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Research paper

Stability analysis of closed loop TRMS with observer based reliable H infinity controller using Kharitonov's stability theorem

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Abstract

The laboratory Twin Rotor Multiple Input Multiple Output System (TRMS) serving as a model of a helicopter has un modeled errors in its model, due to linearization, measurement errors, equipment wear, sensors or/and actuator failures. This mismatch is termed as uncertainties in the model. Due to sensor and actuator failure there would exist a large range of uncertainties. In this paper, the range of robust stability bound for closed loop TRMS along with observer based reliable H infinity controller using Kharitonov's stability theorem is found. The variation in parameters of TRMS from its nominal values are shown. The Kharitonov's stability analysis on TRMS proves that within the mentioned uncertainty limit the TRMS along with observer based reliable H infinity controller gives the closed loop stable response.

Keywords: *TRMS; Uncertainties, kharitonov's polynomials, routh's array, robust stability bound, observer based reliable H infinity controller.*

1. Introduction

Kharitonov's theorem is useful in the field of robust control, which seeks to design systems that will work well despite of uncertainties due to measurement errors, changes in operating conditions, wear and so on. Also Kharitonov's theorem provides a means of performing sensitivity analysis for the roots of polynomials whose coefficients are perturbed. Kharitonov's theorem proves to be one of the simplest and best methods of stability analysis using which the stability bound of the system parameters could be found. Parametric uncertainty model is often used when precise knowledge of the actual parameters is not known. Systems with parametric uncertainties can be described by interval polynomial. Analysis for interval polynomials is performed with the help of the Kharitonov's theorem [1]. In [2] the system stability is proved using Lyapunov criterion. Jung-Hua Yang *et al.* has done Lyapunov stability analysis for the closed loop TRMS in [3].

Here Lyapunov theory is used to guarantee stability for state estimation.

In both the above papers robust stability bound is not found. In the present paper the robust stability bound is aimed to be found. The range of uncertainty signifies range of stability of the system along with the concerned controller.

This range of uncertainty gives the range of variation in the parameters of the system. The robust controller should maintain stability within the upper and lower bound within which the plant parameter can vary. P.K.Rajan*et al.* [4] have given the details of how variation in uncertainties result in parameter variation of the system. The paperclearly describes that for the system with parameter variation the Kharitonov's theorem could be applied efficiently to analyze its closed loop stability along with controller.

Kharitonov's stability theorem is a result used in control theory to assess the stability of a dynamical system when the physical parameters of the system are not known precisely. When the coefficients of the characteristic polynomial are known exactly the Routh-Hurwitz criterion can be used to check the stability of the system. That is if there is no sign change in first column of Routh array, all roots of the equation will have negative real part, then can be concluded that the system is stable. Kharitonov's theorem can be used in the case where the coefficients are only known to be within specified ranges. It provides a test of stability for interval polynomial, while Routh-Hurwitz method is concerned with an ordinary polynomial. In 1978, V. L. Kharitonov published a stability theorem for a class of polynomials of which each of the coefficients vary independently in a specified (but arbitrary) interval. This theorem is known as Kharitonov's theorem which states that the whole class of polynomials with real coefficients is Hurwitz if and only if four special, well defined polynomials are Hurwitz. It also states that the whole class of polynomials with complex coefficients is Hurwitz if and only if eight special, well defined polynomials are Hurwitz[5]. The stability analysis for TRMS is done by constructing four Kharitonov's polynomials in this paper, because coefficients of polynomials defining TRMS are real.

An interval polynomial (1) is the family of all polynomials.

$$I(s) = a_0 + a_1 s^1 + a_2 s^2 + \cdots \quad + a_n s^n \qquad (1)$$

Where each coefficient a_i is real and can take any value in the specified intervals. That is as defined in (2)

$$l_i \leq a_i \leq u_i \quad (2)$$

An interval polynomial shown in (1) is stable, that is all members of the family are stable if and only if the four so called Kharitonov's polynomials $K_{h1}(s), K_{h2}(s), K_{h3}(s)$ and$K_{h4}(s)$ are stable. Where Kharitonov's polynomials are given in (3) to (6).

$$K_{h1}(s) = l_0 + l_1 s^1 + u_2 s^2 + u_3 s^3 + l_4 s^4 + l_5 + \cdots \quad (3)$$
$$K_{h2}(s) = u_0 + u_1 s^1 + l_2 s^2 + l_3 s^3 + u_4 s^4 + u_5 s^5 + \cdots \quad (4)$$
$$K_{h3}(s) = l_0 + u_1 s^1 + u_2 s^2 + l_3 s^3 + l_4 s^4 + u_5 s^5 + \cdots \quad (5)$$
$$K_{h4}(s) = u_0 + l_1 s^1 + l_2 s^2 + u_3 s^3 + u_4 s^4 + l_5 s^5 + \cdots \quad (6)$$

Although in principle an infinite number of polynomials are to be tested for stability, according to Kharitonov's stability analysis only four Kharitonov's polynomials need to be tested. These four Kharitonov's polynomials can be tested for stability using any method like Routh-Hurwitz, polar plot, Nyquist plot or by finding roots of them. In this work, after constructing four Kharitonov's polynomials, Routh's table is formed and stability of TRMS is checked for each Kharitonov's polynomial. Also roots of individual Kharitonov's polynomial are found and whether it is stable or not is analyzed. It takes four times more work to be performed to check the stability of an interval polynomial than it takes to test the stability of any ordinary polynomial.

2. TRMS model with H infinity observer and controller

TRMS is a MIMO system with cross coupling shown in Fig. 1 and Fig. 2. The two inputs to TRMS are voltages $u1$ and $u2$ which are control inputs to TRMS. These are the controller outputs that is the voltages applied to the actuators of TRMS. Outputs of TRMS are $y1$(pitch angle) and $y2$(yaw angle) which are measured using sensors. The system identification technique is successfully applied on TRMS [6] to obtain suitable model for TRMS briefly explained in section 2A.

Fig. 1: TRMS laboratory setup

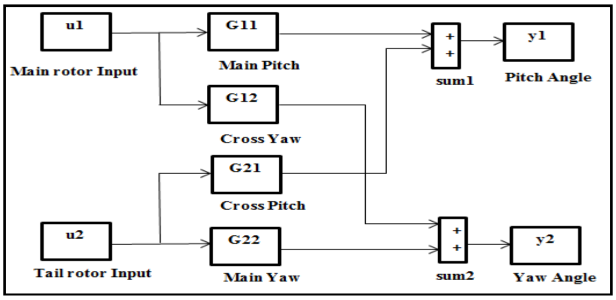

Fig. 2: Open loop model for TRMS

System identification of TRMS

To estimate a model of the TRMS mixed sine waves of varying frequencies between 0-1 Hz with amplitude within the range ±2.5 V is given to TRMS and both the input and output are recorded. This input output data is imported to MATLAB system identification toolbox and time domain data is selected which is the first step for estimating the model for TRMS. The time domain data consists of input and output variables of TRMS that is recorded at sampling period 0.001 s. The whole experimentation on TRMS has been done for system identification several times and the percentage fit for different type of models for main pitch, main yaw, cross pitch and cross yaw are tabulated as shown in Table I. In this work, ARMAX model gave the best fit compared to other models.

Table I: Percentage fit of Different Model Structures for TRMS

Main Pitch	%fit	Main Yaw	%fit	Cross pitch	%fit	Cross Yaw	%fit
amx 54910	83.84	amx 101055	63.56	amx 10333	54.3	amx 101023	50.52
amx 64810	83.8	N9S9	63.51	amx 6111	49.12	amx101033	50.49
amx 74810	83.75	amx 101054	63.5	BJ102221	48.44	amx 6234	45.23
amx 62810	83.73	amx 111054	63.43	OE1021	48.44	amx 4112	42.44
amx 75810	83.69	amx 101056	63.41	amx10133	46.39	N4s4	33.88
amx 6489	83.67	amx 101065	63.4	N4s2	45.66	Nlhw1	21.29
amx 106810	83.55	amx111050	63.39	Nlarx1	43	Oe10103	20.23
amx 72810	83.54	amx 101050	63.38	amx 4212	37.22	arx10109	10.25
amx 128810	83.19	N8S8	61.91	arx1015	36.94	Nlarx1	-16.81
amx 126810	83.03	Amx 10857	61.3	N4S4	22.74		

The best fit model for TRMS is obtained as shown in (7)
Main pitch – amx54910
Main yaw – amx101055 (7)
Cross pitch – amx10333
Cross yaw – amx101023

Equation (7) gives the discrete model for TRMS. This is converted into continuous model of which the order is reduced and given in (8)-(11).

Main pitch(Transfer function of pitch angle to the voltage supplied to main rotor):G11 :

$$\frac{y1(s)}{u1(s)}=\frac{0.0002s^9+0.01569s^8+1.3339s^7+5.0689s^6+14.1751s^5+24.2433s^4+29.8257s^3+23.5613s^2+11.7037s+1.8998}{s^{10}+5.0404s^9+19.7749s^8+51.7518s^7+105.7788s^6+164.6851s^5+196.3385s^4+172.6767s^3+105.6574s^2+38.6068s+5.2402} \quad (8)$$

Cross yaw (Transfer function of pitch angle to the voltage supplied to tail rotor):G12 :

$$\frac{y1(s)}{u2(s)}=\frac{-0.0103s^{10}-0.042016s^9+0.44703s^8+1.6726s^7+7.1798s^6+14.8825s^5+27.5294s^4+33.0653s^3+29.1328s^2+16.1547s+2.7327}{s^{10}+5.0404s^9+19.7749s^8+51.7518s^7+105.7788s^6+164.6851s^5+196.3385s^4+172.6767s^3+105.6574s^2+38.6068s+5.2402} \quad (9)$$

Cross pitch(Transfer function of yaw angle to the voltage supplied to main rotor): G21:

$$\frac{y2(s)}{u1(s)}=\frac{0.048575s^9+0.22063s^8+0.70772s^7+1.5724s^6+2.5211s^5+2.8913s^4+2.3054s^3+1.1773s^2+0.3219s+0.034081}{s^{10}+5.0404s^9+19.7749s^8+51.7518s^7+105.7788s^6+164.6851s^5+196.3385s^4+172.6767s^3+105.6574s^2+38.6068s+5.2402} \quad (10)$$

Main yaw : (Transfer function of yaw angle to the voltage supplied to tail rotor): G22 :

$$\frac{y2(s)}{u2(s)}=\frac{0.0016s^{10}-0.010835s^9+0.08862s^8+0.70031s^7+2.784s^6+8.0362s^5+15.8111s^4+23.5471s^3+24.6452s^2+16.1658s+4.8995}{s^{10}+5.0404s^9+19.7749s^8+51.7518s^7+105.7788s^6+164.6851s^5+196.3385s^4+172.6767s^3+105.6574s^2+38.6068s+5.2402} \quad (11)$$

Observer based reliable H infinity controller design for TRMS

Using TRMS model given in (8)-(11), the observer based reliable H infinity controller is designed which is a robust controller which guarantees that if changes are within the given bound the control law need not be changed presented in [6]. Fig. 3 shows the block diagram of observer based H_∞ controller for TRMS. The control problem is to synthesize a controller gain K which keeps size of performance variable z small in presence of exogenous input w. This influences the size of closed loop transfer function $T_{zw}(s)$ which has to be kept minimum even in presence of uncertainties. In the present work to quantify the size of $T_{zw}(s)$, H infinity norm is used[7]. For implementation of 50 percent actuator failure, the chosen values are as follows, $\bar{\alpha}_j = 1$, $\underline{\alpha_j}$ =0, $\alpha = 0.5$, $\alpha_0 = 0.5$ and γ = 10. The H_∞ controller gain K should satisfy (12),

$$(\alpha^2 P - X)K = \alpha^2 C^T \quad (12)$$

Where X and P are solution for controller and observer Riccati equations respectively which is explained in [6] and [8].

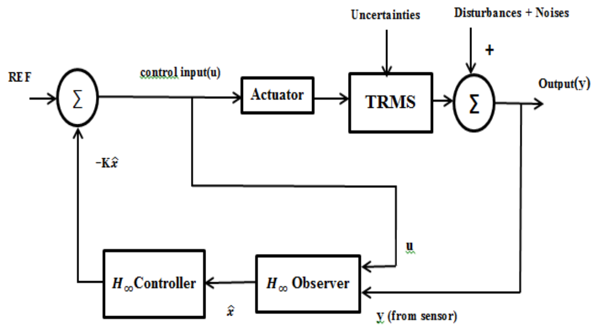

Fig. 3: Observer based H∞ controller for TRMS

The Fig. 4 demonstrates the flowchart of observer based reliable H infinity controller design which is robust as well as reliable. The control algorithm gives stable result without and with sensor or actuator failure [8]. The nominal model of TRMS (given in (8) to (11)) with observer based reliable H infinity controller gives closed loop nominal controller gain K_N.

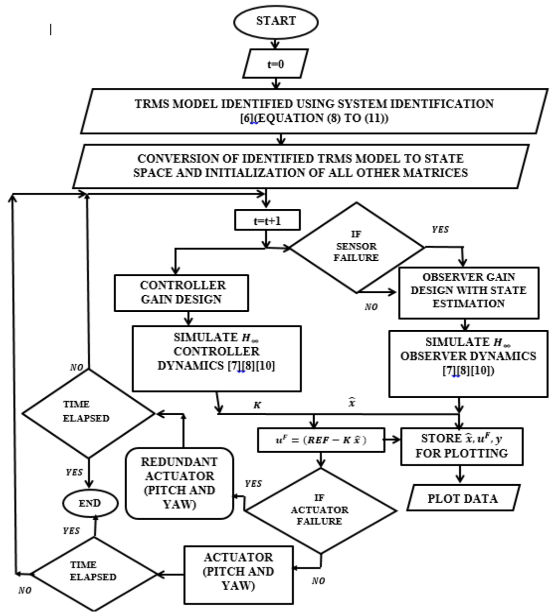

Fig. 4: Flowchart of observer based reliable H infinity controller algorithm designed for TRMS

3. Stability analysis of TRMS with observer based reliable H infinity controller using Kharitonov's stability theorem

The system matrix A of TRMS varies with uncertainties for which the controller has to be made robust by finding suitable controller gain K, which also varies with uncertainties[9]. The robust stability bound is the stability margin within which the closed loop TRMS along with observer based reliable H infinity controller is stable. In [10] authors have mentioned the method to choose the uncertainty bound. The uncertainty in the system is represented by multiplicative function but uncertainty bound is a constant which is a numerical value. In this work, to make TRMS robustly stable after designing the controller for TRMS, the percentage of sensor or actuator failure is varied within certain interval. This varies TRMS state space parameters A, B, C, D. The allowable range before it loses its stability is observed. Practically by trial and error approach the actual TRMS state space parameters are varied from 0.1 to 1.75. This means the uncertainty is varied between 10% of actual parameters on lower side to 75% more than the actual parameters on upper side. During experimentation on TRMS it was found that uncertainty bound between 0.5 to 1.27 of TRMS nominal parameters for which closed loop TRMS along with observer based reliable H infinity controller is stable. These are range of model parameters beyond which TRMS model((8) –(11)) is not valid. Using these uncertainty limits, the characteristic equation of closed loop TRMS is formed which gives the interval polynomial for TRMS. Using this interval polynomial (along with the uncertainty) four Kharitonov's polynomials are formed [11-15] which is explained below.

For TRMS stability analysis, continuous model is used. This is the identified uncertain model of TRMS. Using extension of Kharitonov's theorem for MIMO system[16], the transfer function for MIMO system is a matrix which is shown in (8) to (11).

$$\begin{bmatrix} y1 \\ y2 \end{bmatrix} = \begin{bmatrix} G11 & G12 \\ G21 & G22 \end{bmatrix} \quad (13)$$

Where TRMS open loop transfer function $G(s) = \begin{bmatrix} G11 & G12 \\ G21 & G22 \end{bmatrix}$ in MIMO case is represented as in (13) where

$$G11 = \frac{NT11}{DT1};\ G12 = \frac{NT12}{DT1};\ G21 = \frac{NT21}{DT1};\ G22 = \frac{NT22}{DT1}$$

Which could be written as, $G(s) = \frac{1}{DT1}\begin{bmatrix} NT11 & NT12 \\ NT21 & NT22 \end{bmatrix}$

$$G(s) \quad = C(sI - A)^{-1}B + D \quad (14)$$

Also the characteristic equation of TRMS is given in (15)

$$|sI - A| = 0 \quad (15)$$

With observer based reliable H infinity controller gain K the characteristic equation of TRMS will be as shown in (16)

$$|sI - (A - BK)| = 0 \quad (16)$$

which is also given by equation (17).

$$1 + G(s)K = 0 \quad (17)$$

Using uncertainty lower limit as 0.5, the observer based reliable H infinity controller gain $K_{0.5}$ is given in (18)

$$K_{0.5} = 10^{-3} * \begin{bmatrix} 0.205 & 0.119 & 0.274 & 0.063 & 0.056 & 0.205 & 0.365 & -0.277 & -0.484 & -0.527 \\ -0.062 & 0.014 & -0.041 & -0.039 & 0.0144 & -0.103 & -0.106 & 0.189 & 0.171 & -0.038 \end{bmatrix} \quad (18)$$

Using the value of $K_{0.5}$, the characteristic equation of TRMS along with observer based reliable H infinity controller with uncertainty 0.5, is formed as in (19) and (20)

$$|sI - (A - B * K_{0.5})| = 0 \quad (19)$$

$$0.1s^{10} + 1.45s^9 + 10.65s^8 + 47.2s^7 + 169.1s^6 + 246.6s^5 + 270.7s^4 + 341.9s^3 + 173s^2 + 0.005s + 0.0005 = 0 \quad (20)$$

With uncertainty upper limit 1.27, the observer based reliable H infinity controller gain $K_{1.27}$ is given in (21)

$$K_{1.27} = \begin{bmatrix} 0.0005 & 0.0003 & 0.0007 & 0.0002 & 0.0001 & 0.0005 & 0.0009 & -0.0007 & -0.0012 & -0.0013 \\ -0.0002 & 0 & -0.0001 & -0.0001 & 0 & -0.0002 & -0.0003 & 0.0005 & 0.0004 & 0.0001 \end{bmatrix} \quad (21)$$

Using the value of $K_{1.27}$, the characteristic equation of TRMS along with observer based reliable H infinity controller is formed as in (22) and (23)

$$|sI - (A - B * K_{1.27})| = 0 \quad (22)$$

$$0.1s^{10} + 1.48s^9 + 11.84s^8 + 48.7s^7 + 138.9s^6 + 332.2s^5 + 422.1s^4 + 169.98s^3 + 45.77s^2 + 49.8s + 5.7 = 0 \quad (23)$$

For nominal TRMS model given in [8], the observer based reliable H infinity controller gainis computed as shown in (24)

$$K_N = \begin{bmatrix} 0.0004 & 0.0002 & 0.0006 & 0.0001 & 0.0001 & 0.0004 & 0.0008 & 0.0006 & -0.001 & -0.0011 \\ -0.0001 & 0 & -0.0001 & -0.0001 & 0 & -0.0002 & -0.0002 & 0.0004 & 0.0004 & -0.0001 \end{bmatrix} \quad (24)$$

The characteristic equation using nominal parameters of TRMS computed as in (25) and is given in (26)

$$|sI - (A - B * K_N)| = 0 \quad (25)$$

$$0.1s^{10} + 1.47s^9 + 10.9s^8 + 47.6s^7 + 142.8s^6 + 272.1s^5 + 302.1s^4 + 198.2s^3 + 105.8s^2 + 38.4s + 3.9 = 0 \quad (26)$$

(20) and (23) give interval polynomial for TRMS. Using this interval polynomial, Kharitonov's polynomials for TRMS are constructed for stability analysis as shown in (27) to (30).

$$K_{h1}(s) = 0.1s^{10} + 1.45s^9 + 10.65s^8 + 48.7s^7 + 138.8s^6 + 246.6s^5 + 270.7s^4 + 169.9s^3 + 45.8s^2 + 0.005s + 0.0005 = 0 \quad (27)$$

$$K_{h2}(s) = 0.1s^{10} + 1.47s^9 + 11.8s^8 + 47.2s^7 + 169.1s^6 + 332.2s^5 + 422.1s^4 + 341.9s^3 + 173s^2 + 49.8s + 5.74 = 0 \quad (28)$$

$$K_{h3}(s) = 0.1s^{10} + 1.45s^9 + 11.84s^8 + 48.8s^7 + 169.1s^6 + 246.6s^5 + 422.1s^4 + 169.9s^3 + 173s^2 + 0.005s + 5.74 = 0 \quad (29)$$

$$K_{h4}(s) = 0.1s^{10} + 1.47s^9 + 10.65s^8 + 47.2s^7 + 138.9s^6 + 332.2s^5 + 270.7s^4 + 341.9s^3 + 45.8s^2 + 49.8s + 0.0005 = 0 \quad (30)$$

Constructing Routh's array, Kharitonov'spolynomials[17][18], (27) to (30) are tested individually for its Hurwitz stability. For first Kharitonov's polynomial (27) Routh's table is constructed and result is demonstrated as shown in Table II.

Analysing the first column of theRouth's table, it is found that there is no sign change in first column of Routh's array, hence the Kharitonov's polynomial (27) is stable. Similarly Routh's test for other Kharitonov's polynomials (28) to (30) also gives the result that they are Hurwitz stable.

Table II: Routh's Table for First Kharitonov's Polynomial Formed for Closed Loop TRMS with the Controller

s^{10}	0.1	10.65	138.8	270.7	45.8	0.0005
s^9	1.45	48.7	246.6	169.9	0.005	
s^8	7.29	121.79	258.98	45.79	0.005	
s^7	24.48	195.1	160.79	0.0049		
s^6	63.68	211.09	45.79	0.0005		
s^5	113.95	143.19	0.0047			
s^4	131.07	45.79	0.0005			
s^3	103.37	0.0043				
s^2	45.79	0.0005				
s^1	0.0031					
s^0	0.0005					

Also the roots of four Kharitonov's polynomials (27) to (30) are found and are tabulated in Table III. All the Kharitonov's polynomials have roots which are negative which proves that the Kharitonov's polynomials constructed for TRMS do not have right hand side poles. Hence proves that closed loop TRMS along with observer based reliable H infinity controller for the variation in the coefficients of the characteristic polynomial as in (20) and (23) with the uncertainty bound 0.5 to 1.27 is stable. Thus Kharitonov's stability theorem has been successfully applied and verified for closed loop TRMS along with observer based reliable H infinity controller. The stability of TRMS demonstrates that it is robust stability since it maintains the stability under uncertainties like TRMS parameter variation mainly due to sensor actuator failure. The robust stability bound for TRMS is obtained which is discussed in section 4.

Table III: Roots of Kharitonov's Polynomials of TRMS

No.	Kharitonov's polynomials formed for TRMS	Roots of Kharitonov's polynomials formed
K_{h1}	$0.1s^{10} + 1.45s^9 + 10.65s^8 + 48.7s^7 + 138.8s^6 + 246.6s^5 + 270.7s^4 + 169.9s^3 + 45.8s^2 + 0.005s + 0.0005 = 0$	-2.0716 ±4.1491i -4.0056 -0.9310 ±1.0266i -1.9891 -1.6652 -0.8349 -0.0001 ± 0.0033i
K_{h2}	$0.1s^{10} + 1.47s^9 + 11.8s^8 + 47.2s^7 + 169.1s^6 + 332.2s^5 + 422.1s^4 + 341.9s^3 + 173s^2 + 49.8s + 5.74 = 0$	-5.5413± 5.3116i -0.3798± 3.5644i -0.5357± 0.8840i -0.4479± 0.4715i -0.6200 -0.2707
K_{h3}	$0.1s^{10} + 1.45s^9 + 11.84s^8 + 48.8s^7 + 169.1s^6 + 246.6s^5 + 422.1s^4 + 169.9s^3 + 173s^2 + 0.005s + 5.74 = 0$	-0.2638± 5.6789i -1.2575± 1.3719i -2.1484± 0.1672i -1.0976 -0.5901 -0.2364 ±0.3377i
K_{h4}	$0.1s^{10} + 1.47s^9 + 10.65s^8 + 47.2s^7 + 138.9s^6 + 332.2s^5 + 270.7s^4 + 341.9s^3 + 45.8s^2 + 49.8s + 0.0005 = 0$	-6.0258 -3.7645± 4.0395i -0.3681± 3.4325i -0.2494± 1.1083i -0.0051± 0.4198i -0.0001

4. Robust stability bound for TRMS

Referring to the characteristic polynomials of TRMS (20), (23) and (26), the coefficients of characteristic polynomial of TRMS are termed as 'a' as shown in (31). After doing the stability analysis on TRMS using Kharitonov's stability theorem the range of coefficients for TRMS are shown in (32).

$$a_{10}s^{10} + a_9 s^9 + a_8 s^8 + a_7 s^7 + a_6 s^6 + a_5 s^5 + a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0 = 0 \quad (31)$$

Where,

$$a_{10} = 0.1; \quad 1.45 < a_9 < 1.48; \quad 10.65 < a_8 < 11.84; 47.2 < a_7 < 48.7; 138.9 < a_6 < 169.1 \,; 246.6 < a_5 < 332.2; 270.7 < a_4 < 422.1; \; 169.98 < a_3 < 341.9 \,; \; 45.77 < a_2 < 173 \,; 0.005 < a_1 < 49.8 \text{ and } 0.0005 < a_0 < 5.7 \quad (32)$$

The nominal values of a's lie between $\underline{a}$ and $\overline{a}$ shown in (32). Table IV shows the range of TRMS model parameter variation which is robust stability bound of closed loop TRMS along with observer based reliable H infinity controller. The observer based H infinity controller is proved to be robust because it gives the same performance in absence and in presence of uncertainties like model parameter variation which occurs due to modelling errors or due to sensor, actuator failure. The closed loop step response of TRMS with the observer based H infinity controller with and without failure of sensor, actuator failure is shown in Fig. 5 and Fig. 6. The pitch output of TRMS is shown in Fig. 5. The yaw output of TRMS is shown in Fig. 6. At time t=40s and t=60s the sensor failure and actuator failure occur respectively during which the TRMS model characteristic polynomial parameters would reach extreme bound which is shown as min and max in Table IV. It is observed that even under failure conditions of sensor, actuator both pitch output and yaw output of TRMS are stable. If the characteristic polynomial coefficients are taken out of range of extreme bound mentioned in (32), the pitch output and yaw output are going to be unbounded and hence TRMS will be unstable which is shown in Fig. 7 and Fig. 8. Therefore the range of TRMS model parameter variation given in Table IV is the robust stability bound for TRMS beyond which the TRMS loses its stability.

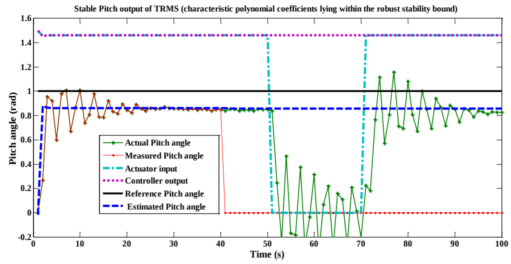

Fig. 5: TRMS pitch output under failure of sensor, actuator (Characteristic polynomial lying within the robust stability bound as shown in (32))

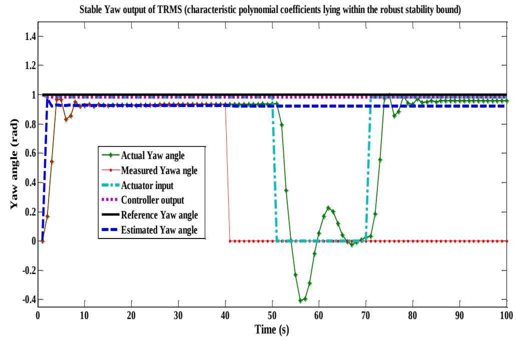

Fig. 6: TRMS yaw output under failure of sensor, actuator (Characteristic polynomial coefficients lying within the robust stability bound as shown in (32))

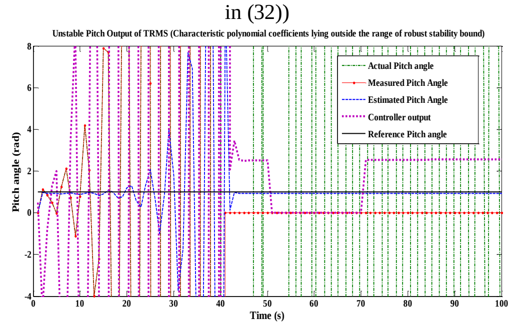

Fig. 7: TRMS pitch output under failure of sensor,actuator(Characteristic polynomial coefficients lying outside the range of robust stability bound)

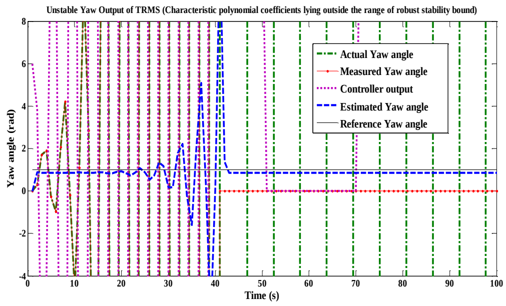

Fig. 8: TRMS yaw output under failure of sensor, actuator(Characteristic polynomial coefficients lying outside the range of robust stability bound)

Table IV: Range of TRMS Model Parameter Variation (Robust Stability Bound for TRMS)

Parameter		Minimum	Nominal	Maximum
TRMS Characteristic polynomial	(coefficients of s^{10})	0.1	0.1	0.1
	(coefficients of s^{9})	1.45	1.47	1.48
	(coefficients of s^{8})	0.45	10.99	11.84
	(coefficients of s^{7})	47.2	47.6	48.7
	(coefficients of s^{6})	138.9	142.8	169.1
	(coefficients of s^{5})	246.6	272.1	332.2
	(coefficients of s^{4})	270.7	302.1	422.1
	(coefficients of s^{3})	169.98	198.2	341.9
	(coefficients of s^{2})	45.77	105.8	173
	(coefficients of s^{1})	0.005	38.4	49.8
	(coefficients of s^{0})	0.0005	3.9	5.7

5. Conclusion

In this paper, the closed loop stability of TRMS with observer based reliable H infinity controller is analyzed using Kharitonov's stability theorem to find out its stability bound. The variation in uncertainty range means the variation in the physical parameters of TRMS which reflects in the variation of TRMS model. This will result in variation of transfer function coefficients of TRMS. This is reflected in the change in coefficients of its characteristic polynomial. Varying uncertainties is achieved by varying the amount of percentage failure of sensor and actuator of TRMS. The extreme values for the coefficients of characteristic polynomial of TRMS are obtained using Kharitonov's stability theorem for which TRMS maintains its stability. In this paper the demonstration of finding robust stability bound of TRMS is done. That is the extreme uncertainty limits beyond which TRMS loses its closed loop stability has been found.

References

[1] Tesfey M, Degang C & Brad P, "Applications of Kharitonov's Theorem to mechanical systems", *IEEE transactions on Automatic control*, Vol.38, No.3, (1993), pp.488-491.

[2] Mondal S & Mahanta C, "Adaptive second order sliding mode controller for twin rotor multiple input and multiple output system", *IET Control Theory and Applications*, Vol.6, No.14, (2012), pp.2157-2167.

[3] Jung-Hua Y & Wen Chun H, "Adaptive back stepping control for electrically driven unmanned helicopter", *Control Engineering Practice*, Vol.17, No.8, (2009), pp.903-913.

[4] Rajan PK & Reddy HC, "An alternate circuit theoretic proof for Kharitonov's stability criterion", *IEEE*, (1989), pp.1784-1787.

[5] Kharitonov VL, "Asymptotic stability of an equilibrium position of a family of systems of linear differential equations", *Differential equations*, Vol.14, (1979), pp.1483-1485.

[6] Vidya S.R, George VI, Surekha K, Shreesha C, "Implementation of Reliable H infinity Observer-Controller for TRMS with sensor and actuator failure", *10th Asian Control Conference, Malaysia*, (2015).

[7] Sonlin H, Dong Y, Jinliang L & Zhaoping D, "Robust H_∞control for networked systems with parameter uncertainties and multiple stochastic sensors and actuators faults", *International Journal of Innovative computing Information and Control*, Vol.8, No.4, (2012), pp.2693-2704.

[8] Vidya SR, George VI, Surekha K & Shreesha C, "Performance Evaluation of Reliable H infinity observer controller with robust PID controller designed for TRMS with sensor, actuator failure", *Far East Journal of Electronics and communication*, Vol.16, No.2, (2016), pp.355-380

[9] Shen Y, Huijun G, "Descriptor reduced order sliding mode observers design for switched systems with sensor and actuator faults", *Automatica*, Vol.76, (2017), pp.282-292.

[10] Gene, FF, *Digital Control of Dynamic Systems*, Pearson Education, Third edition, (2003).

[11] Joaquimblesa, "FDI and FTC of wind turbines using the interval observer approach and virtual actuators/sensors", *Control Engineering Practice*, Vol.24, (2014), pp.138-155.

[12] Minnichelli RJ, "An elementary proof of Kharitonov's stability theorem with extensions", *IEEE Transactions on Automatic control*, Vol.34, No.9, (1989).

[13] Vidya SR, George VI, Surekha K, "Comparison of Kalman Observer and H infinity Observer designed for TRMS", *International Journal of Control and Automation*, Vol.9, No.10, (2016), pp.275-292.

[14] Xin Y, Ye Y, Zhiqiang L, Jorge G & Patrick RP, "Robust Stability Analysis of Active Voltage Control for High-power IGBT Switching by Kharitonov's Theorem", *IEEE Transactions on Power Electronics*, (2015).

[15] Mahmud Iwan S.W, Ari L & Rini A, "Robust PID Anti-Swing control of Automatic gantry crane based on Kharitonov's stability", *Proc. 4th IEEE conference on Industrial Electronics and Applications*, (2009), pp.275-280.

[16] Long W, "Edge Theorems for MIMO systems", *IEEE Transactions on Circuits and Systems-I: Fundamental Theory and Applications*, Vol.50, No.12, (2003), pp.1577-1580.

[17] Toulabi MR, "Robust analysis and design of power system load frequency control using Kharitonov's theorem", *Electrical Power and Energy Systems*, Vol.55, (2014), pp.51-58.

[18] Yogesh VH, Gupta JRP & Roy Choudhury D, "Kharitonov's Theorem and Routh Criterion for Stability Margin of Interval Systems", *International Journal of Control Automation and Systems*, Vol.8, No.3, (2010), pp. 647-654.