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Research paper

Indentation and flattening of rough surfaces spherical asperities

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Abstract

The paper indicates that the application of roughness models and the theories of contacting rough surfaces developed by Greenwood-Williamson and N.B. Demkin for solving the problems of hermetology leads to significant errors. This is explained by much greater contact pressures than for the tribology problems, by describing only the initial part of the reference surface curve, the lack of allowance for the plastic extrusion of the material. A brief review of methods for describing the introduction of a sphere into an elastoplastic reinforced half-space is given. The properties of the elastoplastic reinforced material are described by the power law of Hollomon. To describe the indentation and flattening of single spherical asperity, the results of finite element modeling are used. The cases of contacting a rigid rough surface with an elastoplastic half-space and a rigid smooth surface with a rough surface are considered. To determine the relative contact area, the discrete roughness model is used in the form of a set of spherical segments distributed along the height in accordance with the curve of the reference surface.

Keywords: *Rough Surface, Relative Contact Area, Spherical Asperity, Elastoplastic Contact,, Indentation Of The Sphere, Flattening Of The Sphere.*

1. Introduction

At present, the models of roughness and the theory of contacting of rough surfaces developed by Greenwood-Williamson [1], N.B. Demkin [2] and their followers. However, the use of such models to solve the problems of hermetology leads to significant errors, which is explained by the following: 1) contact pressure of sealing is about 1-2 orders of magnitude higher than that of friction; 2) in the sealing joint, contacting of all asperities is possible and it is required the description of the entire profile bearing curve and not only its initial part; 3) extrusion of the material into the intercontact space under elastic-plastic contact is not taken into account.
Therefore, to describe the sealing joint, a rough surface model that adequately describes the real surface and corresponds to the entire bearing curve and not just its initial part. In addition, in order to improve the accuracy of the calculation of the contact characteristics in a discrete model of a rough surface, the real distribution of microroughness dimensions must be taken into account.
Tightness of sealing joints is ensured by loading their sealing surfaces with contact pressures and depends to a large extent on the contact interaction of rough surfaces, which is characterized by the type of contact, the convergence of surfaces, the relative contact area and the gap density in the joint [1]. To seal media with high energy parameters (pressure over 40 MPa and a temperature above 300^{o}C), metal materials are mainly used.
To calculate the contact characteristics listed above, a discrete roughness model is widely used in the form of a set of spherical segments whose height distribution corresponds to the bearing surface's curve of the rough surface [1], for which a regulated incomplete beta function is used. In most cases, when contacting metallic rough surfaces, the contact is elastoplastic [2], therefore, in determining the contact characteristics, it is necessary to take into account the parameters of material hardening [3]. In this case, it is possible indentation spherical asperities into a less solid surface or flattening of spherical asperities by a harder surface. In [3], the author believes that the parameters of the contact interaction in these cases are approximately the same. With purely elastic deformation in contact, the validity of such an approach is undoubted, but with elastoplastic deformation it is not obvious and needs additional investigation [4]. In this connection, it is of practical interest for the problems of hermetology to compare the dependences of the relative areas of the contact on the load during the indentation and flattening of spherical asperities of rough surfaces. Since the contact of two rough surfaces can be regarded as the contact of an equivalent rough surface with a smooth surface [3], in the present paper we compare the contacts of a rigid rough surface with an elastoplastic half-space and a rigid smooth surface with a rough surface. First, consider the indentation and flattening of single spherical asperities.

2. Contacting the single spherical asperity

A detailed analysis of the methods for calculating elastoplastic deformation during the indentation of spherical asperities was considered in [3], where it was noted that the regularities of elastoplastic contact have not been sufficiently studied, and some proposed solutions require refinement and improvement. One of the important problems in this case is the consideration of hardening of the material and the description of the effects of "sink-in / pile-up" (elastic punching and plastic material displacement). With the growth of the applied load, the region of limited elastoplasticity and the region of developed elastoplasticity are distinguished [6, 7], but there is no common opinion about the boundaries of the regions. Therefore, a description of the elastoplasticity regions by a single expression is of some interest.

In the description of elastic-plastic hardening material used by the Hollomon's power law.

$$\sigma = \begin{cases} \varepsilon E, & \varepsilon \le \varepsilon_y, \\ \sigma_y (\varepsilon/\varepsilon_y)^n, & \varepsilon > \varepsilon_y, \end{cases} \quad (1)$$

where n is the exponent of hardening; $\varepsilon_y = \sigma_y / E$, σ_y is the yield strength, E is the elastic modulus.
The value of the hardening exponent can be determined from the parameters of the conditional tension diagram according to [3]:
The approach of the authors for describing the indentation of the sphere in various elastic-plasticity regions by a single expression is described in [8, 9]. The essence of the method is the application of the kinetic indentation diagram and the similarity method for deformation characteristics. In this case, the concept of "plastic hardness" [10] is used, as the resistance characteristics of the material of contact plastic deformation. Plastic hardness is represented in the form:

$$HD = K_h(\varepsilon_y, n) \cdot \sigma_y , \quad (2)$$

where $K_h(\varepsilon_y, n)$ is the parameter defined by the "double indentation" method [3] using the results of finite element analysis, the introduction of a sphere into an elastoplastic hardening half-space [11, 12].
In [8], in order to generalize the results of the study of the dependence *P–h* (force - displacement) are reduced to the form *k*–δ, where $k = P/P_y$, $\delta = h/h_y$. Critical force P_y the value of the indentation h_y correspond to the appearance of plastic deformation in the near-surface layer. According to [13], this occurs when the maximum pressure in the center of the contact area

$$p_{\max} = K_y \sigma_y , \quad (3)$$

where $K_y = 1.613$ for Poisson's ratio $\nu = 0.3$.
Then when the sphere is indented by the radius *R* critical force P_y and the value of the indentation h_y are equal

$$\bar{h}_y = \frac{h_y}{R} = \left(\frac{\pi}{2} K_y \varepsilon_y\right)^2, \quad \bar{P}_y = \frac{P_y}{E^* R^2} = \frac{4}{3} h_y^{\frac{3}{2}} , \quad (4)$$

where $\varepsilon_y = \sigma_y / E^*$, E^* is the reduced modulus of elasticity.
A simpler way to calculate the contact characteristics for the indentation of a sphere with radius R is the expression from [12]:

$$\bar{P}_{ep} = \frac{P_{ep}}{E^* R^2} = e^{-B} \left(\frac{h}{R}\right)^A = e^{-B} \left(\bar{h}\right)^A , \quad (5)$$

where P_{ep} is the applied force, *h* is the the value of the indentation, $A = A(\varepsilon_y, n)$, $B = B(\varepsilon_y, n)$, $\bar{h} = h/R$.
The dependences obtained in [9] k – δ for different elastic-plasticity regions are represented in the form

$$k = B_i(\varepsilon_y, n) \cdot \delta^{\gamma_i(\varepsilon_y, n)} . \quad (6)$$

To determine the depth h_c, on which the sphere comes into contact with the material of the half-space, the results of [14] should be used:

$$c^2 = \frac{h_c}{h} = M^{\frac{2}{N}} \left(2\bar{h}\right)^{\frac{2}{N}-1} , \quad (7)$$

where $M = M(\varepsilon_y, n)$, $N = N(\varepsilon_y, n)$.
The actual contact area for the indentation

$$A_r = 2\pi R h c^2 . \quad (8)$$

A number of works [15-17] used the empirical Mayer law to account for material hardening during elastoplastic contact, which establishes the relationship between the force when the sphere is pressed in and the diameter of the print. In [16, 17], the influence of the single physico-mechanical properties of real materials on the features of the formation of contact elastoplastic deformations is emphasized. However, it explicitly features of the elastoplastic hardening body are not taken into account, which is a disadvantage of this approach. This defect was eliminated in [18, 19], in which the authors used the interrelation of the Mayer index with the hardening exponent [20, 21].The obtained dependences are in good agreement with the results of finite element analysis [12] and experimental data [22].
The flattening of the sphere by a rigid flat surface is less well studied. Basically, finite element modeling is used for this. For an elastically ideally plastic material, the authors of [23, 24] proposed convenient expressions for practical use

$$\frac{P}{P_y} = B_i \left(\frac{h}{h_y}\right)^{\gamma_i}, \quad \frac{A}{A_y} = C_i \left(\frac{h}{h_y}\right)^{\lambda_i} , \quad (9)$$

where B_i, γ_i, C_i, λ_i are the constants for different ranges of values h/h_y; the critical values used in [24] P_y and h_y practically coincide with those determined from Eq. 4.
The proposed approach was developed by the authors [25], which received a similar dependence for elastoplastic hardenable material described expressions (1).When the exponent of hardening changes from 0 to 1, the properties of the material vary from elastically ideally plastic to elastic.
Relative force when the sphere is flattened

$$\bar{P} = \bar{P}_y B_i \left(\bar{h}/\bar{h}_y\right)^{\gamma_i} , \quad (10)$$

where

$$B_1 = B_1(n) = -0.07598n + 0.96081 ,$$
$$\gamma_1 = \gamma_1(n) = 0.10725n + 1.43352 , \text{ for } 1 \le \bar{h}/\bar{h}_y \le 6 ;$$
$$B_2 = B_2(n) = -0.82815n + 1.68998 ,$$
$$\gamma_2 = \gamma_2(n) = 0.31831n + 1.21111 , \text{ for } 6 \le \bar{h}/\bar{h}_y \le 110 .$$

The actual contact area in the flattening

$$A_r = A_y C_i \left(\bar{h}/\bar{h}_y\right)^{\lambda_i}, \quad A_y = \pi R h_y , \quad (11)$$

Where

$$C_1 = C_1(n) = -0.01763n + 1.13173 ,$$
$$\lambda_1 = \lambda_1(n) = 0.04715n + 1.03997 , \text{ for } 1 \le \bar{h}/\bar{h}_y \le 6 ;$$
$$C_2 = C_2(n) = 0.23235n + 0.94066 ,$$
$$\lambda_2 = \lambda_2(n) = 0.18325n + 1.14559 , \text{ for } 6 \le \bar{h}/\bar{h}_y \le 110 .$$

Eqs. 10-11 describe the corresponding characteristics for different regions of elastoplasticity and are similar in form to Eq. 6.

In what follows, in order to simplify the comparison of the characteristics of the contact during the indentation and flattening of the sphere, we use Eq. 5, Eqs. 7-8 and Eqs. 10-11.
Fig. 1 shows the dependence of the reduced contact area $\overline{A}_r = A_r/\left(\pi R^2\right)$ on the relative load $\overline{P}$ during the indentation and flattening of the sphere for different values of the hardening parameters ε_y and n.

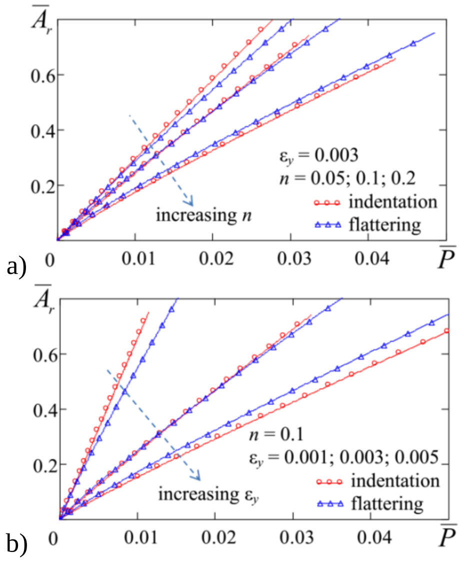

Fig. 1. Dependencies $\overline{A}_r - \overline{P}$ for different values of hardening parameters.

3. Contacting the single spherical asperity

We use the discrete roughness model given in [3]. The asperities can be represented in the form of a set of identical spherical segments of radius $R = a_c^2/(2\omega R_{\max})$, base a_c and height $\omega R_{\max}$. To describe the curve of the support surface, we use a regularized incomplete beta function. The density function of the height distribution of the asperities is described by the expression:

$$\varphi_n'(u) = \frac{u^{p-2}(1-u)^{q-2}[(p-1)(1-u)(q-1)u]}{\varepsilon_s^{p-1}(1-\varepsilon_s)^{q-1}}, \tag{12}$$

where p and q are parameters of the beta-function, which are determined by the height parameters of the roughness; $\varepsilon_s = p/(p+q)$; $\omega = 1-\varepsilon_s$.
When using Eqs. (5), (7), (10) and (11) for the i-th asperity of a rough surface, it should be taken into account that

$$h_i = (\varepsilon - u)R_{\max},$$
$$\frac{h_i}{R} = \frac{(\varepsilon-u)\cdot 2\omega R_{\max}^2}{a_c^2} = \left(\frac{\varepsilon-u}{2\omega}\right)\cdot\left(\frac{2\omega R_{\max}}{a_c}\right)^2, \tag{13}$$

where ε is the relative convergence, u is the initial distance to the vertex of the i-th asperity; number of vertices in the layer du

$$dn_r = n_c\varphi_n'(u)du,\ n_c = \frac{A_c}{\pi a_c^2}. \tag{14}$$

Summing up the efforts and the areas over all asperities with the indentation of a rough surface, we obtain

$$\frac{q_c a_c}{\omega R_{\max}E^*} = f_{q1} =$$
$$= \frac{2^{2(A-1)}e^{-B}}{\pi}\left(\frac{\omega R_{\max}}{a_c}\right)^{2A-3}\int_0^{\varepsilon}\left(\frac{\varepsilon-u}{2\omega}\right)^A\varphi_n'(u)du,$$
$$\overline{q}_{\sigma 1} = \frac{q_c}{\sigma_y} = \frac{f_{q1}}{f_y}, \tag{15}$$
$$f_y = \frac{a_c\varepsilon_y}{\omega R_{\max}}.$$

$$\eta_1 = (2M)^{\frac{2}{N}}\left(\frac{2\omega R_{\max}}{a_c}\right)^{2\left(\frac{2}{N}-1\right)}\int_0^{\varepsilon}\left(\frac{\varepsilon-u}{2\omega}\right)^{\frac{2}{N}}\varphi_n'(u)du. \tag{16}$$

When the rough surface is flattened:

$$\overline{q}_{\sigma 2} = 1.5K_y h_y\left(\frac{a_c}{2\omega R_{\max}}\right)^2\times$$
$$\times\int_0^{\varepsilon}\left(\frac{2\omega R_{\max}}{a_c}\right)^{2\gamma_i}\overline{h}_y^{-\gamma_i}B_i\left(\frac{\varepsilon-u}{2\omega}\right)^{\gamma_i}\varphi_n'(u)du. \tag{17}$$

$$\eta_2 = \int_0^{\varepsilon}\left(\frac{2\omega R_{\max}}{a_c}\right)^{2(\lambda_i-1)}\overline{h}_y^{1-\lambda_i}C_i\left(\frac{\varepsilon-u}{2\omega}\right)^{\lambda_i}\varphi_n'(u)du. \tag{18}$$

The condition for the transition of the boundary of the elasto-plastic region $\overline{h}/\overline{h}_y \le 6$ in the Eqs. (10) and (11) for a separate spherical asterity in the Eqs. (17) and (18) should be represented in the form

$$\frac{\varepsilon-u}{2\omega} \le \frac{3}{8}\left(\pi K_y f_y\right)^2. \tag{19}$$

In Fig. 2 shows the relative contact area η relative to the relative load $\overline{q}_\sigma = q/\sigma_y$ when a rigid rough surface is introduced into the elastoplastic hardened half-space and when the unevenness of the rough surface is flattened by a rigid smooth surface for different values of the hardening parameters ε_y and n. The values of the parameters of the curve of the reference surface: $p = 3.5$, $q = 3.5$.

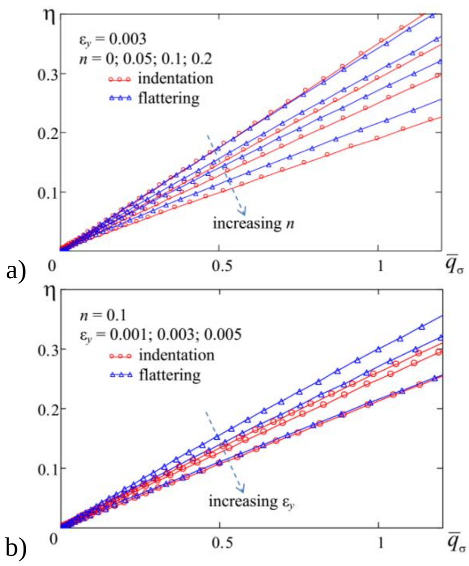

Fig. 2. Dependencies $\eta - \overline{q}_\sigma$ for different values of hardening parameters.

4. Conclusion

1. As it follows from Fig. 1, the values of the reduced contact area $\overline{A}_r$ during the indentation and flattening of the sphere differ insignificantly. And, depending on the combination of the values of the hardening parameters, the reduced contact area during the indentation can be more or less than when flattening. As the plasticity of the material increases (by decreasing the values of the parameters ε_y and n), the values of the reduced contact area increase. More effective is to reduce the value of ε_y.
2. When the rough surface interacts with a smooth surface with the same load, the relative contact area during flattening is always greater than during the indentation. With the same adjustment parameters, the range of variation in the relative contact area is significantly less than the similar range of variation in the values of the reduced contact area of a particular asperity.
3. An increase in the relative contact area improves the tightness of the joints. However, the effectiveness of this contact characteristic requires a comprehensive analysis in conjunction with another important contact characteristic - the density of gaps in the sealing joint. Such an analysis for indentation of a rigid rough surface into an elastoplastic hardened half-space is given by the authors of [3]. Therefore, one of the problems of future research of the authors will be to determine the gap density in the joint when the asperity of the rough surface is flattened.
4. An important problem of hermetology is to predict the tightness of joints with possible unloading of the sealing joint. Such investigations have been carried out for the case of the indentation of a rigid rough-hovered surface into an elastoplastic hardened half-space [26, 27]. A solution of a similar problem is also planned for the case of flattening roughness of a rough surface by a rigid smooth surface.

References

[1] Greenwood JA & Williamson JBR (1966), Contact of nominally flat sufaces. *Proc Roy Soc*, A295, 301-313.
[2] Demkin NB (1970), Contacting rough surfaces. Nauka, Moscow.
[3] Ogar PM, Gorokhov DB, & Kozhevnikov AS (2017), Contact tasks in hermetic sealing studies of fixed joints. BrSU, Bratsk. 242 p.
[4] Ogar PM, Gorokhov DB & Elsukov VK (2017), Criteria for the appearance of plastic deformations at contacting rough surfaces of joints in technological equipment. *Systems Methods Technologies* 3(35) , 32-39. doi: 10.18324/2077-5415-2017-3-32-39.
[5] Kazankin VA (2016), Development of a technique for calculating the strength of fixed joints, taking into account the contact stiffness of matched parts of close hardness. *Dissertation*, VolSTU, Volgograd. 145 p.
[6] Lankov AA (2009), The probability of elastic and plastic deformations during compression of metallic rough surfaces. *Trenie-i-smazka-v-mashinah-i-mekhanizmah* 3, 3-5.
[7] Voronin NA (2003), Theoretical model of elastic-plastic introduction of rigid sphere. *Friction and Wear* 24, 16-26.
[8] Ogar PM &Tarasov VA (2013) Kinetic indentation application to determine contact characteristics of sphere and elastoplastic half-space. *Advanced Materials Research* 664, 625-631. doi: 10.4028/www.scientific.net/AMR.664.625.
[9] Ogar PM, Tarasov VA & Turchenko AV (2013), Tribomechanics of elastoplastic contact. *Modern technologies. System analysis. Modeling* 2(18), 116-122.
[10] Drozd MS, Matlin MM & Sidyakin YI (1986), Engineering methods for calculating elastoplastic contact deformation. *Mechanical Engineering*, Moscow.
[11] Lee H, Lee J & Pharr GM (2005), A numerical approach to spherical indentation techniques for material property evaluation. *J Mech Phys Solids* 53, 2037-2069.
[12] Collin JM, Mauvoisin G & Pilvin P (2010), Materials characterization by instrumented indentation using two different approaches. *Materials and Desing* 31, 636-640.
[13] Johnson K.L (1985), Contact mechanics. *University Press*, Cambridge.
[14] Hernot X, Bartier O, Bekouche Y, El Abdi R & Mauvoisin G (2006), Influence of penetration depth and mechanical properties on contact radius determination for spherical indentation. *Int J of Solids and Struct* 43, 4136-4153.
[15] Myshkin NK & Petrokovec MI (2007) Friction, lubrication, wear. Physical bases and technical applications of tribology. *Fizmatlit*, Moscow.
[16] Bolotov AN, Meshkov VV, Sutyagin OV & Vasiliev MV (2013), Study of elastoplastic contact of a spherical indenter with metals and solid lubricating coatings: Part 1. Critical loads. *Friction and Wear* 34, 1-5.
[17] Bolotov AN, Meshkov VV, Sutyagin OV & Vasiliev MV (2013) Study of elastoplastic contact of a spherical indenter with metals and solid lubricating coatings: Part 2. Contact characteristics. *Friction and Wear* 34, 129-133.
[18] Ogar P & Gorokhov D (2017), Meyer law application to account of material hardening under rigid spherical indentation. *In: Proc. of 22nd int. conf. «MECHANIKA 2017»*. KUT, Kaunas, 287-290.
[19] Ogar P, Gorokhov D & Belokobylsky S (2017), The elastic-plastic contact of a single asperity of a rough surface. MATEC Web of Conferences 129, 06017. doi: 10.1051/matecconf/201712906017.
[20] Bulychev SI (2010), The transition from the indentation diagrams to the tensile diagrams with allowance for the hardened surface layer. Deformation and fracture of materials 2, 43-48.
[21] Bulychev SI (2011), Hardness and hysteresis at the yield strength. *Deformation and fracture of materials* 1, 41-45.
[22] Matlin M, Kazankina E, Kazankin V (2009), Mechanics of unitial contact. *Mechanika* 2, 20-23.
[23] Etision I, Kligerman Y & Kadin Y (2005), Unloading of an elastic–plastic loaded spherical contact. *Int J Solids Struct*, 42, 3716–29.
[24] Kogut L & Etision I (2002), Elastic–plastic contact analysis of a sphere and a rigid flat. *ASME J Appl Mech* , 69, 657–62.
[25] Zhao JH, Nagao S & Zhang ZI (2012), Loading and anloading of a shtrical contact: From elastic to elastic-perfectly plastic materials. *Int. J. of Mech. Ssience,s* 56, 70-76.
[26] Ogar PM & Gorokhov DB (2017), Influence of Materials Hardenability Parameters on the Machine Parts Characteristics after Unloading. *Key Engineering Materials* 723, 369-375.
doi: 10.4028/www.scientific.net/KEM.723.369.
[27] Ogar P, Gorokhov D, Ugryumova E (2017), Mechanics of unloading of a rough surfaces pre-loaded joint. MATEC Web of Conf, 129, 06016. doi: 10.1051/matecconf/201712906016.