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Research paper

About one decision of the quasiclassical kinetic equation

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Abstract

It has been proved that the solution of the quasi-classical kinetic equation for Bose and Fermi statistics can be represented in the general form, using the relaxation time approximation. The general solution found for the distribution function $f(\mathbf{r},\mathbf{p},t)$ helps calculate any non – equilibrium characteristics of metals, magnets, and dielectrics in any order of the perturbation theory according to the relaxation time τ .

Keywords: *Quasi-Classical Kinetic Equation; Quasi-Equilibrium Distribution Function; Heat – Conductivity; Current Density.*

1. Introduction

The problem in question appeared in the result of studying monographs and original articles [1-6] relating to application of the quasi-classical kinetic equation (QKE for short) to various problems of theoretical physics: calculation of the thermal conductivity coefficient, susceptibility, conductivity of metals, etc. It could be that we lost sight of some paper, which are related to the solution of the problem of the finding precise common decision of the quasi-classical kinetic equation (QKE) in the "tau approximation". However, we quite realize the fact that this problem hasn't been found reflected in an external literature, that's why the purpose of this paper is the elimination of this problem. In this paper, we don't touch upon subject of the well-known and popular specific methods of finding solutions of the linear differential equations in partial derivatives, such as expansion in Fourier integrals, Laplas integrals and the wavelet integrals, which are used in a solving of the different problems in the theory of nonequilibrium phenomene. Also, we don't touch upon subject of the similar purely mathematical approaches, which are related to the application other integral transforms and other useful differential equations, when solving the linear differential ones. We would like to pinpoint our attention upon other, but very effective representation of the general solution of QKE in the "tau approximation". As null approximation, we have only the function of distribution of particles or (quasi-particles), which is denoted as $f_0(\mathbf{r},\mathbf{p},t)$. This statement means that we hypothetically assume fulfillment of condition, which is imposed on all the relaxation times, existing in these subsystems is to strengthen the inequality $\tau_{ss} << \tau'_s$, where s index – some subsystem of particles (for example, electrons, photons, etc.) or quasi-particles (for example, phonons, magnons, etc.), the relaxation time τ_{ss} – its own the same name inside relaxation time. Shaded relaxation time refers to any kind of interaction between the oppositely charged participles or (quasi-particles), it means between electrons and phonons, for example.

2. The generalized solution QKE by "tau approximation"

We are writing QKE for the function of distribution of Fermi – particle $f(\mathbf{r},\mathbf{p},t)$ in the general form

$$\frac{\partial f}{\partial t}+\mathbf{F}\frac{\partial f}{\partial \mathbf{p}}+\mathbf{u}\nabla f=L\{f\}, \tag{1}$$

where $\mathbf{u}$ - the electron's velocity and $\mathbf{F}=e\left(\mathbf{E}+\frac{[\mathbf{u}\times\mathbf{H}]}{c}\right)$ Lorenz' force acting on the electron in the electromagnetic field, $\mathbf{H}-$ the intensity of magnetic field, $\mathbf{E}-$ the intensity of electric field, $c-$ the speed of light in vacuum, $e-$ the electron - charge. We will find the solution of equation (1) in the expanded form $f=f_0+\lambda f_1+\lambda^2 f_2+...=\sum_{n=0}^{\infty}\lambda^n f_n$, where $\lambda-$ the small parameter (see below). The quasi – equilibrium function of the distribution of Fermi – particle can be represented as $f_0(\mathbf{r},\mathbf{p},t)=\frac{1}{e^{\frac{\varepsilon(\mathbf{p})-\mu}{T(\mathbf{r},t)}}+1}$, where the electron - energy is $\varepsilon(\mathbf{p})=\frac{p^2}{2m}$, $\mu-$ the chemical potential, $T(\mathbf{r},t)$ the quasi – equilibrium temperature, which depends on the coordinates and the time. Therefore, our problem is to find the exact different $f-f_0$, which is a full addition $\delta f=\lambda f_1+\lambda^2 f_2+...$ to the quasi – equilibrium distribution function. We are writing the collision integral in the right side of the equation (1) in the following form $L\{f\}=-\frac{\lambda f_1}{\tau_p}$, where $\tau_p -$ the relaxation time of the electron with the impulse $\mathbf{p}$, which connected with its interaction with phonons. In the result, we are getting from the equation (1) the following one

$$\lambda f_1 = -\tau_p\left(\frac{\partial f_0}{\partial t} + \mathbf{F}\frac{\partial f_0}{\partial \mathbf{p}} + \mathbf{u}\nabla f_0 \right). \qquad (2)$$

Substituting further the solution (6) in the equation (1) we are finding the following amendment

$$\lambda^2 f_2 = -\lambda\tau_p\left(\frac{\partial f_1}{\partial t} + \mathbf{F}\frac{\partial f_1}{\partial \mathbf{p}} + \mathbf{u}\nabla f_1 \right). \qquad (3)$$

Substituting (2) here, we are finding that

$$\lambda^2 f_2 = \tau_p^2\left(\frac{\partial}{\partial t}\left(\frac{\partial f_0}{\partial t} + \mathbf{F}\frac{\partial f_0}{\partial \mathbf{p}} + \mathbf{u}\nabla f_0 \right) + \mathbf{F}\frac{\partial}{\partial \mathbf{p}}\times \right. \qquad (4)$$
$$\left.\times\left(\frac{\partial f_0}{\partial t} + \mathbf{F}\frac{\partial f_0}{\partial \mathbf{p}} + \mathbf{u}\nabla f_0 \right) + \mathbf{u}\nabla\left(\frac{\partial f_0}{\partial t} + \mathbf{F}\frac{\partial f_0}{\partial \mathbf{p}} + \mathbf{u}\nabla f_0 \right) \right)$$

Since $\frac{\partial f_0}{\partial t} = \frac{\partial f_0}{\partial T}\dot{T},\ \frac{\partial f_0}{\partial \mathbf{p}} = \frac{\partial f_0}{\partial \varepsilon}\frac{\partial \varepsilon}{\partial \mathbf{p}} = \mathbf{u}\frac{\partial f_0}{\partial \varepsilon},\ \nabla f_0 = \frac{\partial f}{\partial T}\nabla T$, we can easy get the linear approximation on $\delta T = T - T_0$ from (4)

$$\lambda^2 f_2 = \tau_p^2\left(\frac{\partial}{\partial t}\left(\frac{\partial f_0}{\partial t} + \mathbf{F}\frac{\partial f_0}{\partial \mathbf{p}} + \mathbf{u}\nabla f_0 \right) + \mathbf{F}\frac{\partial}{\partial \mathbf{p}}\times \right.$$
$$\left.\times\left(\frac{\partial f_0}{\partial t} + \mathbf{F}\frac{\partial f_0}{\partial \mathbf{p}} + \mathbf{u}\nabla f_0 \right) + \mathbf{u}\nabla\left(\frac{\partial f_0}{\partial t} + \mathbf{F}\frac{\partial f_0}{\partial \mathbf{p}} + \mathbf{u}\nabla f_0 \right) \right) = \qquad (5)$$
$$= \tau_p^2\left[\left(\frac{\partial f_0}{\partial T}\ddot{T} + \mathbf{F}\mathbf{u}\dot{T}\frac{\partial^2 f_0}{\partial T\partial \varepsilon} + \mathbf{u}\nabla\dot{T}\frac{\partial f_0}{\partial T} \right) + \right.$$
$$+\mathbf{F}\left(\mathbf{u}\dot{T}\frac{\partial f_0}{\partial T\partial \varepsilon} + \mathbf{u}(\mathbf{u}\mathbf{F})\frac{\partial^2 f_0}{\partial \varepsilon^2} + \mathbf{u}(\mathbf{u}\nabla T)\frac{\partial^2 f_0}{\partial T\partial \varepsilon} \right) +$$
$$+\mathbf{u}\left(\frac{\partial f_0}{\partial T}\nabla\dot{T} + (\mathbf{u}\mathbf{F})\nabla T\frac{\partial^2 f_0}{\partial T\partial \varepsilon} + (\mathbf{u}\nabla)(\mathbf{u}\nabla T)\frac{\partial f_0}{\partial T} \right)$$

As a result of a simple grouping of relevant summands the approximation solution (2) and (3) can be represented as

$$f \approx f_0 - \tau_p\left(\frac{\partial f_0}{\partial T}(\dot{T} + \mathbf{u}\nabla T) + \mathbf{u}\mathbf{F}\frac{\partial f_0}{\partial \varepsilon} \right) +$$
$$+\tau_p^2\left[\frac{\partial f_0}{\partial T}\left(\ddot{T} + 2\mathbf{u}\nabla\dot{T} + (\mathbf{u}\nabla)(\mathbf{u}\nabla T) \right) + \right. \qquad (6)$$
$$\left. +2(\mathbf{u}\mathbf{F})(\dot{T} + \mathbf{u}\nabla T)\frac{\partial^2 f_0}{\partial T\partial \varepsilon} + (\mathbf{u}\mathbf{F})^2\frac{\partial^2 f_0}{\partial \varepsilon^2} \right].$$

We would like to point up that the formula (6) is obtained in the second order of the time of relaxation on τ_p and the function of distribution is the equilibrium function with constant and homogeneous temperature T_0, i.e. $f_0 = \frac{1}{e^{\frac{\varepsilon(p)-\mu}{T_0}} + 1}$. In the opinion of many authors seeking the approximate solutions for explanation the experimentally found different weakly anomalous properties of the metal, the solution (6) in the second – order approximation on the relaxation time τ_p must provide an answer for possible amendments to any of the physical characteristics of the metal. However, it turns out not so exactly, we will make sure of it after finding the exact solution of QRE by "tau approximation". Note, the symbols of the quasi – equilibrium function of the distribution and the equilibrium function are the same, but this fact does not cause confusion. As we could see from (6) the parameter λ must be proportional to the time of the relaxation τ_p. So, the n-th term of series must be proportional to the time $(\tau_p)^n$ in each order in the method of sequential approximations. The general iterative formula in the series it easy to write the following expr. $\lambda^2 f_2 = -\lambda\tau_p\left(\frac{\partial}{\partial t} + \mathbf{F}\frac{\partial}{\partial \mathbf{p}} + \mathbf{u}\nabla \right) f_1 = \tau_p^2\left(\frac{\partial}{\partial t} + \mathbf{F}\frac{\partial}{\partial \mathbf{p}} + \mathbf{u}\nabla \right)^2 f_0$, it is obvious that,

$$\lambda^n f_n = (-1)^n\tau_p^n\left(\frac{\partial}{\partial t} + \mathbf{F}\frac{\partial}{\partial \mathbf{p}} + \mathbf{u}\nabla \right)^n f_0 \qquad (7)$$

Therefore, the exact solution of the quasi – classical kinetic equation in "tau approximation" can be written as the sum of an infinite number of the summands

$$f = f_0 + \lambda f_1 + \lambda^2 f_2 + \ldots = \sum_{n=0}^{\infty}(-1)^n\tau_p^n\left(\frac{\partial}{\partial t} + \mathbf{F}\frac{\partial}{\partial \mathbf{p}} + \mathbf{u}\nabla \right)^n f_0, \qquad (8)$$

where the relaxation time τ_p is traditionally determined by the following approximate formula

$$\frac{1}{\tau_p} = -\left.\frac{\delta L\{f_p\}}{\delta f_p}\right|_{f_p = f_0}, \qquad (9)$$

In which the right side is the functional derivative of the collision integral of distribution function. It is clear to see that the solution (8) in the approximation (9) is approximate, but we could predict a number of effects, which in the first approximation on the relaxation time couldn't demonstrate themselves. For the practical and more convenient application of the formula (8) it is conveniently written it in the following form

$$f = \sum_{n=0}^{\infty}(-1)^n\tau_p^n\left(\frac{\partial}{\partial t} + \mathbf{F}\frac{\partial}{\partial \mathbf{p}} + \mathbf{u}\nabla \right)^n f_0 = \sum_{n=0}^{\infty}(-1)^n\hat{A}^n f_0, \qquad (10)$$

where for the linear operator was used the symbol $\hat{A} = \tau_p\left(\frac{\partial}{\partial t} + \mathbf{F}\frac{\partial}{\partial \mathbf{p}} + \mathbf{u}\nabla \right)$. As it can be seen from (10), the written expression is the alternating series of the geometric progression with the ratio $\hat{A}$. So, we can write the series by the following integral

$$f = \sum_{n=0}^{\infty}(-1)^n\hat{A}^n f_0 = \frac{1}{1+\hat{A}}f_0 = \int_0^{\infty} e^{-(1+\hat{A})\xi} f_0(\mathbf{r},\mathbf{p},t)\,d\xi. \qquad (11)$$

We are accepting (11) to the shorthand notation for the operators: $\hat{B} = \tau_p\frac{\partial}{\partial t},\ \hat{C} = \tau_p\mathbf{F}\frac{\partial}{\partial \mathbf{p}},\ \hat{D} = \tau_p\mathbf{u}\nabla$, i.e. $\hat{A} = \hat{B} + \hat{C} + \hat{D}$. Using a know rule

$$e^{-\xi(1+\hat{A})} = e^{-\xi}e^{-\xi(\hat{B}+\hat{C}+\hat{D})} =$$
$$= e^{-\xi}e^{\frac{1}{2}\xi^2[\hat{B},\hat{C}]}e^{\frac{1}{2}\xi^2[\hat{B},\hat{D}]}e^{\frac{1}{2}\xi^2[\hat{C},\hat{D}]}e^{-\xi\hat{B}}e^{-\xi\hat{C}}e^{-\xi\hat{D}},$$

where the expressions in the square brackets are the commutators of the corresponding operators and the operator $\hat{A}$ is the linear derivative operator, namely, the operator $\hat{A}$ must get the all property of the operator of the translation, for example $e^{\mathbf{a}\frac{\partial}{\partial \mathbf{r}}}\varphi(\mathbf{r}) = \varphi(\mathbf{r}+\mathbf{a})$, where $\mathbf{a}$ – some constant vector, we are getting immediately the exact solution in the following integral form

$$f = \int_0^{\infty} e^{-\xi}e^{\frac{1}{2}\xi[\hat{B},\hat{C}]}e^{\frac{1}{2}\xi[\hat{B},\hat{D}]}e^{\frac{1}{2}\xi[\hat{C},\hat{D}]} f_0(\mathbf{r} - \tau_p\mathbf{u}, \mathbf{p} - \xi\mathbf{F}\tau_p, t - \xi\tau_p)\,d\xi. \qquad (12)$$

In the accordance commutations rules we are getting the following commutators

$$\left[\hat{B},\hat{C}\right] = \tau_p^2\left[\frac{\partial}{\partial t}, \mathbf{F}\frac{\partial}{\partial \mathbf{p}}\right] = \tau_p^2\dot{\mathbf{F}}\frac{\partial}{\partial \mathbf{p}},$$

$$\left[\hat{C},\hat{D}\right]=\tau_p^2\left[\mathbf{F}\frac{\partial}{\partial\mathbf{p}},\mathbf{u}\nabla\right]=\frac{\tau_p^2}{m}\left(\mathbf{F}\cdot\nabla-(\mathbf{p}\cdot\nabla)\left(\mathbf{F}\cdot\frac{\partial}{\partial\mathbf{p}}\right)\right),$$

$$\left[\hat{B},\hat{D}\right]=\tau_p^2\left[\frac{\partial}{\partial t},\mathbf{u}\nabla\right]=0\cdot$$

Substituting their into (12), we are founding that

$$f=\int_0^\infty e^{-\xi}e^{\frac{\xi^2\tau_p^2}{2m}\left[\mathbf{F}\cdot\nabla-(\mathbf{p}\cdot\nabla_{\mathbf{F}})\left(\mathbf{F}\cdot\frac{\partial}{\partial\mathbf{p}}\right)\right]}f_0\left(\mathbf{r}-\tau_p\mathbf{u},\mathbf{p}-\xi\mathbf{F}\tau_p+\frac{\xi^2}{2}\tau_p^2\dot{\mathbf{F}},t-\xi\tau_p\right)d\xi, \tag{13}$$

where $\mathbf{F}$ the index of the gradient operator means that it is acting on its right the force $\mathbf{F}$. And, hence

$$f=\int_0^\infty e^{-\xi}e^{-\frac{\xi^4\tau_p^4}{8m^2}\left[\mathbf{F}\cdot\nabla,(\mathbf{p}\cdot\nabla_{\mathbf{F}})\left(\mathbf{F}\cdot\frac{\partial}{\partial\mathbf{p}}\right)\right]}e^{\frac{\xi^2\tau_p^2}{2m}\mathbf{F}\cdot\nabla}e^{-\frac{\xi^2\tau_p^2}{2m}(\mathbf{p}\cdot\nabla_{\mathbf{F}})\left(\mathbf{F}\cdot\frac{\partial}{\partial\mathbf{p}}\right)}\times$$
$$\times f_0\left(\mathbf{r}-\tau_p\mathbf{u},\mathbf{p}-\xi\mathbf{F}\tau_p+\frac{\xi^2}{2}\tau_p^2\dot{\mathbf{F}},t-\xi\tau_p\right)d\xi=$$
$$=\int_0^\infty e^{-\xi}e^{-\frac{\xi^4\tau_p^4}{8m^2}\left[\mathbf{F}\cdot\nabla,(\mathbf{p}\cdot\nabla_{\mathbf{F}})\left(\mathbf{F}\cdot\frac{\partial}{\partial\mathbf{p}}\right)\right]}\times$$
$$\times f_0\left(\mathbf{r}-\tau_p\mathbf{u}+\frac{\xi^2\tau_p^2}{2m}\mathbf{F},\mathbf{p}-\xi\mathbf{F}\tau_p+\frac{\xi^2}{2}\tau_p^2\left(\dot{\mathbf{F}}-(\mathbf{u}\cdot\nabla)\mathbf{F}\right),t-\xi\tau_p\right)d\xi .$$

Finally, calculating the last commutator in the exponent we are getting that

$$f=\int_0^\infty e^{-\xi}f_0\left(\begin{array}{l}\mathbf{r}-\tau_p\mathbf{u}+\frac{\xi^2\tau_p^2}{2m}\mathbf{F}-\frac{\xi^4\tau_p^4}{8m^2}p_k\frac{\partial F_s}{\partial x_k}\frac{\partial\mathbf{F}}{\partial p_s},\mathbf{p}-\xi\mathbf{F}\tau_p+\\+\frac{\xi^2}{2}\tau_p^2\left(\dot{\mathbf{F}}-(\mathbf{u}\cdot\nabla)\mathbf{F}\right)+\frac{\xi^4\tau_p^4}{8m^2}F_ip_k\frac{\partial^2\mathbf{F}}{\partial x_i\partial x_k},t-\xi\tau_p\end{array}\right)d\xi\cdot \tag{14}$$

In the case of homogeneous fields $\mathbf{F}$ force does not depend on the coordinates, the formula (14) is simplified. So, we are getting that

$$f=\int_0^\infty e^{-\xi}f_0\left(\mathbf{r}-\tau_p\mathbf{u}+\frac{\xi^2\tau_p^2}{2m}\mathbf{F},\mathbf{p}-\xi\mathbf{F}\tau_p+\frac{\xi^2}{2}\tau_p^2\dot{\mathbf{F}},t-\xi\tau_p\right)d\xi\cdot \tag{15}$$

Through the substitution in the (15) the quasi – equilibrium distribution function $f_0=\frac{1}{e^{\frac{\varepsilon(p)-\bar{\mu}}{T(\mathbf{r},t)}}+1}$ we are getting easy for Fermi – particles

$$f(\mathbf{r},\mathbf{p},t)=\int_0^\infty\frac{e^{-\xi}}{\exp\left(\frac{\varepsilon\left(\mathbf{p}-\xi\mathbf{F}\tau_p+\frac{\xi^2\tau_p^2}{2m}\dot{\mathbf{F}}\right)-\mu}{T\left(\mathbf{r}-\mathbf{u}\xi\tau_p+\frac{\xi^2\tau_p^2}{2m}\mathbf{F},t-\xi\tau_p\right)}\right)+1}d\xi \tag{16}$$

Analogical distribution take place and for Bose – particles.

3. The application of the formulas (15) and (16)

We would like to consider some examples of calculation to demonstrate the functional ability of the formulas (15) and (16). First, we are starting with the current density. According to the definition, the electron current is calculated by this formula

$$\mathbf{j}=2e\int\mathbf{u}f\frac{d^3p}{(2\pi\hbar)^3}, \tag{17}$$

where the factor "2" is finding, due to the degeneracy of states on the spin election. In the case when it is considered the interaction of the spin with the magnetic field, the expression (17) is disported into the sum of the two summands, one of which considers the polarization of the electron along the magnetic field, and the other is not. Considering the magnetic field is constant and typing the abbreviation $\varepsilon=\frac{p^2-2\xi\tau pF\cos\theta+\xi^2\tau^2F^2}{2m}$, where $\theta-$ the angle between the fixed vector $\mathbf{F}$ and the impulse $\mathbf{p}$. Choosing the direction of the current density along the vector $\mathbf{F}$ and using the common solution (15), we get it of (17)

$$j=\frac{e}{2\pi^2\hbar^3m}\int_0^\infty e^{-\xi}d\xi\int_0^\pi\cos\theta\sin\theta d\theta\int_0^\infty\frac{p^3dp}{e^{\frac{\varepsilon-\mu}{T}}+1}\cdot$$

After the integration of the parts of the inner integral, we are getting that

$$j=-\frac{e}{8\pi^2\hbar^3m}\int_0^\infty e^{-\xi}d\xi\int_0^\pi\cos\theta\sin\theta d\theta\int_0^\infty p^4\frac{\partial f_0}{\partial\varepsilon}d\varepsilon\cdot \tag{18}$$

Since we are learning a degenerate electron gas, in the accordance with the general properties of the Fermi distribution function, we are entitled to assume that the partial derivative of the energy distribution function is approximately equal to the delta function, i.e. $\frac{\partial f_0}{\partial\varepsilon}\approx-\delta(\varepsilon-\varepsilon_F)$, where ε_F – the energy of Fermi. According to the equation (18), we are getting that $p=\xi\tau\cos\theta+\sqrt{2m\varepsilon-\xi^2\tau^2F^2\sin^2\theta}$. Due to this fact and the great property of delta function the triple integral is simplified to the double integral

$$j=\frac{e}{8\pi^2\hbar^3m}\int_0^\infty e^{-\xi}d\xi\int_0^\pi\cos\theta\sin\theta\left(\xi\tau F\cos\theta+\sqrt{2m\varepsilon_F-\xi^2\tau^2F^2\sin^2\theta}\right)^4d\theta\cdot$$

By the designation $x=\cos\theta$ the integral on x is founded in the symmetrical limits and we will get only the X odd - degree. It means as a result we are getting the following one

$$j=\frac{e\tau F}{\pi^2\hbar^3m}\int_0^\infty\xi e^{-\xi}d\xi\int_0^1x^2\left(2\xi^2\tau^2F^2x^2+2m\varepsilon_F-\xi^2\tau^2F^2\right)\times. \tag{19}$$
$$\times\sqrt{2m\varepsilon_F-\xi^2\tau^2F^2+\xi^2\tau^2F^2x^2}dx$$

Introducing one more designation $\alpha=\frac{\xi^2\tau^2F^2}{2m\varepsilon_F-\xi^2\tau^2F^2}$, we are finding that

$$j=\frac{e\tau F}{\pi^2\hbar^3m}\int_0^\infty\xi e^{-\xi}\left(2m\varepsilon_F-\xi^2\tau^2F^2\right)^{\frac{3}{2}}J(\xi)d\xi, \tag{20}$$

where $J(\xi)=\int_0^1x^2\left(1+2\alpha x^2\right)\sqrt{1+\alpha x^2}dx$. We can see that if $\tau\to0$ $(\alpha\to0)$, it is the classical expression for the current density from the (20)

$$j=\frac{e\tau Fp_F^3}{3\pi^2\hbar^3m}\cdot \tag{21}$$

But the most surprising thing is elsewhere. As a result of the integral (21) of calculation, we are getting the following $J(\xi)=\frac{1}{3}(\alpha+1)^{\frac{3}{2}}$. Substituting this expression in (20), we are getting the exact formula for the current density as in (37)! It means that in any order of the perturbation theory on the relaxation time τ, the current density is $j=\frac{e\tau Fp_F^3}{3\pi^2\hbar^3m}$. Using the differential law of Ohm $j=\sigma E$, where $\sigma-$ conduction, we are getting well-known formula of Drude $\sigma=\frac{e^2\tau p_F^3}{3\pi^2\hbar^3m}$. Here we are using the equality $F=eE$. Thereby, we have shown that the formula of Drude is exact one and it has no any corrections for the electron degenerating gas in a precise accounting of all the summands of the iterative solution for the relaxation time τ .

4. Conclusion

1. The exact expression for metal conductivity in all orders of the perturbation theory according to the relaxation time τ has been found. It has been proofed that the Drude's formulae in tau approximation is exact. Its has no additions according to relaxation time;
2. The exact expression for the thermal conductivity coefficient of metals has been calculated;
3. The general formula of conductivity in the general nonlinear case has been found when the amplitude of the external oscillating field is not small.

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