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***Research paper***

# Analysis of modern modeling methods in problems of stabilization of motions of mechatronic systems with differential constraints

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**Abstract**

The most important problem of controlling mechatronic systems is the development of methods for the fullest possible application of the properties of our own (without the application of controls) motions of the object for the optimal use of all available resources. The basis of this can be a non-linear mathematical model of the object, which allows to determine the degree of minimally necessary interference in the natural behavior of an object with the purpose of stable implementation of a given operating mode. The operating modes of the vast majority of modern mechatronic systems are realized due to the steady motions (equilibrium positions and stationary motions) of their mechanical components, and often these motions are constrained by connections of various kinds. The paper gives an analysis of methods for obtaining nonlinear mathematical models in stabilization problems of mechanical systems with differential holonomic and non-holonomic constraints.

***Keywords***: *differential constraints; stability; stabilization; vector-matrix equations.*

## 1. Introduction

Mechatronic systems, due to the availability of advanced information processing systems in the control loop, have the potential to implement in the current time algorithms of control of almost any complexity. Thus, there are prerequisites for creating control methods that can more fully use the properties of their natural, free (in the absence of controls) behavior when implementing the specified mode of operation of these devices. Such organization of their functioning could both reduce the number of actuators involved and reduce the amount of measurement information needed to form a control, simplifying the structure of the control system and thereby increasing its reliability. This requires the development of general methods for obtaining, by rigorous methods of analytical mechanics, such nonlinear mathematical models of mechanical components of mechatronic systems that would allow a much more detailed (in comparison with the existing ways of obtaining models) analysis of the structure of not only the linear approximation but also the nonlinear terms of the equations of motion.

For the operating modes of most modern mechatronic systems, steady motions (equilibrium positions and stationary motions) take place in their mechanical components, and often these motions are limited to the constraints of different kinds. The question of the sustainable implementation of this modes of operation is the question of the operability of any automatic control system. The stability of stationary motions of systems with homogeneous differential constraints is possible only in critical cases, when the characteristic equation of the first approximation system necessarily has roots on the imaginary axis.

Differential constraints in mechanical systems are of two types: non-holonomic and differentiated geometric (holonomic).

## 2. Non-holonomic mechanical systems

The first class of mechanical systems with differential constraints - non-holonomic mechanical systems - is a special class of mechanical systems, limited non-integrable constraints that impose conditions on the speed of the system without imposing conditions on the position of the system [1-4]. Non-holonomic systems serve as models of many technical objects, not just mechanical ones, but also electromechanical ones. In connection with the development of robotics and mechatronics, the study of the dynamics, stability and stabilization of non-holonomic mechanical systems, which are the main models of various wheeled [5-8], and not only wheeled [9-12] robots, are becoming increasingly relevant. The structure of the equations of motion of nonholonomic systems has its own peculiarities, which considerably complicates the study of their dynamics. Therefore, the study of the various theoretical problems of non-holonomic systems has been and remains a great interest and is the subject of research by many specialists (see, for example, [13], with a larger list of literature, and also [14] -18]).

First of all, it is necessary to establish whether the steady-state motions (as a rule, operating modes of many technical objects with non-holonomic constraints) considered are stable and what is the nature of this stability. However, in many cases, these motions either do not possess the necessary stability properties, or (in the case of stability) the nature of the transient processes does not satisfy the specified technical requirements. Thus, it naturally becomes necessary to solve the stabilization problem stabilization to stability or asymptotic stability in all or part of variables.

The study of the stability of steady motions of non-holonomic systems presents considerable difficulties. It suffices to point out that the steady motions of nonholonomic systems with homogeneous constraints are not isolated, but are located on manifolds of dimensions not less than the number of nonintegrable constraints. The steady motions of systems with inhomogeneous constraints can be either isolated or located on manifolds. If for holonomic systems such coordinates are cyclic called, on which the Lagrange function of the system does not depend, then for non-holonomic systems there are several definitions [1,4,19-23]. In this case, the problems of searching for stationary motions and integrals of the equations of motion in the case of nonholonomic systems are not closely related [19, 21]; the equations of motion of the system may not admit cyclic integrals, but admit a stationary motion. As a consequence, for non-holonomic systems various statements of stability problems (conditional or unconditional stability, separate steady motion or the whole variety of such motions as a whole, etc.) are possible.

All this causes much more variety of possible types of perturbed motion equations than for holonomic systems, and the analytical difficulties of their obtaining equations, the cumbersomeness of these equations and the ensuing complexity of their transformation and analysis of their structure significantly increase in comparison with analogous problems for holonomic systems. Therefore, the study of stability and stabilization of steady motions of non-holonomic systems, despite the efforts of numerous researchers, continues to be a complex abstract-theoretical problem, the study of which is far from complete.

In the study of stationary motions of non-holonomic systems, two approaches are usually used. The first of them [13, 24-27] allows us to give a complete analysis of the problem if the linear integrals corresponding to the symmetries of the system are known in explicit form. The second approach is more universal, using the theory of critical cases of Lyapunov-Malkin-Kamenkov [28-30]. It will be discussed in this article. In this approach, a detailed analysis of the structure of the nonlinear terms is required, which is associated with complex analytic transformations. When using the equations of motion in scalar form, such a procedure becomes extremely laborious. Much more convenient is the use of vector-matrix equations.

Vector-matrix equations of disturbed motion are widely used [13, 31-55] to consider the problems of stability and stabilization of motions of complex mechanical systems. It should be noted that the systematic application of such a form of equations for the study of the dynamics of nonholonomic systems began quite a long time [34-38] (compare [13, 39-41]). In the method developed by the authors of this article, unlike [13,39-41], the equations have a form that allows one to analyze in detail the structure of the linear and nonlinear terms of the perturbed equations of motion after replacing the theory of critical cases. Therefore, the use of the equations obtained in [34-38, 42-56] is not limited (compare [13, 39-41]) only for locating the roots of the characteristic equation or for studying controllability and observability (compare [13, 40]), but also from the point of view of the theory of critical cases [28-30], the structure of nonlinear terms is analyzed.

Since the method developed by the authors is oriented toward automating the process of studying the dynamics of complex systems, perhaps of a wider class, the P.V. Voronets equations in Routh variables were originally chosen to describe the system, in contrast to, for example, [13, 40]. The choice of these variables is due to the fact that they use rather general and well established procedures. For example, it was proved that the use of Routh variables is more advantageous not only for studying the dynamics of systems with cyclic coordinates. The advantages of using variables of this type were found [36] when studying the stability problems of such equilibrium positions of nonholonomic systems without cyclic coordinates in the neighborhood of which the number of roots of the characteristic equation with zero real parts is greater than the number of non-holonomic constraints.

Equations in the form of Voronets not only have a general character, but also are one of the simplest forms of the equations of motion and in principle can describe the dynamics of any mechanical system with arbitrary linear differential constraints. With the chosen solution method [42], the Routh variables are most convenient for the considerated problems, since it is in these variables, under certain conditions, that the equations of perturbed motion immediately obtain a special form [28-30] for the theory of critical cases. This, in turn, facilitates the reverse transition from a special form of the theory of critical cases to the original equations of perturbed motion in the original variables. In addition, the application of the results of mathematical control theory is significantly simplified in comparison with Lagrangian variables. This is due to the fact that the equations for the Hamiltonian variables are resolved with respect to the derivatives, i.e. immediately have a normal form used in control theory.

It should be emphasized that the developed form of the vector-matrix equations will allow us to consider stabilization problems for nonholonomic systems that differ significantly from known ones (for example, [13, 40]). It is possible to consider the problem of ensuring in the general case of non-asymptotic stability of unperturbed motion by applying linear control with the maximum possible use of the properties (possibly, non-asymptotic and conditional) of the stability of the system's own motions to reduce the dimension of control, simplify its structure, and reduce the volume of measurement information. Moreover, using [56], the problem of estimating the minimum dimensionality of the measurement vector is possible to investigate, which provides the estimates of the phase vector for the formation of such a control.

The use of P.V. Voronets equations for modeling the dynamics of nonholonomic systems in the proposed method is also justified by the fact that all the results that will be obtained for nonholonomic systems can easily be transferred to systems with geometric constraints. In the case of integrability of differential constraints, the Voronets equations become equations in the form of M.F. Shulgin [4] for holonomic systems with redundant coordinates. But, unlike non-holonomic constraints, geometric constraints impose conditions not only on the velocity, but also on the coordinates, including the initial perturbations. The resulting features of stability and stabilization problems for steady-state motions of systems with geometric constraints require special careful study.

## 3. Holonomic mechanical systems

The second class of mechanical systems with differential constraints are systems with geometric constraints. In many problems of modern technical practice (in particular, the management of multi-link manipulators [57-59] and other mechatronic systems) it is advisable [2-4,58,60] to specify the configuration of a mechanical system with parameters taken in a number exceeding the necessary n-number of degrees of freedom system. Then m of these n + m parameters are called redundant coordinates. Between n + m parameters there are m independent relations (which, generally speaking, can also contain time). The exclusion of unnecessary dependent coordinates from these expressions often leads to cumbersome formulas, especially when trigonometric functions are present in the constraint equations ([4], pp. 20-21; [45,46,48]).

The use of redundant coordinates requires another formulation of the dynamics problems [2-4]. There is not possible to use the Lagrange equations of the second kind, since the derivation them are assumed the introduction of independent generalized coordinates whose variations will also be independent. For the systems under consideration, one can use the Lagrange equations of the first kind in Cartesian coordinates or the equations with Lagrange constraints multipliers in redundant curvilinear coordinates [2-4,7]. Another direction involves the elimination of multipliers. In particular, in [3], (pp. 328-31) is proposed the following approach to eliminating joining factors: it is proposed to differentiate the equations of geometric constraints twice, and differentiate the equations of differential constraints, and then to substitute in these conditioned equations the accelerations as linear functions of the joining factors from the equations of motion. Joining factors are found from the obtained linear algebraic non-homogeneous system of equations. In [14], this method of eliminating factors is developing with respect to a new class of problems, and it is noted ([14], p. 27) that analytic expressions for the factors were first obtained and investigated by G.K. Suslov [3] and A.M. Lyapunov [61].

A fundamentally different way of obtaining the equations of motion of systems with redundant coordinates without joining factors, connected with a single differentiation of the bonds, was developed by M.F. Shulgin [4]. It is this form of these equations that, in our opinion, is one of the most suitable for the study of the dynamics of systems with redundant coordinates, especially for the study of problems of stability and stabilization of steady motions of systems with geometric constraints. The application of the Lagrange equations with constraints factors leads [2-4,58,61] to an increase in the dimension of the problem. An alternative way is to go over to time-differentiated equations of geometric constraints and use of equations free from joining factors in the form of M.F. Shulgin [4]. The equations of M.F. Shulgin do not contain unknown joining factors, do not require their determination, and therefore are convenient for investigating stability problems. The procedure for obtaining them is based on the exclusion of dependent velocities from the kinetic energy by means of differentiated constraint eq-uations. These equations can be derived from the equations of motion of nonholonomic systems [1] in the form of P.V. Voronets in the case of integrability of non-holonomic constraints. Using equations in the Shulgin form, it is possible to obtain nonlinear mathematical models of holonomic systems with redundant coordinates with strict account of the nonlinear equations of geometric constraints.

It should be noted that a theoretical study of the stability of systems with geometric constraints, as systems occupying an intermediate position between holonomic and nonholonomic systems, has been launched relatively recently [45-48,51-52,54,55], is a complex problem requiring the application of the results of the theory critical cases [28-30].

In contrast to nonholonomic systems, in spite of the formal reduction of the problem to the special case of the critical case of zero roots, the number of which is equal to the number of differentiated geometric bonds, the equilibrium variety is absent due to the presence of these geometric constraints. For the equilibrium position of systems with redundant coordinates, the possibility of asymptotic stability arises. A theorem on conditional asymptotic stability in the sense of the classical definition of Lyapunov (conditional - due to the conditions imposed by geometric constraints on the initial perturbations) is proved [45]. This theorem was used in [47] in the problem of the stability of stationary motions. In this case, we consider the case when the cyclic coordinates belong to the number of coordinates taken as independent coordinates. Depending on the structure of the acting forces and the type of kinematic bonds, different definitions of the cyclic coordinates are possible, as in the case of nonholonomic systems. For one of the cases the theorem on the stability of the stationary motion of a system with redundant coordinates is proved. In this case, the dimension of the variety of stationary motions is equal to the number of cyclic coordinates.

The theoretical study begun by the authors of the article [45-48,51-52,54,55] on the singularities of the application of vector-matrix equations in the form of M.F. Shulgin (with the equations of geo-metric constraints differentiated once in time) in the Routh and Lagrange variables to problems of stability and stabilization of steady motions of systems with geometric constraints is far from complete. At the same time, the results already obtained have shown their effectiveness in applying to the study of stabilization problems for real mechatronic systems. In particular, it was found that with the correct modeling of the GBB1005 Ball & Beam, the system has, in addition to the well-known, yet another not previously considered equilibrium with a non-zero steering angle of the drive wheel. It is shown that the characteristic equation of the first approximation essentially depends on the choice of the re-dundant coordinate. Moreover, in such an investigation, the method of N.N. Krasovskii simply determines the stabilizing control [62].

## 4. Conclusion

The problems proposed in this article are relevant both from a theoretical point of view and from the point of view of developing techniques for applying the results of abstract theoretical considerations to solving problems of controlling practically existing technical devices.

On the one hand, such problems represent [10] interest from the point of view of classical mechanics and control theory (that is, when a particular system with an explicit solution of the control problem is considered). On the other hand, the results of the project and the equations used can be used by engineers as a theoretical basis for further research that turns model tasks into complex mechatronic and robotic systems with different specifics (stabilization issues, feedbacks, the use of different sensors and other problems, up to artificial intelligence [63]).

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