*International Journal of Engineering &Technology*, 7 (4) (2018) 6781-6785

International Journal of Engineering & Technology

*Website: www.sciencepubco.com/index.php/IJET*
*doi: 10.14419/ijet.v7i4.11794*
***Research paper***

# Soliton solution of the coupled nonlinear Klein Gordon equations with two integration schemes

**Anwar Ja'afar Mohamad Jawad ***

*Al-Rafidain University College, Baghdad-10014, IRAQ*
**Corresponding author E-mail: anwar_jawad2001@yahoo.com*

**Abstract**

This paper established a traveling wave solution by using Sech - Csch function algorithms as well as the Modified Simple Equation Method (MSEM) for nonlinear partial differential equations. These methods will be applied to solving the conservation laws of the coupled Klein-Gordon equations that arise in the quantum field theory. There are two types of nonlinearity that will be considered, namely the cubic and the power law. The conservation laws will be finally determined from these conserved densities from the corresponding soliton solution.

***Keywords***: *Nonlinear PDEs; Klein-Gordon Equations; Exact Solutions; Sech –Csch Function Method.*

## 1. Introduction

Nonlinear evolution equations have a major role in various scientific and engineering fields, such as fluid mechanics, plasma physics, optical fibers, solid state physics, chemical physics and geochemistry. Nonlinear wave phenomena of dispersion, dissipation, diffusion, reaction and convection are very important in nonlinear wave equations. In recent years, quite a few methods for obtaining explicit traveling and solitary wave solutions of nonlinear evolution equations have been proposed. A variety of powerful methods has been used to solve different types of nonlinear systems of PDEs. [1-15]
The nonlinear Klein-Gordon equation (KGE) is the NLEE that is going to be studied. KGE is a very important equation in the area of theoretical and mathematical physics. It is studied in quantum mechanics. This paper is going to take a look at the coupled KGE where cubic and power law nonlinearities [2].

## 2. The travelling wave

Consider the nonlinear partial differential equation in the form

$$F(u, u_t, u_x, u_{tt}, u_{xx}, \dots\dots\dots) = 0 \tag{1}$$

Where u(x, y, t) is a traveling wave solution of nonlinear partial differential equation Eq. (1). We use the transformations,

$$u(x,t) = f(\xi) \tag{2}$$

Where $\xi = kx - \lambda t$ This enables us to use the following changes:

$$\frac{\partial}{\partial t}(.) = -\lambda \frac{d}{d\xi}(.)\,, \frac{\partial}{\partial x}(.) = k\,\frac{d}{d\xi}(.) \tag{3}$$

Using Eq. (3) to transfer the nonlinear partial differential equation Eq. (1) to nonlinear ordinary differential equation

$$Q(f, f', f'', f''', \dots\dots\dots\dots\dots.) = 0 \tag{4}$$

The ordinary differential equation (4) is then integrated as long as all terms contain derivatives, where we neglect the integration constants. The solutions of many nonlinear equations can be expressed in the form: [3]

## 3. Sech and csch methods

Consider the solution of the form [16]

$$f(\xi) = \sigma\, \mathrm{sech}^{\beta}(\mu\xi)$$

$$f'(\xi) = -\sigma\,\beta\,\mu\, \mathrm{sech}^{\beta}(\mu\xi)\,.\tanh(\mu\xi) \tag{5}$$

$$f''(\xi) = -\sigma\,\beta\,\mu^2\left[(\beta+1)\mathrm{sech}^{\beta+2}(\mu\xi) - \beta\, \mathrm{sech}^{\beta}(\mu\xi)\right]$$

$$f'''(\xi) = \sigma\,\beta\,\mu^3\left[(\beta+1)(\beta+2)\, \mathrm{sech}^{\beta+2}(\mu\xi) - \beta^2\, \mathrm{sech}^{\beta}(\mu\xi)\right]\tanh(\mu\xi)$$

And their derivative. Or use

$$f(\xi) = \sigma\, \mathrm{csch}^{\beta}(\mu\xi)$$

$$f'(\xi) = -\sigma\,\beta\,\mu\, \mathrm{csch}^{\beta}(\mu\xi)\,.\coth(\mu\xi) \tag{6}$$

$$f''(\xi) = \sigma\,\beta\,\mu^2\left[(\beta+1)\mathrm{csch}^{\beta+2}(\mu\xi) + \beta\, \mathrm{csch}^{\beta}(\mu\xi)\right]$$

$$f'''(\xi) = -\sigma\,\beta\,\mu^3\left[(\beta+1)(\beta+2)\, \mathrm{csch}^{\beta+2}(\mu\xi) + \beta^2\, \mathrm{csch}^{\beta}(\mu\xi)\right]\coth(\mu\xi)$$

Where σ , μ, and β are parameters to be determined, μ and c are the wave number and the wave speed, respectively. We substitute (5) or (6) into the reduced equation (4), balance the terms of the sech functions when (5) are used, or balance the terms of the csch functions when (6) are used, and solve the resulting system of algebraic equations by using computerized symbolic packages. We next collect all terms with the same power in $\mathrm{sech}^{k}(\mu\xi)$ or $\mathrm{csch}^{k}(\mu\xi)$ and set to zero their coefficients to get a system of algebraic equations among the unknown's α , μ and β, and solve the subsequent system.

## 4. Governing equations

The coupled KGE with the cubic law as well as the power law nonlinearity are going to be studied in this paper. The following two subsections will list the dimensionless form of these equations along with their respective 1-soliton solutions. The constraint conditions or the domain restrictions for the existence of the solitons will also be given.

### 4.1. KGE with cubic law

In this section, the coupled NKGE will be investigated with cubic law of nonlinearity which is on (1+1) dimensions:
The coupled KGE with cubic law nonlinearity, in dimensionless form is given by:

$$q_{tt} - k^2 q_{xx} + a_1 q + b_1 q^3 + c_1 q . r^2 = 0$$
(7)

$$r_{tt} - k^2 r_{xx} + a_2 r + b_2 r^3 + c_2 q^2 . r = 0$$
(8)

Where q(x,t) and r(x,t) in Eq.(1), and (2) are the wave profiles. The independent variables are x and t
This equation studied by Biswas et al [2]. Introduce the transformations

$$q(x,t) = u_1(\xi), r(x,t) = u_2(\xi)$$
(9)

$$\xi = (x - \lambda t + \chi)$$
(10)

Where λ, and χ are real constants. The parameter λ represents the soliton velocity..
Substituting (10) into Equations (8-9) we obtain that

$$[\lambda^2 - k^2] u_1'' + a_1 u_1 + b_1 u_1^3 + c_1 u_1 . u_2^2 = 0$$
(11)

$$[\lambda^2 - k^2] u_2'' + a_2 u_2 + b_2 u_2^3 + c_2 u_1^2 . u_2 = 0$$
(12)

Seeking the solution by csh function method as in (6)

$$u_1(\xi) = \sigma_1 \mathrm{csch}^{\beta_1}(\mu \xi)$$
(13)

$$u_2(\xi) = \sigma_2 \mathrm{csch}^{\beta_2}(\mu \xi)$$
(14)

where $\sigma_1, \sigma_2$ represent the amplitudes of the solitons, μ represents the solitons width. the system of equations in Eqs. (11) and (12) becomes respectively:

$$[\lambda^2 - k^2] \sigma_1 \beta_1 \mu^2 \left[(\beta_1 + 1)\mathrm{csch}^{\beta_1+2}(\mu\xi) + \beta_1 \mathrm{csch}^{\beta_1}(\mu\xi)\right] + a_1 \sigma_1 \mathrm{csch}^{\beta_1}(\mu\xi) + b_1\sigma_1^3 \mathrm{csch}^{3\beta_1}(\mu\xi) + c_1 \sigma_1 . \sigma_2^2 \mathrm{csch}^{\beta_1+2\beta_2}(\mu\xi) = 0$$
(15)

$$[\lambda^2 - k^2] \sigma_2 \beta_2 \mu^2 \left[(\beta_2 + 1)\mathrm{csch}^{\beta_2+2}(\mu\xi) + \beta_2 \mathrm{csch}^{\beta_2}(\mu\xi)\right] + a_2 \sigma_2 \mathrm{csch}^{\beta_2}(\mu\xi) + b_2 \sigma_2^3 \mathrm{csch}^{3\beta_2}(\mu\xi) + c_2 \sigma_1^2 \sigma_2 \mathrm{csch}^{2\beta_1+\beta_2}(\mu\xi) = 0$$
(16)

Equating the exponents and the coefficients of each pair of the csch functions we find from (15)

$$3\beta_1 = \beta_1 + 2\beta_2 \text{ , then } \beta_1 = \beta_2$$
(17)

from Eq. (16)

$$2\beta_1 + \beta_2 = \beta_2 + 2$$
(18)

Then

$$\beta_1 = 1 , \beta_2 = 1$$
(19)

Thus setting coefficients of Equations (15-16) to zero yields

$$[\lambda^2 - k^2] \sigma_1 \mu^2 + a_1 \sigma_1 = 0$$

$$2 [\lambda^2 - k^2] \sigma_1 \mu^2 + b_1\sigma_1^3 + c_1 \sigma_1 \sigma_2^2 = 0$$

$$[\lambda^2 - k^2] \sigma_2 \mu^2 + a_2 \sigma_2 = 0$$

$$2[\lambda^2 - k^2] \sigma_2 \mu^2 + b_2 \sigma_2^3 + c_2 \sigma_1^2 \sigma_2 = 0$$
(20)

Solving the system of equations in (20) we get:

$$\sigma_1 = \sqrt{\frac{2 a_2 [b_2 - c_1]}{[b_1 b_2 - c_1 c_2]}}$$
(21)

$$\sigma_2 = \sqrt{\frac{2 a_1 [b_1 - c_2]}{[b_1 b_2 - c_1 c_2]}}$$
(22)

$$\mu = \sqrt{\frac{a_1}{[k^2 - \lambda^2]}}$$
(23)

$$\mu = \sqrt{\frac{a_2}{[k^2 - \lambda^2]}}$$
(24)

These relations consequently introduce the constraint conditions:

$$a_1 = a_2 = a = [k^2 - \lambda^2] \mu^2$$
(25)

Then

$$\mu = \sqrt{\frac{a}{[k^2 - \lambda^2]}}$$
(26)

Where

$$[k^2 - \lambda^2] > 0$$
(27)

$$[b_1 b_2 - c_1 c_2] > 0$$
(28)

in general form the amplitudes:

$$\sigma_j = \sqrt{\frac{2\,a\,[b_i - c_j]}{[b_1\,b_2 - c_1\,c_2]}}\ , j = 1,2\ ;\ i = 3 - j$$
(29)

Also, the amplitude relations dictate the constraint condition given by:

$$a\left[b_i - c_j\right] > 0$$
(30)

Then:

$$u_j(x,t) = \sqrt{\frac{2\,a\,[b_i - c_j]}{[b_1\,b_2 - c_1\,c_2]}}\ csch\left(\sqrt{\frac{a}{[k^2 - \lambda^2]}}\ (x - \lambda\,t + \chi)\right)\ ,, j = 1,2\ ;\ i = 3 - j$$
(31)

Then

$$q(x,t) = \sqrt{\frac{2\,a\,[b_2 - c_1]}{[b_1\,b_2 - c_1\,c_2]}}\ csch\left(\sqrt{\frac{a}{[k^2 - \lambda^2]}}\ (x - \lambda\,t + \chi)\right)$$
(32)

Where:

$$a\,[b_2 - c_1] > 0$$
(33)

And

$$r(x,t) = \sqrt{\frac{2\,a\,[b_1 - c_2]}{[b_1\,b_2 - c_1\,c_2]}}\ csch\left(\sqrt{\frac{a}{[k^2 - \lambda^2]}}\ (x - \lambda\,t + \chi)\right)$$
(34)

Where

$$a\,[b_1 - c_2] > 0$$
(35)

For $a = 1, k = \sqrt{2}, \lambda = 1, b_1 = b_2 = 2, c_1 = c_2 = 1, \chi = 0$

$$q(x,t) = r(x,t) = \sqrt{\frac{2}{3}}\ csch((x - t))$$
(36)

Figure 1 represents the solitary wave solution of $q(x,t), and\ r(x,t)$ in Eq.(36).

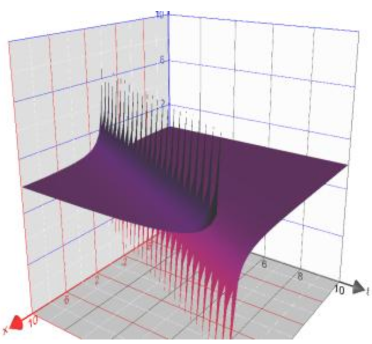

**Fig. 1:** The Solitary Wave Solution of Eq.(36).

### 4.2. Power law nonlinearity

For power law nonlinearity, the dimensionless form of the coupled KGE is given:

$$q_{tt} - k^2\,q_{xx} + a_1\,q + b_1\,q^{m+n} + c_1\,q^m\,.\,r^n = 0$$
(37)

$$r_{tt} - k^2\,r_{xx} + a_2\,r + b_2\,r^{m+n} + c_2\,q^n\,.\,r^m = 0$$
(38)

Where n ≠1. While the coefficients have the same interpretation as in the previous subsection, the additional parameters in this case are the power law nonlinearity parameters m and n and here m > 0 as well as n > 0. This equation studied by Biswas et al [2]. Assume the transformations in Equation (9-10) and Substituting into Equations (31-32) we obtain that

$$[\lambda^2 - k^2]\,u_1'' + a_1\,u_1 + b_1\,u_1^{m+n} + c_1\,u_1^{m}\,.\,u_2^{n} = 0$$
(39)

$$[\lambda^2 - k^2]\ u_2'' + a_2\,u_2 + b_2\,u_2^{m+n} + c_2\,u_1^{n}\,.\,u_2^{m} = 0$$
(40)

Seeking the solution by sech function method as in (5)

$$u_1(\xi) = \sigma_1\,sech^{\beta_1}(\mu\,\xi)$$
(41)

$$u_2(\xi) = \sigma_2\,sech^{\beta_2}(\mu\,\xi)$$
(42)

Where $\sigma_1, \sigma_2$ represent the amplitudes of the solitons, $\mu$ represents the solitons width. The system of equations in Eqs. (40) and (41) becomes respectively:

$$[\lambda^2 - k^2]\,\sigma_1\,\beta_1\,\mu^2\left[-(\beta_1 + 1)sech^{\beta_1+2}(\mu\xi) + \beta_1\,sech^{\beta_1}(\mu\xi)\right] + a_1\,\sigma_1\,sech^{\beta_1}(\mu\xi) + b_1\sigma_1^{(m+n)}\,sech^{(m+n)\beta_1}(\mu\xi) + c_1\,\sigma_1^{m}\,.\,\sigma_2^{n}\,sech^{m\beta_1+n\beta_2}(\mu\xi) = 0$$
(43)

$$[\lambda^2 - k^2]\,\sigma_2\,\beta_2\,\mu^2\left[-(\beta_2 + 1)\,sech^{\beta_2+2}(\mu\xi) + \beta_2\,sech^{\beta_2}(\mu\xi)\right] + a_2\,\sigma_2\,sech^{\beta_2}(\mu\xi) + b_2\,\sigma_2^{(m+n)}\,sech^{(m+n)\beta_2}(\mu\xi) + c_2\,\sigma_1^{n}\,\sigma_2^{m}\,sech^{n\beta_1+m\beta_2}(\mu\xi) = 0$$
(44)

Equating the exponents and the coefficients of each pair of the sech functions we find the following algebraic system from Eq. (43)

$$m\beta_1 + n\beta_2 = (m+n)\beta_1$$
(45)

From Eq. (45)

$$n\beta_1 + m\beta_2 = \beta_2 + 2$$
(46)

Then

$$\beta_1 = \beta_2 = \frac{2}{m+n-1}$$
(47)

Thus setting coefficients of Equations (43-44) to zero yields

$$[\lambda^2 - k^2]\frac{4}{(m+n-1)^2}\,\mu^2 + a_1 = 0$$

$$-[\lambda^2 - k^2]\,\frac{2\,(m+n+1)}{(m+n-1)^2}\,\mu^2 + b_1\sigma_1^{(m+n-1)} + c_1\,\sigma_1^{m-1}\,.\,\sigma_2^{n} = 0$$

$$[\lambda^2 - k^2]\frac{4}{(m+n-1)^2}\,\mu^2 + a_2 = 0$$

$$-[\lambda^2 - k^2]\,\frac{2\,(m+n+1)}{(m+n-1)^2}\,\mu^2 + b_2\,\sigma_2^{(m+n-1)} + c_2\,\sigma_1^{n}\,\sigma_2^{m-1} = 0$$
(48)

Solving the system of equations in (44) we get:

$$\mu = \frac{(m+n-1)}{2}\sqrt{\frac{a_1}{[k^2-\lambda^2]}} = \frac{(m+n-1)}{2}\sqrt{\frac{a_2}{[k^2-\lambda^2]}}$$
(49)

These relations consequently introduce the constraint conditions:

$$a_1 = a_2 = a = \frac{4\,[k^2-\lambda^2]}{(m+n-1)^2}\,\mu^2$$
(50)

$$a\,[k^2-\lambda^2] > 0$$
(51)

Then in general form:

$$\mu = \sqrt{\frac{a}{[k^2-\lambda^2]}}$$
(52)

the amplitudes $\sigma_j$; ( j =1,2 ) are connected to each other by the coupled relations:

$$2\,b_j\,\sigma_j^{(m+n-1)} + 2\,c_j\,\sigma_j^{m-1}\,.\,\sigma_{3-j}^{n} + (m+n+1)\,a_j = 0\,,$$
(53)

Then:

$$u_j(x,t) = \sigma_j\ sech^{\left(\frac{2}{m+n-1}\right)}\left(\frac{(m+n-1)}{2}\sqrt{\frac{a}{[k^2-\lambda^2]}}\,(x-\lambda\,t+\chi)\right),\, j = 1,2$$
(54)

For $m = 1\,, n = 2\,, a = 1, k = 2\,, \lambda = 1\,, \chi = 0$

$$u_j\,(x,t) = \sigma_j\ sech\left(\sqrt{\frac{1}{3}}\,(x-\ t+\chi)\right),\, j = 1,2$$
(55)

$$\sigma_1 = \sigma_2 = \mp\, i\,\sqrt{\frac{2}{3}}$$
(56)

$$q(x,t) = r(x,t) = \mp i\,\sqrt{\frac{2}{3}}\ sech\left(\sqrt{\frac{1}{3}}\,(x-\ t)\right)$$
(57)

Figure 2 represents the solitary wave solution of $q(x,t), and\ r(x,t)$ in Eq.(57).

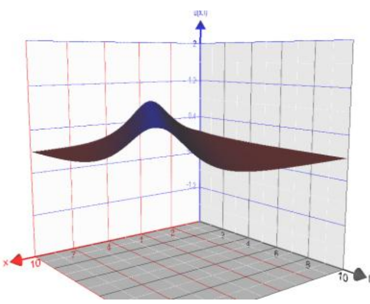

**Fig. 2:** The Solitary Wave Solution of Eq.(57).

## 5. Modified simple equation method

Consider for solutions of the form [17]:

$$r(\xi) = \left(A_0 + A_1\frac{\psi_\xi}{\psi}\right)$$
(58)

$$q(\xi) = \left(B_0 + B_1\frac{\psi_\xi}{\psi}\right)$$
(59)

Then Eqsn. (11-12 ) become the following equations:

$$(\lambda^2-k^2)\,A_1\left\{\frac{\psi_{\xi\xi\xi}}{\psi} - 3\frac{\psi_\xi\,\psi_{\xi\xi}}{\psi^2} + 2\frac{{\psi_\xi}^3}{\psi^3}\right\} + a_1\left\{A_0 + A_1\frac{\psi_\xi}{\psi}\right\} + b_1\left\{{A_0}^3 + 3\,{A_0}^2\,A_1\frac{\psi_\xi}{\psi} + 3A_0\,{A_1}^2\,\frac{{\psi_\xi}^2}{\psi^2} + {A_1}^3\,\frac{{\psi_\xi}^3}{\psi^3}\right\} + c_1\left\{A_0 + A_1\frac{\psi_\xi}{\psi}\right\}\left\{{B_0}^2 + 2\,B_0\,B_1\frac{\psi_\xi}{\psi} + {B_1}^2\,\frac{{\psi_\xi}^2}{\psi^2}\right\} = 0$$
(60)

$$(\lambda^2-k^2)\,B_1\left\{\frac{\psi_{\xi\xi\xi}}{\psi} - 3\frac{\psi_\xi\,\psi_{\xi\xi}}{\psi^2} + 2\frac{{\psi_\xi}^3}{\psi^3}\right\} + a_2\left\{B_0 + B_1\frac{\psi_\xi}{\psi}\right\} + b_2\left\{{B_0}^3 + 3\,{B_0}^2\,B_1\frac{\psi_\xi}{\psi} + 3B_0\,{B_1}^2\,\frac{{\psi_\xi}^2}{\psi^2} + {B_1}^3\,\frac{{\psi_\xi}^3}{\psi^3}\right\} + c_2\left\{B_0 + B_1\frac{\psi_\xi}{\psi}\right\}\left\{{A_0}^2 + 2\,A_0\,A_1\frac{\psi_\xi}{\psi} + {A_1}^2\,\frac{{\psi_\xi}^2}{\psi^2}\right\} = 0$$
(61)

Equating expressions (60-61) at ψ−1, ψ−2, ψ−3, and ψ−4 to zero we have the following system of equations

$$a_1 + b_1\,{A_0}^2 + c_1\,.\,{B_0}^2 = 0$$

$$a_2 + b_2\,{B_0}^2 + c_2\,{A_0}^2 = 0$$

$$[\lambda^2-k^2]\,A_1\psi_{\xi\xi\xi} + a_1A_1\psi_\xi + 3b_1\,{A_0}^2\,A_1\psi_\xi + c_1[{B_0}^2A_1 + 2\,B_0\,B_1A_0]\psi_\xi = 0$$

$$[\lambda^2-k^2]\,B_1\psi_{\xi\xi\xi} + a_2\,B_1\,\psi_\xi + 3b_2\,{B_0}^2\,B_1\psi_\xi + c_2\,[{A_0}^2B_1 + 2\,A_0\,A_1B_0\,]\psi_\xi = 0$$

$$-3[\lambda^2-k^2]\,A_1\psi_\xi\,\psi_{\xi\xi} + [3b_1A_0\,{A_1}^2 + 2c_1\,A_1\,B_0\,B_1 + c_1A_0{B_1}^2]{\psi_\xi}^2 = 0$$

$$-3\,[\lambda^2-k^2]\,B_1\,\psi_\xi\,\psi_{\xi\xi} + [3b_2\,B_0\,{B_1}^2 + 2\,c_2A_0\,A_1B_1 + c_2\,{A_1}^2B_0\,]{\psi_\xi}^2 = 0$$

$$2\,[\lambda^2-k^2]\,A_1\,{\psi_\xi}^3 + b_1\,{A_1}^3{\psi_\xi}^3 + c_1A_1{B_1}^2\,{\psi_\xi}^3 = 0$$

$$2\,[\lambda^2-k^2]B_1\,{\psi_\xi}^3 + b_2{B_1}^3\,{\psi_\xi}^3 + c_2\,B_1\,{A_1}^2\,{\psi_\xi}^3 = 0$$
(62)

These relations consequently introduce the constraint conditions:

$$A_1 = B_1\,,\, A_0 = B_0\,,\, a_1 = a_2 = a\,,\, b_1 = b_2 = b\,,\, c_1 = c_2 = c$$
(63)

System of equations in (62) reduce to the following system:

$$a + (b+c)\,{A_0}^2 = 0$$

$$[\lambda^2-k^2]\,\psi_{\xi\xi\xi} + [\,a + 3(b+c)\,A_0]\,\psi_\xi = 0$$

$$[\lambda^2-k^2]\,\psi_{\xi\xi} - A_0A_1\,(b+c)\,\psi_\xi = 0$$

$$2\,[\lambda^2-k^2] + (b+c)\,{A_1}^2 = 0$$
(64)

Solving system (64), then

$A_0 = \mp i\sqrt{\frac{a}{(b+c)}}, A_1 = \mp i\sqrt{\frac{2[\lambda^2 - k^2]}{[b+c]}}$

(65)

Case 1

$\psi_{\xi\xi\xi} - \frac{2a}{[\lambda^2 - k^2]}\psi_\xi = 0$

(66)

Solving the differential equation in (66) to get

$\psi = \alpha + \beta\,\xi + \mu\,exp(\frac{2a}{[\lambda^2 - k^2]}\xi)$

(67)

Then

$\psi_\xi = \beta + \mu\,\frac{2a}{[\lambda^2 - k^2]}\,exp(\frac{2a}{[\lambda^2 - k^2]}\xi)$

(68)

Finally the solution

$r(x,t) = q(x,t) = \mp i\sqrt{\frac{1}{(b+c)}}\left(\sqrt{a} + \sqrt{2}\,\frac{\beta\sqrt{[\lambda^2 - k^2]} + 2\,\mu\,a\,exp(\frac{2a}{[\lambda^2 - k^2]}(x - \lambda\,t + \chi))}{\alpha + \beta(x - \lambda\,t + \chi) + \mu\,exp(\frac{2a}{[\lambda^2 - k^2]}(x - \lambda\,t + \chi))}\right)$

(69)

Case 2

$\psi_{\xi\xi} + \sqrt{\frac{2\,a}{[\lambda^2 - k^2]}}\,\psi_\xi = 0$

(70)

Solving the differential equation in (70) to get

$\psi = \rho + \sigma\,exp(-\sqrt{\frac{2\,a}{[\lambda^2 - k^2]}}\,\xi)$

(71)

Then

$\psi_\xi = -\sigma\sqrt{\frac{2\,a}{[\lambda^2 - k^2]}}\,exp(-\sqrt{\frac{2\,a}{[\lambda^2 - k^2]}}\,(x - \lambda\,t + \chi))$

(72)

The solution

$r(x,t) = q(x,t) = \mp i\sqrt{\frac{a}{[b+c]}}\left(1 - 2\,\sigma\,\frac{exp\left(-\sqrt{\frac{2\,a}{[\lambda^2 - k^2]}}(x - \lambda\,t + \chi)\right)}{\rho + \sigma\,exp\left(-\sqrt{\frac{2\,a}{[\lambda^2 - k^2]}}(x - \lambda\,t + \chi)\right)}\right)$

(73)

Where:

$[b + c]\,[\lambda^2 - k^2] > 0$

(74)

## 6. Conclusion

In this paper, the Sech-Csch function methods as well as the MSEM have been successfully applied to find solitons solutions for the coupled KGE. Both the cubic and the power law are considered. It was observed that for power law nonlinearity the conserved quantities exist only if the exponents are connected by a simple relation. This is one of the very many integration algorithms to locate soliton solutions. Such results will be reported in future publications.

## References


[1] Wazwaz, A.M. (2005). The tanh-function method: Solitons and periodic solutions for the Dodd-Bullough-Mikhailov and the Tzitzeica-Dodd-Bullough equations, Chaos Solitons and Fractals, Vol. 25, No. 1, pp. 55-63. https://doi.org/10.1016/j.chaos.2004.09.122.

[2] Anjan Biswas, Abdul H. Kara, Luminiţa Moraru, Ashfaque H. Bokhari, F. D. Zaman, (2014),"Conservation Laws of Coupled Klein-Gordon equations with cubic and power law nonlinearities" Proceeding of the Romanian Academy, Series A, Volume 15, Number 2 , pp. 123–129.

[3] Anwar Ja'afar Mohamed Jawad, Soliton Solutions for the Boussinesq Equations, (2013) J. Math. Comput. Sci. 3 , No. 1, 254-265

[4] El-Wakil, S.A, Abdou, M.A. (2007). New exact travelling wave solutions using modified extended tanh-function method, Chaos Solitons Fractals, Vol. 31, No. 4, pp. 840-852. https://doi.org/10.1016/j.chaos.2005.10.032.

[5] Jawad A J M, (2012), Soliton Solutions for Nonlinear Systems (2+ 1)-Dimensional Equations, IOSR Journal of Mathematics (IOSRJM) ISSN: 2278-5728 vol.1 no. 6. https://doi.org/10.9790/5728-0162734.

[6] Biswas A, Ebadi G, Fessak M, Johnpillai AG, Johnson S, Krishnan EV, Yildirim A. (2012), Solutions of the perturbed Klein-Gordon equations. Iran J Sci Tech no l Trans A; 36:431–52.

[7] Fan, E. (2000). Extended tanh-function method and its applications to nonlinear equations. Phys Lett A, Vol. 277, No.4, pp. 212-218. https://doi.org/10.1016/S0375-9601(00)00725-8.

[8] Feng, Z.S. (2002). The first integer method to study the Burgers-Korteweg-de Vries equation, J Phys. A. Math. Gen, Vol. 35, No. 2, pp. 343-349. https://doi.org/10.1088/0305-4470/35/2/312.

[9] Inc, M., Ergut, M. (2005). Periodic wave solutions for the generalized shallow water wave equation by the improved Jacobi elliptic function method, Appl. Math. E-Notes, Vol. 5, pp. 89-96.

[10] Malfliet, W. (1992). Solitary wave solutions of nonlinear wave equations, Am. J. Phys, Vol. 60, No. 7, pp. 650-654. https://doi.org/10.1119/1.17120.

[11] Khater, A.H., Malfliet, W., Callebaut, D.K. and Kamel, E.S. (2002). The tanh method, a simple transformation and exact analytical solutions for nonlinear reaction–diffusion equations, Chaos Solitons Fractals, Vol. 14, No. 3, PP. 513-522. https://doi.org/10.1016/S0960-0779(01)00247-8.

[12] Zhang, Sheng. (2006), the periodic wave solutions for the (2+1) -dimensional Konopelchenko Dubrovsky equations, Chaos Solitons Fractals, Vol. 30, pp. 1213-1220. https://doi.org/10.1016/j.chaos.2005.08.201.

[13] Wazwaz, A.M. (2006). Two reliable methods for solving variants of the KdV equation with compact and noncompact structures, Chaos Solitons Fractals, Vol. 28, No. 2, pp. 454-462. https://doi.org/10.1016/j.chaos.2005.06.004.

[14] Xia, T.C., Li, B. and Zhang, H.Q. (2001). New explicit and exact solutions for the Nizhnik- Novikov-Vesselov equation. Appl. Math. E-Notes, Vol. 1, pp. 139-142.

[15] Yusufoglu, E., Bekir A. (2006), Solitons and periodic solutions of coupled nonlinear evolution equations by using Sine-Cosine method, Internat. J. Comput. Math, Vol. 83, No. 12, pp. 915-924. https://doi.org/10.1080/00207160601138756.

[16] Anwar Ja'afar Mohamad Jawad and Mahmood Jawad Abu-AlShaeer, (2018), Soliton Solutions of the Coupled Schrödinger-Boussinesq Equations for Kerr Law Nonlinearity, Abstract and Applied Analysis. https://doi.org/10.1155/2018/8325919.

[17] Anwar Ja'afar Mohamad Jawad, Marko D. Petkovic´, Anjan Biswas, (2010), Modified Simple Equation Method for Evolution Equations, Applied Mathematics and Computation journal,Vol. 217, No. 1. https://doi.org/10.1016/j.amc.2010.06.030.