] and it is considered a special case of a four parameters kappa distribution [4, 19]. This distribution has a three parameters μ, α and β where μ is a location, α is a shape and β is a scale for $\mu \leq \min_{1\leq i\leq n}(x_i)$, $\alpha > 0$, $\beta > 0$ and $x > 0$. The quantile function of the three parameter kappa type-II distribution is written as:

$$x(F) = \mu + \beta\left(\frac{\alpha F^\alpha}{1-F^\alpha}\right)^{\frac{1}{\alpha}} , \quad 0 < F < 1. \quad (7)$$

3.2. L-moments of K3D-II distribution

The first four sample L-moments of K3D-II distribution are:

$$\lambda_1 = \mu + \beta\alpha^{\frac{1}{\alpha}-1}Beta\left(\frac{2}{\alpha}, 1-\frac{1}{\alpha}\right), \quad \lambda_2 = \beta\alpha^{\frac{1}{\alpha}-1}\left\{2Beta\left(\frac{3}{\alpha}, 1-\frac{1}{\alpha}\right) - Beta\left(\frac{2}{\alpha}, 1-\frac{1}{\alpha}\right)\right\}, \quad (8)$$
$$\lambda_3 = \beta\alpha^{\frac{1}{\alpha}-1}\left\{6Beta\left(\frac{4}{\alpha}, 1-\frac{1}{\alpha}\right) - 6Beta\left(\frac{3}{\alpha}, 1-\frac{1}{\alpha}\right) + Beta\left(\frac{2}{\alpha}, 1-\frac{1}{\alpha}\right)\right\}$$
$$\lambda_4 = \beta\alpha^{\frac{1}{\alpha}-1}\left\{20Beta\left(\frac{5}{\alpha}, 1-\frac{1}{\alpha}\right) - 30Beta\left(\frac{4}{\alpha}, 1-\frac{1}{\alpha}\right) + 12Beta\left(\frac{3}{\alpha}, 1-\frac{1}{\alpha}\right) - Beta\left(\frac{2}{\alpha}, 1-\frac{1}{\alpha}\right)\right\}$$

The L-skewness and L-kurtosis of K3D-II distribution can be defined respectively as:
$\tau_3 = \frac{\lambda_3}{\lambda_2}$, $\tau_4 = \frac{\lambda_4}{\lambda_2}$
Now, can be estimated the L-moments parameter of α depending on the equation of a sample L-skewness (τ_3). We can note that the equation of a sample L-skewness depends on α only. The parameter β *and* μ can be estimated respectively by

$$\hat{\beta} = \frac{l_2}{\hat{\alpha}^{\frac{1}{\hat{\alpha}}-1}\left\{2Beta\left(\frac{3}{\hat{\alpha}}, 1-\frac{1}{\hat{\alpha}}\right) - Beta\left(\frac{2}{\hat{\alpha}}, 1-\frac{1}{\hat{\alpha}}\right)\right\}} \quad (9)$$

$$\hat{\mu} = l_1 - \hat{\beta}\hat{\alpha}^{\frac{1}{\alpha}-1}Beta\left(\frac{2}{\hat{\alpha}}, 1-\frac{1}{\hat{\alpha}}\right) , \quad (10)$$

3.3. LH-moments of K3D-II distribution

The LH-skewness and LH-kurtosis of K3D-II distribution can be defined respectively as:
$\tau_3^\eta = \frac{\lambda_3^\eta}{\lambda_2^\eta}$, $\tau_4^\eta = \frac{\lambda_4^\eta}{\lambda_2^\eta}$
The LH-skewness is only a function of α, the LH-moments of α is acquired by τ_3^η. The parameter β and μ can be estimated respectively as:

$$\hat{\beta} = \frac{\hat{\lambda}_2^\eta}{\frac{(\eta+2)}{2!}\left\{\hat{\alpha}^{\frac{1}{\hat{\alpha}}-1}\left[(\eta+2)Beta\left(\frac{\eta+3}{\hat{\alpha}}, 1-\frac{1}{\hat{\alpha}}\right) - (\eta+1)\,Beta\left(\frac{\eta+2}{\hat{\alpha}}, 1-\frac{1}{\hat{\alpha}}\right)\right]\right\}} \quad (12)$$

$$\hat{\mu} = \hat{\lambda}_1^\eta - (\eta+1)\,\hat{\beta}\hat{\alpha}^{\frac{1}{\hat{\alpha}}-1}Beta\left(\frac{\eta+2}{\hat{\alpha}}, 1-\frac{1}{\hat{\alpha}}\right) \quad (13)$$

4. Case study

This study used the characteristics of precipitation records obtained from Department of Irrigation and Drainage, Terengganu for the

station of Kemaman, Terengganu. The data contains a maximum monthly precipitation data of the Kemaman site for the period 2011-2016. The objective of the analysis, to demonstrate the effect of the L-moments method and the LH-moments method in estimating of the K3D type II distribution for the rainfall frequency analysis.
In this study, real data was studied to compare the performance of L-moments and LH-moments based on mean absolute deviation index (MADI), that suggested by [8] for comparison among the probability distribution in fitting the data. The MADI value was computed as following:

$$MADI = \frac{1}{N}\sum_{i=1}^{N}\left|\frac{x_i - w_i}{x_i}\right| \quad (14)$$

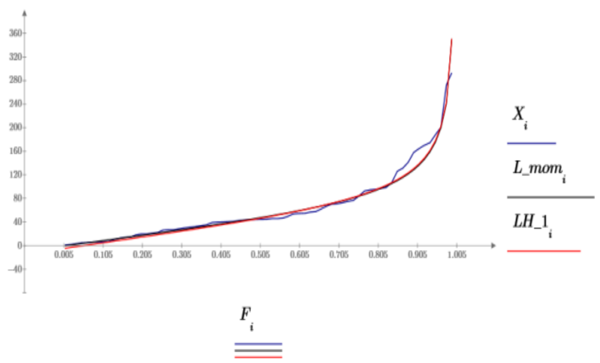
5. Results and discussion

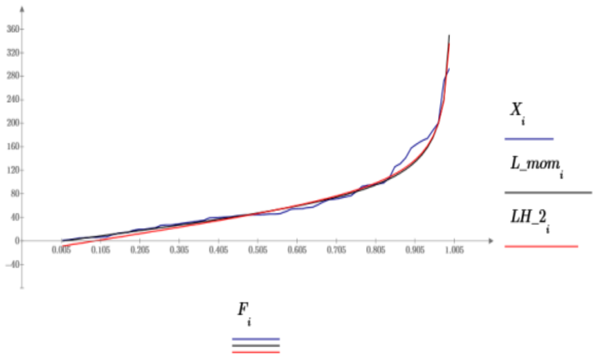
The values of monthly maximum rainfall for Kemaman station were analyzed to compare between L-moments (L0) methods and LH-moments (L1, L2, L3 and L4) methods for K3D-II distribution. Table 1 presents the values of MADI for a complete data series $(0 \le F \le 1)$ and the upper part of distribution $(0.9 \le F \le 1)$ and $(0.95 \le F \le 1)$ for K3D-II distribution.

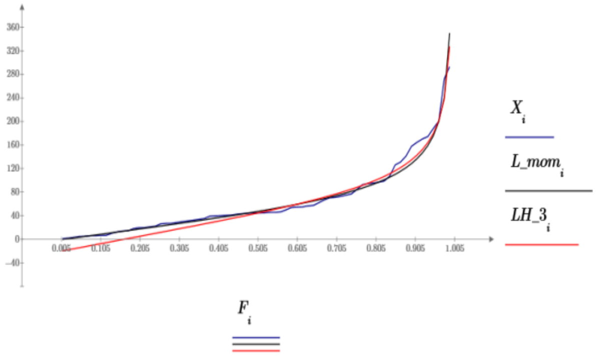
Table 1: Values of MADI for $(0 \le F \le 1)$, $(0.9 \le F \le 1)$ and $(0.95 \le F \le 1)$ for K3D-type II distribution

	$(0 \le F \le 1)$	$(0.9 \le F \le 1)$	$(0.95 \le F \le 1)$
L0	0.114	0.015	0.007
L1	0.287	0.014	0.006
L2	0.475	0.013	0.006
L3	0.984	0.012	0.005
L4	1.667	0.010	0.004

From Table 1, the bold values are the smallest in each column. So, it is clear that L-moments methods have the smallest MADI values in a complete data. Hence, the K3D-II distribution using L-Moments is the best methods for a complete data, while L4-Moments method is the best at the upper part of the distribution at $(0.9 \le F \le 1)$ and $(0.95 \le F \le 1)$ and can be noted that L4-Moments at$(0.95 \le F \le 1)$ better than L4-Moments at $(0.9 \le F \le 1)$.
Figure 1 shows the K3D-II distribution curves fit to the complete data series of Kemaman station by L-moments and at different levels of LH-moments. Based on Figure 1, the curves fitted by LH-moments method better capture the trends shown by high quantiles. Whereas, results of high level of LH-moments (L3 and L4) are poor for low quantiles. So, the LH-moments method yields a satisfactory result that fitted the observed data better than the L-moments method in high quantile estimation. Thus, this seems to suggest that the LH-moments method could improve the estimation of floods of larger return periods.







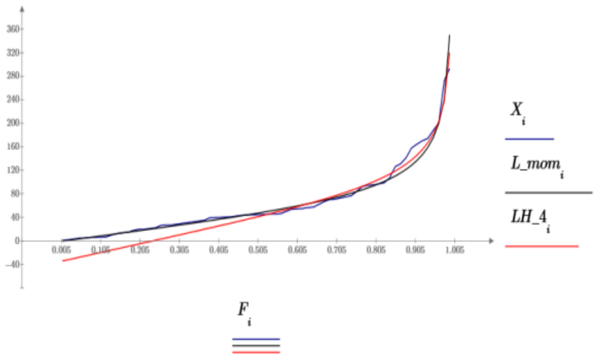


Fig. 1: Fitting the K3D-II distribution for different levels of L-moments and LH-moments

6. Conclusion

LH-moments are based on linear combination of higher order statistics can be used for describing the upper part of distributions and larger events in a sample. L-moments are a special case of LH moments. Analysis of monthly maximum rainfall series data of Kemaman station shows that the LH-moments method is very effective in fitting K3D-II distribution to large rainfall data and even produces better performance than the L-moments method. The diagrams of the maximum rainfall series data showed superiority LH-moments over L-moments at high quantiles. In conclusion, it is therefore recommended to use LH-moments method for estimating rainfall of large return periods.

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