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Research paper

On the stabilization problems of non-holonomic systems with inhomogeneous constraints with incomplete information on the state

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Abstract

The systematic application of Voronetz vector-matrix equations in Routh variables is developed to describe the dynamics of non-holonomic systems with inhomogeneous constraints. The obtained form of the equations of disturbed motion in problems of stability and stabilization makes it possible to analyze the structure of the model for a reasonable choice of the application of control actions. The developed technique makes it possible to automate the solution of problems of stabilization of operating modes of complex technical devices.

Keywords: *Inhomogeneous nonholonom constraints;, Voronets's equation; stability; stabilization.*

1. Introduction

Despite the fact that in recent decades the theory and methods for solving stabilization problems have been intensively developed, most of the known results pertain to the problem of stabilizing the motion under investigation to asymptotic stability with respect to all phase variables.

At the same time, for many practically important systems, there is no need for such stabilization, it is sufficient to ensure non-asymptotic stability of the unperturbed motion. In addition, for certain classes of steady-state motions, it is the non-asymptotic stability that is most easily achieved. This is the case, in particular, in problems of stability and stabilization of equilibrium positions, and in many cases - stationary motions of non-holonomic systems. Proceeding from this, the formulation of the problem of stabilization of steady motions to non-asymptotic stability and the development of rigorous methods for their solution with the fullest possible use of the properties of the stability of proper motions to reduce the dimension of the vector of stabilizing control and the volume of measuring information sufficient for its formation are relevant and important.

The steady motion of mechanical systems can be divided into two categories, for which the stabilization problems have essentially different character. To one of these we classify those motions which, in principle, can not be stabilized to asymptotic stability with respect to the first approximation in all variables. Such a situation occurs in problems of stabilizing the equilibrium positions of nonholonomic systems with linear homogeneous constraints (and, under certain conditions, also stationary motions of non-holonomic systems). In problems of this category, the preservation of critical variables after stabilization does not depend on our desire.

For another category of steady motions-stationary motions of holonomic systems, as well as equilibrium positions of non-holonomic systems with inhomogeneous constraints and (in the general case) stationary motions of such non-holonomic systems-in many cases, in principle, stabilization can be ensured up to asymptotic stability with respect to the first approximation with respect to all phase variables. But for many applied problems of this category it is sufficient to achieve non-asymptotic stability. In this case, it is possible to solve the problem with preservation (already at our will) after stabilizing some of the critical [1-3] variables. This approach makes it possible to reduce both the dimension of the control problem and the dimension of the estimation system.

In problems of the first category, the critical variables are the coordinates whose velocities are dependent on the strength of the constraint equations. Since the non-holonomic constraints in the problems considered here are not violated, and their equations, as a rule, have a rather simple form, there are no special problems connected with the construction of mathematical models of problems of this category.

When the preservation of critical variables is arbitrary enough, there is, generally speaking, the problem of the accuracy of constructing the mathematical model of the problem, connected with the choice of those variables that are left critical. The isolation of critical variables and the very possibility of solving the stabilization problem by the method developed here are determined in each particular case by the extent to which such a model is acceptable.

The application of nonlinear vector-matrix Voronetz equations in Routh variables [4-5] to the modeling of the dynamics of non-holonomic systems with inhomogeneous constraints is discussed in this paper. This form of the mathematical model allows us to analyze the structure of the equations of disturbed motion in the study of stability and can serve as the basis for the development of various methods of stabilizing steady-state motions. The use of methods of analytical mechanics is proposed to simplify the study of stability and stabilization due to the rational choice of the type of variables, forms of equations, linear subsystems and, accordingly, linear control actions. In the presence of zero roots, stability for a complete nonlinear system closed by the control found follows from the Lyapunov-Malkin theorem [2] when the system is reduced to a special case. The developed methodology allows the most complete use of stability properties with respect to some variables of the system itself and automates the solution of stabilization problems of steady motions in different software environments [6,7] for complex technical devices. The control law, and the coefficients of the estimation system for the systems with incomplete information, are found by solving the corresponding linearly quadratic problems by Krasovskii's method [8] for an isolated subsystem that does not include critical variables corresponding to zero roots.

2. Vector-matrix Equations of Motion in the general case.

We consider a nonholonomic system whose position is determined by the generalized coordinates $q_1, \ldots, q_n$, and the generalized velocities are connected by m non-integrable nonhomogeneous (in contrast to [9,10]) constraints

$$\dot{q}_\mu = B_{\mu\rho}(q)\dot{q}_\rho + B_{\mu r}(q)\dot{q}_r + B_\mu(q) \qquad (1)$$

Here and below, summation is carried out over repeated indices; Indices take the following values:

$$w = 1,\ldots,n;\ i, j, s = 1,2,\ldots,n-m;\quad \alpha,\rho = 1,2,\ldots,k;$$
$$r,t,u,v = k+1,\ldots,n-m;\ h = n-m+l+1,\ldots,n$$
$$\mu,\sigma,\tau = n-m+1,\ldots,n;\ \delta = n-m+1,\ldots,n-m+l;$$

Suppose that the system is under the action of potential forces with energy $\Pi(q)$ and nonpotential generalized forces $\tilde{Q}_w(q,\dot{q})$ (This may include control forces) related to the coordinates. Let us assume that in some open region of the phase space the potential energy, the coefficients in the expression for the kinetic energy and in the constraints equations are at least twice continuously differentiable with respect to q, and the nonpotential generalized forces are continuously differentiable with respect to $q_i, \dot{q}_i$ and the quadratic part of the kinetic energy Is a positive definite function of the velocities.

Suppose that the kinetic energy, the potential energy $\Pi(q)$ and the constraints coefficients $B_{\mu i}(q)$, $B_\mu(q)$ do not depend explicitly on time. Let the kinetic energy have the most general form (compare [9,10])

$$\tilde{T} = \tilde{T}_2 + \tilde{T}_1 + \tilde{T}_0 = \frac{1}{2}\dot{q}'\tilde{a}(q)\dot{q} + \tilde{d}'(q)\dot{q} + \tilde{T}^0(q);$$

In accordance with the different nature of the dependence of the kinetic and potential energies, the coefficients of the constraint equations, the nonpotential generalized forces, and also the type of connections (Chaplygin [11, 12] or not), the vector of generalized coordinates is divided into 4 vectors. It is possible to subdivide this vector into 5 vectors. The need to introduce the fifth vector component is due to the fact that the introduction of pulses along all the cyclic coordinates turned out to be not always beneficial [13].

$q' = (q_1,\ldots,q_n)$; $\alpha' = (q_1,\ldots,q_k)$; $\beta' = (q_{k+1},\ldots,q_{n-m})$; $s' = (q_{n-m+1},\ldots,q_n)$; $\delta' = (q_{n-m+1},\ldots,q_{n-m+l})$; $h' = (q_{n-m+l+1},\ldots,q_n)$; $q' = (\alpha', \beta', \delta', h')$;

In the matrices of the kinetic energy coefficients and the coefficients of the coupling equations, submatrices of the corresponding dimensions are distinguished, using which, using the known technique, after excluding the dependent velocities, one can introduce impulses and the Routh function

$$p = \frac{\partial T}{\partial \dot{\beta}} = a_{21}(q)\dot{\alpha} + a_2(q)\dot{\beta} + d_\beta(q);\ \dot{\beta} = -\gamma'(q)\dot{\alpha} + b(q)(p - d_\beta);$$
$$b(q) = a_2^{-1}(q);\ \gamma'(q) = b(q)a_{21}(q);\ R = R_2 + R_1 + R_0 = L^* - p'\dot{\beta};$$
$$R_2 = \frac{1}{2}\dot{\alpha}'a^*(q)\dot{\alpha} = \frac{1}{2}\dot{\alpha}'(a_1 - \gamma a_{21})\dot{\alpha};$$
$$L^* = L(q,\dot{\alpha},-\gamma'(q)\dot{\alpha} + b(q)(p - d_\beta));\quad R_1 = \left(d'_\alpha + \left(p' - d'_\beta\right)\gamma'\right)\dot{\alpha};$$
$$R_0 = T_0 - \Pi - \frac{1}{2}\left(p' - d'_\beta\right)b\left(p - d_\beta\right);$$
$$Q_w(q,\dot{\alpha},p) = \tilde{Q}_w(q,\dot{\alpha}; -\gamma'(q)\dot{\alpha} + b(q)(p - d_\beta));$$

Introducing standard expressions

$$\omega_{\mu i} = -\frac{\partial B_\mu}{\partial q_i} + B_\sigma\frac{\partial B_{\mu i}}{\partial q_\sigma} - B_{\sigma i}\frac{\partial B_\mu}{\partial q_\sigma};$$
$$\Omega_{\mu ji} = \frac{\partial B_{\mu j}}{\partial q_i} - \frac{\partial B_{\mu i}}{\partial q_j} + B_{\sigma i}\frac{\partial B_{\mu j}}{\partial q_\sigma} - B_{\sigma j}\frac{\partial B_{\mu i}}{\partial q_\sigma};$$

We obtain the Voronets equations in the Routh variables

$$\dot{q}_r = -\frac{\partial R}{\partial p_r};\quad \dot{p}_r = \frac{\partial R}{\partial q_r} + B_{\mu r}\frac{\partial R}{\partial q_\mu} + \left(\frac{\partial \tilde{T}}{\partial \dot{q}_\mu}\right)_{\dot{q}_\mu \to \dot{q}_i}\left(\Omega_{\mu ri}\dot{q}_i + \omega_{\mu r}\right) +$$
$$+ Q_r + B_{\mu r}Q_\mu;\ \frac{d}{dt}\frac{\partial R}{\partial \dot{q}_\rho} - \frac{\partial R}{\partial q_\rho} - B_{\mu\rho}\frac{\partial R}{\partial q_\mu} = \left(\frac{\partial \tilde{T}}{\partial \dot{q}_\mu}\right)_{\dot{q}_\mu \to \dot{q}_i}\left(\Omega_{\mu\rho i}\dot{q}_i + \omega_{\mu\rho}\right) + \qquad (2)$$
$$+ Q_\rho + B_{\mu\rho}Q_\mu;$$
$$\dot{q}_\mu = \left(B_{\mu\rho} - B_{\mu r}\gamma_{\rho r}\right)\dot{q}_\rho + B_{\mu r}b_{ru}\left(p_u - d_{\beta u}\right) + B_\mu.$$

Analytic difficulties in the construction of equations of perturbed motion, the cumbersome nature of these equations and the complications of their transformation and analysis of their structure for non-holonomic systems, as compared with analogous problems for holonomic systems, increase substantially. To a large extent, these complications are associated with the need to include the so-called non-holonomic terms in the equations of motion [11,12,14]:

$$\left(\frac{\partial \tilde{T}}{\partial \dot{q}_\mu}\right)_{\dot{q}_\mu \to \dot{q}_i}\left(\Omega_{\mu ji}\dot{q}_i + \omega_{\mu j}\right)$$

Carrying out this one of the most laborious procedures in the compilation of the equations of motion of non-holonomic systems, introducing some notation [13,15,16] and carrying out the necessary transformations (provided that the last *m-l* constraints are of the Chaplygin type), we can obtain vector-matrix Voronets equations:

$$\dot{\alpha} = \alpha_1;$$
$$a^*\dot{\alpha}_1 = -\Pi_\alpha - \gamma\dot{p} - \alpha'_1\Big[a_{(\alpha)}^* - a_{(\beta)}^*\gamma' + a_{(\delta)}^*\left(B_{11} - B_{12}\gamma'\right) - \frac{1}{2}a_{[\alpha]}^* -$$
$$\frac{1}{2}B_{11}a_{(\delta)}^* - \theta'_{\delta\alpha}\left(\Omega_{11}{}^\alpha - \Omega_{12}{}^\alpha\gamma'\right) - \theta'_{h\alpha}\left(\Omega_{21}{}^\alpha - \Omega_{22}{}^\alpha\gamma'\right)\Big]\alpha_1 - \alpha'_1\Big[\frac{\partial d_\alpha}{\partial \alpha} -$$
$$-\gamma\frac{\partial d_\beta}{\partial \alpha} - \frac{\partial d_\alpha}{\partial \beta}\gamma' + \gamma\frac{\partial d_\beta}{\partial \beta}\gamma' + \frac{\partial d_\alpha}{\partial \delta}(B_{11} - B_{12}\gamma') - \gamma\frac{\partial d_\beta}{\partial \delta}\left(B_{11} - B_{12}\gamma'\right) +$$
$$+ a_{(\delta)}^*B_\delta - d_{\alpha[\alpha]} - B'_{11}d_{\alpha[\delta]} + \gamma d_{\beta[\alpha]} + B'_{11}\gamma d_{\beta[\delta]} - \theta'_{\delta\alpha}\omega_{\delta\alpha} -$$
$$-\theta'_{h\alpha}\omega_{h\alpha} - \left(\Omega_{11}{}^\alpha - \Omega_{12}{}^\alpha\gamma'\right)s'_\delta - \left(\Omega_{21}{}^\alpha - \Omega_{22}{}^\alpha\gamma'\right)s'_h\Big] + \alpha'_1\Big[a_{(\beta)}^*b + \left(\gamma'_{(\alpha)}\right)' -$$
$$-\gamma\gamma'_{(\beta)} + (B'_{11} - \gamma B'_{12})(\gamma'_{(\delta)})' +$$

$$+a_{(\delta)}{}^{*}B_{12}b-\gamma_{[\alpha]}-B'_{11}\gamma_{[\delta]}-\theta'_{\delta\alpha}\Omega_{12}{}^{\alpha}-\theta'_{h\alpha}\Omega_{22}{}^{\alpha}-\left(\Omega_{11}{}^{\alpha}-\Omega_{12}{}^{\alpha}\gamma'\right)'\theta_{\delta\beta}-\left(\Omega_{21}{}^{\alpha}-\Omega_{22}{}^{\alpha}\gamma'\right)'\theta_{h\beta}\Big](p-d_{\beta})-$$
$$[d_{\alpha(\beta)}b-d_{\beta(\beta)}\gamma b+d_{\alpha(\delta)}B_{12}b+\gamma_{(\delta)}B_{\delta}-\gamma d_{\beta(\delta)}B_{12}b-d_{\beta[\alpha]}b-B'_{\alpha}d'_{\beta[\delta]}b-\left(s'_{\delta}\Omega_{12}{}^{\alpha}+s'_{h}\Omega_{22}{}^{\alpha}\right)b-$$
$$-\omega'_{\delta\alpha}\theta_{\delta\beta}-\omega'_{h\alpha}\theta_{h\beta}](p-d_{\beta})-\left(p'-d'_{\beta}\right)[\frac{1}{2}b_{[\alpha]}+\gamma_{(\beta)}b+\gamma_{(\delta)}B_{12}b-\frac{1}{2}B'_{11}b_{(\delta)}-\left(\theta'_{\delta\beta}\Omega_{12}{}^{\alpha}+\theta'_{h\beta}\Omega_{22}{}^{\alpha}\right)b]\cdot$$
$$\cdot(p-d_{\beta})+\frac{\partial T_0}{\partial\alpha}+B'_{11}\frac{\partial T_0}{\partial\delta}-d_{\alpha(\delta)}B_{\delta}-B'_{11}\Pi_{\delta}+s'_{\delta}\omega_{\delta\alpha}+s'_{h}\omega_{h\alpha}+Q_{\alpha}+B'_{11}Q_{\delta}+B'_{21}Q_{h}$$
$$\dot{\beta}=-\gamma'\alpha_1+b\left(p-d_{\beta}\right);$$
$$\dot{p}=-\Pi_{\beta}+\left[d'_{b[\beta]}+B'_{12}d'_{\beta[\delta]}+\left(s'_{\delta}\Omega_{12}{}^{\beta}+s'_{h}\Omega_{22}{}^{\beta}+\omega'_{\delta\beta}\theta_{\delta\beta}-\omega'_{h\beta}\theta_{h\beta}\right)b\right]\left(p-d_{\beta}\right)+$$
$$\alpha'_1\cdot\left[\frac{1}{2}a_{[\beta]}{}^{*}+\frac{1}{2}B'_{12}a_{[\delta]}{}^{*}+\theta'_{\delta\alpha}\left(\Omega_{11}{}^{\beta}-\Omega_{12}{}^{\beta}\gamma'\right)+\theta'_{h\alpha}\left(\Omega_{21}{}^{\beta}-\Omega_{22}{}^{\beta}\gamma'\right)\right]\alpha_1+$$
$$+\alpha'_1[d_{\alpha[\beta]}-\gamma d_{\beta[\beta]}++B'_{12}\left(d_{\alpha[\delta]}-\gamma d_{\beta[\delta]}\right)+\theta'_{\delta\alpha}\omega_{\delta\beta}+\theta'_{h\alpha}\omega_{h\beta}+\left(\Omega_{11}{}^{\beta}-\Omega_{12}{}^{\beta}\gamma'\right)'s'_{\delta}+ \quad (3)$$
$$+\left(\Omega_{21}{}^{\beta}-\Omega_{22}{}^{\beta}\gamma'\right)s'_{h}+(\gamma'_{[\beta]}+B'_{\beta}\gamma'_{[\delta]}+B'_{12}\frac{\partial T_0}{\partial\delta}+Q_{\beta}+B'_{12}Q_{\delta}+B'_{22}Q_{h}$$
$$\left(\theta'_{\delta\alpha}\Omega_{12}{}^{\beta}+\theta'_{h\alpha}\Omega_{22}{}^{\beta}+\left(\Omega_{11}{}^{\beta}-\Omega_{12}{}^{\beta}\gamma'\right)\theta_{\delta\beta}-\left(\Omega_{21}{}^{\beta}-\Omega_{22}{}^{\beta}\gamma'\right)'\theta_{h\beta})b\right)\left(p-d_{\beta}\right)-$$
$$-(p'-d'_{\beta})\left[\frac{1}{2}b_{[\beta]}+\frac{1}{2}B'_{12}b_{[\delta]}-\left(\theta'_{\delta\beta}\Omega_{12}{}^{\beta}+\theta'_{h\beta}\Omega_{22}{}^{\beta}\right)b\right]\left(p-d_{\beta}\right)+s'_{\delta}\omega_{\delta\beta}+$$
$$+s'_{h}\omega_{h\beta}+\frac{\partial T_0}{\partial\beta}-B'_{12}\Pi_{\delta}$$

$$\dot{\delta}=\left(B_{11}-B_{12}\gamma'\right)\alpha_1+B_{12}b\left(p-d_{\beta}\right)+B_{\delta};$$
$$\dot{h}=\left(B_{21}-B_{22}\gamma'\right)\alpha_1+B_{22}b\left(p-d_{\beta}\right)+B_{h}.$$

Equations (3) are the equations of motion of nonholonomic systems with inhomogeneous constraints in the Voronetz form in the Routh variables. These equations are deduced in the general case without assumptions about the presence of cyclic coordinates for systems with the most general form of kinetic energy under the action of potential forces with energy $\Pi(q)$ and arbitrary nonpotential generalized forces that do not violate the conditions of existence and uniqueness theorems for solutions of differential equations.

3. Vector-matrix equations of perturbed motion in a neighborhood of equilibrium.

From equations (3) it is possible to obtain relations

$$-\Pi_{\alpha}+\frac{\partial T_0}{\partial\alpha}+B'_{11}\frac{\partial T_0}{\partial\delta}-d_{\alpha(\delta)}B_{\delta}-B'_{11}\Pi_{\delta}+s'_{\delta}\omega_{\delta\alpha}+s'_{h}\omega_{h\alpha}+Q_{\alpha}+B'_{11}Q_{\delta}+B'_{21}Q_{h}=0$$
$$-\Pi_{\beta}+\frac{\partial T_0}{\partial\beta}+B'_{12}\frac{\partial T_0}{\partial\delta}-d_{\beta(\delta)}B_{\delta}-B'_{12}\Pi_{\delta}+s'_{\delta}\omega_{\delta\beta}+s'_{h}\omega_{h\beta}+Q_{\beta}+$$
$$+B'_{12}Q_{\delta}+B'_{22}Q_{h}=0;\ B_{\delta}=0;\quad p^{0}=d_{\beta}{}^{0};$$

for the determination of equilibrium positions States of equilibrium

$$\alpha=\alpha_0=const;\ \beta=\beta_0=const;\ p=p_0=const;\ \delta=\delta_0=const \quad (4)$$

According to this, the equilibrium positions of non-holonomic systems with inhomogeneous bonds can be isolated. Consequently, in the problem of stability of the point (4) with respect to all coordinates and independent velocities (unlike systems with homogeneous constraints), asymptotic stability with respect to the first approximation is possible. Introducing perturbations,

$$\alpha=\alpha_0+x;\ \beta=\beta_0+x_2;\ \alpha_1=x_1;\ \delta=\delta_0+z;\ p=p_0+y;$$

one can obtain the equations of perturbed motion with the first approximation selected in the neighborhood of equilibrium (4):

$$\dot{x}=x_1$$
$$a^{*\prime 0}\dot{x}_1=-(C_1+P_1+H_1)x-(D_1+G_1+H_2)x_1-(P_{12}+H_3)x_2-$$
$$-(K_{12}+H_4)y--(Z_1+H_5)z-\gamma^{0}\dot{y}+N_1 \quad (5)$$
$$\dot{x}_2=-\gamma'^{0}x_1-P_2x-P_3x_2-K_2y-P_4z+N_2$$
$$\dot{y}_2=-(C_2+P_{22}+H_{22})x_2-(P_{21}+H_{21})x-(K_{21}+H_{24})x_1-$$
$$(D_2+G_2+H_{25})y-(Z_2+H_{26})z+N_3$$
$$\dot{z}=W_1x+W_2x_1+W_3x_2+W_4y+W_5z+Z^{(2)}$$

Here, the coefficient matrices for linear terms can be expressed in a known manner [5,13,15] in terms of the parameters of the system. Thus, in the absence of external non-potential generalized forces, the first approximation of the equations of the perturbed motion of nonholonomic systems with inhomogeneous constraints in a neighborhood of the equilibrium position can contain terms that have the character of linear non-potential positional, dissipative accelerating, and gyroscopic forces. In the equations for perturbations of coordinates whose velocities are dependent, linear terms over all phase variables can be present.

Hence it is obvious that in the case of instability of equilibrium (4) there are sufficiently broad possibilities for its stabilization. Control actions can be applied both in coordinates α and in β. In systems with differential constrains there is the possibility of stabilizing unstable motions by means of controls acting on coordinates δ, h, corresponding to the dependent velocities. The corresponding sufficient conditions for the solvability of stabilization problems can be obtained starting from the structure of equations (5), similarly to the statements of the papers [17-19], where a large set of theorems on the stabilization of unperturbed motion is proved, including with incomplete information on the state.

For example, we consider the case of the action of controls on a part of the coordinates corresponding to independent velocities. We rewrite the first approximation of the system of equations of the perturbed motion (5) in the form

$$\dot{\xi}=\tilde{P}\xi+\tilde{Q}u;\quad \xi'=\left(x',x'_1,x'_2,y',z'\right);$$

$$\breve{P}=\begin{pmatrix}0 & E_k & 0 & 0 & 0\\ p_{11} & p_{12} & p_{13} & p_{14} & p_{15}\\ p_{21} & p_{22} & p_{23} & p_{24} & p_{25}\\ p_{31} & p_{32} & p_{33} & p_{34} & p_{35}\\ W_1 & W_2 & W_3 & W_4 & W_5\end{pmatrix}$$

$$\tilde{Q}'=\left(0,-\gamma^{0/}a^{*_0-1},0,E_{n-m-\kappa},0\right)$$
$$p_{11}=-a^{*_0-1}\left(C_1+P_1+H_1-\gamma^{0}\left(P_{21}+H_{21}\right)\right)$$
$$p_{12}=a^{*_0-1}\left(D_1+G_1+H_2-\gamma^{0}\left(K_{21}+H_{24}\right)\right)$$
$$p_{13}=-a^{*_0-1}\left(P_{12}+H_3-\gamma^{0}\left(C_2+P_{22}+H_{22}\right)\right)$$
$$p_{14}=-a^{*_0-1}\left(K_{12}+H_4-\gamma^{0}\left(D_2+G_2+H_{25}\right)\right)$$
$$p_{15}=-a^{*_0-1}\left(Z_1+H_5-\gamma^{0}\left(Z_2+H_{26}\right)\right)$$

$$p_{21}=-P_{21},\ p_{22}=-\gamma^{0'},\ p_{23}=-P_3,\ p_{24}=-K_2,\ p_{25}=-P_4$$
$$p_{31}=-P_{21}-H_{21},\ p_{32}=-K_{21}-H_{24},\ p_{35}=-Z_2-H_{26}$$
$$p_{33}=-C_2-P_{22}-H_{22},\ p_{34}=-D_2-G_2-H_{25}$$

A sufficient condition for the stabilizability of the equilibrium (4) of a non-holonomic system with inhomogeneous constraints to asymptotic stability with respect to the first approximation with respect to all phase variables, according to the structure of system (5) is the condition

$$rank\left(\tilde{Q}\tilde{P}\tilde{Q}\ldots\tilde{P}^{2n-3m+\ell}\tilde{Q}\right)=2(n-m)+\ell$$

if the controls

$$U=m_1x+m_2x_1+m_3x_2+m_4y+m_5z$$

are applied over the whole vector β.

In the case of stabilization of the equilibrium (4) of a non-holonomic system with inhomogeneous constraints (1) by applying linear equations with respect to coordinates whose velocities are dependent on general relations by a sufficient condition for stabilizability to asymptotic stability in the first approximation with respect to all the phase variables according to and the structure of the system (5) is the condition

$$rank\left(\tilde{Q}_1 \tilde{P} Q_1 \ldots \tilde{P}^{2(n-m)+\ell-1} \tilde{Q}_1\right) = 2(n-m) + \ell$$
$$\tilde{Q}_1' = \left(0,\ B_{11}^{0/} a^{*_0 -1},\ B_{12}^{0/},\ 0\right)$$

The matrices of the stabilizing control

$$U = m_6 x + m_7 x_1 + m_8 x_2 + m_9 y + m_{10} z$$

an be determined unambiguously by setting a quadratic quality criterion and solving a problem optimal in the sense of a minimum of this stabilization criterion.

Let us discuss the question of the amount of measurement information sufficient for the formation of stabilizing influences in the form of feedback on the estimates of the state vectors (or their parts) of the corresponding systems.

After the replacement of Aizerman-Gantmakher, the first approximation of the system of equations of the perturbed motion will take the form (we retain the previous notation for the components of the matrices of the coefficients of linear terms different from zero):

$$\dot{\xi} = \breve{P}_1 \xi + Qu$$

$$\breve{P}_1 = \begin{pmatrix} & 0 \\ \breve{P}_2 & p_{15} \\ & p_{25} \\ & p_{35} \\ 0 & 0 \end{pmatrix},\ \breve{P}_2 = \begin{pmatrix} 0 & E_\kappa & 0 & 0 \\ p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{pmatrix}$$

If for a nonholonomic system with inhomogeneous constraints (1) in a neighborhood of the equilibrium position (4) satisfy conditions

$$rank\left(Q_2 \breve{P}_2 Q_2 \ldots \breve{P}^{2(n-m)-1} Q_2\right) = 2(n-m)$$

$$rank\left(p_{13} p_{14}\right) = 2(n - m - \kappa)$$

$$Q'_2 = \left(0,\ \gamma^{0/} a^{*_0 -1},\ 0,\ E_{n-m-k}\right)$$

The equilibrium position (4) is stabilized to asymptotic stability in terms of velocities and stability with respect to the remaining variable applications with respect to the coordinates β of the linear control $u = M\hat{\xi}^1$, where the matrix is determined by solving the problem of optimal stabilization of the zero solution of the subsystem

$$\dot{\xi}^1 = \breve{P}_2 \xi^1 + Q_2 u\ ;\ \ \xi^{1/} = (x', x_1', x_2', y')$$

The estimate $\hat{\xi}^1$ obtained for the state vector of the subsystem from the estimation system

$$\dot{\hat{\xi}}^1 = \breve{P}_2 \hat{\xi}^1 + L(\sigma \xi^1 - \sigma \hat{\xi}^1) + Q_2 u$$

$$\sigma = (E_k, 0, 0, 0)$$

by measuring the perturbation of coordinates α.

4. Vector-matrix eqution in a neighborhood of steady motion.

We give some results on the stabilization of stationary motions otthe class of systems under consideration. Suppose that for a system with constraints (1) the conditions

$$\frac{\partial R}{\partial \beta} = 0,\ \frac{\partial B_{11}}{\partial \beta} = 0,\ \frac{\partial B_{12}}{\partial \beta} = 0,\ \frac{\partial B_\delta}{\partial \beta} = 0$$
$$\frac{\partial \sum_\alpha}{\partial \beta} = 0,\ \frac{\partial \sum_\beta}{\partial \beta} = 0$$

and this nonholonomic system has steady motions

$$\alpha = \alpha_0 = const,\ \delta = \delta_0 = const,\ p = c = const \qquad (6)$$

Which are defined by equations

$$-[d_{\alpha(\delta)} B_{12} b + \gamma_{(\delta)} B_\delta - \gamma d_{\beta(\delta)} B_{12} b - d_{\beta[\alpha]} b - B'_\alpha d'_{\beta[\delta]} b -$$
$$\left(S'_\delta \Omega_{12}^\alpha + S'_\hbar \Omega_{22}^\alpha\right) b \omega'_{\delta\alpha} \theta_{\delta\alpha} - - \omega'_{\hbar\alpha} \Omega_{\hbar\beta}]\left(p - d_\beta\right) -$$
$$-\left(p' - d'_\beta\right)\left[\frac{1}{2} b_{[\alpha]} + \gamma_{(\delta)} B_{12} b - \frac{1}{2} B'_{11} b_{(\delta)} - \left(\theta'_{\delta\beta} \Omega_{12}^\alpha + \theta_{\hbar\beta} \Omega_{22}^\alpha\right) b\right]\left(p - d_\beta\right) +$$
$$+ \frac{\partial T_0}{\partial \alpha} + B'_{11} \frac{\partial T_0}{\partial \delta} - d_{\alpha(\delta)} B_\delta - B'_{11} \Pi_\delta + S'_\delta \omega_{\delta\alpha} + + S'_\hbar \omega_{\hbar\alpha} + Q_\alpha + B'_{11} Q_\alpha +$$
$$+ B'_{11} Q_\delta + B'_{21} Q_\hbar - \Pi_\alpha = 0;$$
$$-\left[B'_{12} d'_{[\delta]} + \left(S'_\delta \Omega_{12}^\beta + S'_\hbar \Omega_{22}^\beta + \omega'_{\delta\beta} \theta_{\delta\beta} + \omega'_{\hbar\beta} \theta_{\hbar\beta}\right) b\right]\left(p - d_\beta\right) -$$
$$\left(p' - d'_\beta\right)\left[\frac{1}{2} B'_{12} b[\delta] - \left(\theta'_{\delta\beta} \Omega_{12}^\beta + \theta'_{\hbar\beta} \Omega_{22}^\beta\right) b\right]\left(p - d_\beta\right) -$$
$$- B'_{12} \Pi_\delta + B'_{12} \frac{\partial T_0}{\partial \delta} + Q_\beta + B'_{12} Q_\delta + B'_{22} Q_\hbar = 0;$$
$$B_{12} b\left(p - d_\beta\right) + B_\delta = 0.$$

We obtain $n - m + \ell$ equations for the determination of $n - m + \ell$ constants. Consequently, in the general case, the motion (6) may turn out to be isolated (in contrast to stationary motions of systems with homogeneous constraints). Introducing perturbations

$$\alpha = \alpha_0 + x,\ \delta = \delta_0 + z,\ p = c + y\,.$$

We compose the equations of perturbed motion in the neighborhood of the motion (6):

$$\dot{x} = x_1$$
$$a_*^* \dot{x}_1 = -(\hat{C}_1 + \hat{P}_1 + \hat{H}_1) x - (\hat{D}_1 + \hat{G}_1 + H_2) x_1 - (\hat{K}_{12} + \hat{H}_4) y - \gamma_* \dot{y} - \qquad (7)$$
$$-(\hat{Z}_1 + \hat{H}_5) z + N_1$$
$$\dot{y} = -(\hat{P}_{21} + H_{21}) x - (\hat{K}_{21} + H_{24}) x_1 - (\hat{D}_2 + \hat{G}_2 + \hat{H}_{25}) y -$$
$$-(\hat{Z}_2 + \hat{H}_{26}) z + \hat{N}_3;\ \ \dot{z} = \hat{W}_1 x + \hat{W}_2 x_1 + \hat{W}_4 y + \hat{W}_5 z + Z^{(2)}$$

The differences of the matrices of the coefficients of the linear terms in these equations from the corresponding matrices in equations (5) the icon $p = d_{\beta\otimes}$ below shows that the corresponding expression is calculated on the motion (6).

In the problem of stabilizing unstable steady motion, in the same way as in the problem of stabilizing equilibrium, various variants of applying control actions are possible. The resulting general equations give the possibility of choosing a particular method based on the structure of the system. In general, it is possible to include in the model the description of the dynamics of the actuator.

5. Conclusion

In the article, a systematic application of Voronetz vector-matrix equations in Routh variables is developed to describe the dynamics of non-holonomic systems with inhomogeneous constraints. Nonlinear equations of perturbed motion are obtained in stability and stabilization problems in a form that allows one to analyze the structure of nonlinear terms. The possibilities of various ways of applying control actions are discussed. The developed technique makes it possible to automate the consideration of problems of stabilization of complex technical devices.

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