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Research paper

Analysis of multi server Markovian queue with functioning vacation and intolerance of customer

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Abstract

In this article, we analyze the operating behavior of two server Markovian queueing model with functioning vacation and infinite population. If the server is halt his service suddenly in a normal busy period and repair work is done immediately and service starts. The server failure and repair rates are follow exponential distribution, when the system become vacation the server takes functioning during this period the customer wait in the queue and server serves the customer with the lower service rate. The steady state behavior is also obtained, the various performance measures are also determined. The numerical example is given to test the feasibility of the model.

Keywords: *Vacation, busy period, customer, server, markovian queueing model, distribution.*

AMS subject classification*:60K25,68M20,90B22,60K*

1. Introduction

Queueing theory was introduced by the Danish Mathematician A.K.Erlang(1909) physically a queue is a waiting line of customers which demands service from a service station. The theory permits the derivation and calculation of several performance measures including the average waiting time in the queue or in the system. Queueing systems with server vacations are characterized by using idle time of the server for subsidiary jobs. These models arise in telecommunications system and production process, dispensing patients in hospitals and clinics. The heterogeneous service mechanisms are invaluable scheduling methods that allow customers to receive different quality of service. The role of quality and service performance are crucial aspects in customer perception and firms must dedicate special attention to them. Most of the multi-server queueing models tackled in the literature assume the services to be identical. However, this situation is not very realistic and can prevail only when the service process is highly mechanically controlled. In fact many real life situations involve servers working at different rates. In this paper we study the queueing models with server vacation have been discussed by some researchers due to its wide range.
Boxma, O.J and de waal, P.R. [1] has discussed with Multi server queues with impatient customers. In [2] Gautam Choudhury and Mitali de.ka have dealt with A batch arrival unreliable server Bernoulli vacation queue with two phases of service and delayed repair. Analysis of a bulk queuing system with server breakdown and vacation interruption [3]has studied by M.Haridass and R.P.Nithya. In [4] O.C.Ibe,have discussed with Two queues with alternating service and server breakdowns. An M/M/2 queueing system with heterogeneous servers and multiple vacations [5] has investigated by B.K.Kumar and S.P.Madheswari. Madhu Jain and Anamika Jain [6] has investigated by Working vacations queueing models with multiple types of server breakdowns. In [7] Batch arrival priority queueing model with second optional service and server breakdown have been investigated by Madhu Jain and Anamika Jain. Nai-shu Tian, Ji-houg Li and Zhe George zhang [8] have been discussed with Matrix Analytic Method and working vacations queues-A survey. Matrix Geometric solutions in stochastic Models [9] have been studied by M.F.Neuts.In [10] Matrix-Geometric method for queueing model with multiple vacation, n-policy, server break down, repair and interruption vacationhave been studied by Renisagayaraj. M and Chandrasekar. In [11] Renisagayaraj. M and Chandrasekar dealt with Matrix-Geometric Method for queueing model with state-dependent arrival of an unreliable server and PH-service.In [12] Roshli.Aniyeri and Dr.C. Ratnam Nadar dealt with A multiphase queueing system with Assorted servers by using Matrix Geometric Method.
The present paper as follows. In division 2Model Description is given. In division 3 Balance equations are given. Steady state analysis of the model is given section 4.In division 5 analysis of busy period. In division 6 deriving the waiting time in the queue. Queue length have been computed on division 7. In division 8 some performance indices are established. Numerical results are shown in division9, and concluding remarks are made in division 10.

2. Model description

Consider M/M/2 queuing model with heterogeneous service if the customer arrives for a service, if the server1 is free we get the service otherwise he goes to the server2 for the service. The arrival rate is λ follows poisson distribution, the service rates are $\mu_1$and μ_2 for two servers respectively. The two servers providing exponentially service to the customer with the heterogeneous service on FCFS basis. If the server2 return to the service after vacation with lower service rate, at the rate of $\xi\mu_2$.The vacation rates follows exponential with parameterβ.If the system is vacant the two servers starts a working vacation with rate ψ.The server may suffer a halt at any point during the busy period with rate b and is instantly sent for repair to get a recovery with rate r.

3. Balance Equation

Let L(t) be the number of customers in the system and K(t) is the server position at time t, Then

$$K(t)=\begin{Bmatrix} 0, if\ the\ server\ 2\ is\ on\ vacation \\ 1, if\ the\ server\ 2\ is\ functioning\ vacation\ segment \\ 2, if\ the\ server\ 2\ is\ functioning\ in\ normal\ segment \end{Bmatrix}$$

Assuming X(t)=(L(t), K(t)), t≥0with state space

$$S=\{(0,0),(1,0),(1,1),(1,2)\}\cup\bigcup_{i=2}^{\infty} g(i),\quad where\ g(i)=\{(i,j): i\ge 2, j=1 or 2\}$$

When L(t)=0,the only probable valuation of K(t) =0 and L(t)=1, K(t) having probable values of 0,1,2.
For formulating the model mathematically, we define the steady state probabilities:

$$P_{VVn}\equiv\begin{Bmatrix} Probabilty\ that\ the\ 'n'customers\ in\ the\ system \\ where\ in\ the\ server 1\ and\ server\ 2\ is\ vacation \end{Bmatrix}$$

$$P_{BVn}\equiv\begin{Bmatrix} Probabilty\ that\ the\ 'n'\ customers\ in\ the\ system \\ where\ \text{in the}\ server 1\ \text{is}\ Busy\ and\ server\ 2\ is\ vacation \end{Bmatrix}$$

$$P_{VBn}\equiv\begin{Bmatrix} Probabilty\ that\ the\ 'n'\ customers\ in\ the\ system \\ where\ in\ the\ server 1\ \text{is}\ vacation\ and\ server\ 2\ is\ busy \end{Bmatrix}$$

$$P_{BBn}\equiv\begin{Bmatrix} Probabilty\ that\ the\ 'n'\ customers\ in\ the\ system \\ where\ in\ the\ \text{both servers are busy} \end{Bmatrix}$$

$$P_{DDn}\equiv\begin{Bmatrix} Probabilty\ that\ the\ 'n'\ customers\ in\ the\ system \\ where\ in\ the\ \text{service halt occurs during busy state,n=1,2....} \end{Bmatrix}$$

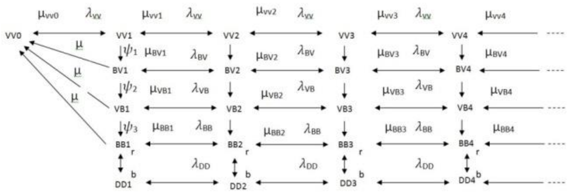


Figure 1: State transition rate diagram

The transition flows for different system states can be balanced to build the steady state equations as
:

$$\lambda_{VV}P_{VV0}=\mu_{VV0}P_{VV1}+\mu_{BV1}P_{BV1}\to(a)$$

$$(\lambda_{VV}+\mu_{VV,n-1}+\psi_1)P_{VV,n}=\lambda_{VV}P_{VV,n-1}+\mu_{VV,n}P_{VV,n+1,}\to(b)$$

$$(\lambda_{BB}+\mu_{BV1}+\psi_1)P_{BV1}=\psi_1P_{VV1}+\mu_{BV1}P_{BB2}+\psi_1P_{VB1}+\mu_{VV0}P_{VV0}\to(c)$$

$$(\lambda_{VB}+\mu_{VB1}+\psi_2)P_{VB1}=\psi_2P_{BV1}+\mu_{BV1}P_{VB1}+\psi_2P_{BB1}+\mu_{VV0}P_{VV0}\to(d)$$

$$(\lambda_{BB1}+\mu_{BB1}+\psi_3+b)P_{BB1}=\psi_2P_{VB1}+\mu_{VB1}P_{BB1}+rP_{DD1}\to(e)$$

$$(\lambda_{BB1}+\mu_{BB1}+b+\mu_3)P_{BBn}=\lambda_{BB}P_{VB,n-1}+\psi P_{VB,n}+rP_{DD,n}+\mu_{BB}P_{BB,n+1}\ ,\ n\ge 2\to(f)$$

$$(\lambda_{DD}+r)P_{DD1}=bP_{BB1}\to(g)$$

The matrix Q defined by

$$Q=\begin{bmatrix} J_{00} & J_{01} & & & \\ J_{10} & J_{11} & J_{12} & & \\ & J_{21} & M_1 & M_0 & \\ & & M_2 & M_1 & M_0 \\ & & .. & .. & .. \end{bmatrix}$$

Where the partitioned matrices are becoming in Q as follows

$$J_{00}=-\lambda,\ J_{01}=[\lambda\ 0\ 0],\ J_{10}=\begin{bmatrix}\mu_1\\ \xi\mu_2\\ \mu_2\end{bmatrix},$$

$$J_{11}=\begin{bmatrix}-\lambda-\mu_1 & 0 & 0\\ 0 & -\lambda-\xi\mu_2-\beta & \beta\\ 0 & 0 & -\lambda-\mu_2\end{bmatrix},\ J_{12}=\begin{bmatrix}\lambda & 0\\ \lambda & 0\\ 0 & \lambda\end{bmatrix},\ J_{21}=\begin{bmatrix}\xi\mu_2 & \mu_1 & 0\\ \mu_2 & 0 & \mu_1\end{bmatrix},$$

$$M_0=\begin{bmatrix}\lambda & 0\\ 0 & \lambda\end{bmatrix},M_1=\begin{bmatrix}-\lambda-\mu_1-\xi\mu_2-\beta & \beta\\ 0 & -\lambda-\mu_1-\mu_2\end{bmatrix},M_2=\begin{bmatrix}\mu_1+\xi\mu_2 & 0\\ 0 & \mu_1+\mu_2\end{bmatrix}$$

4. Steady state analysis

The queueing model represent the above is established if and only if $\frac{\lambda}{\mu}<1 where \frac{\lambda}{\mu}=\frac{\lambda}{\mu_1+\mu_2}$

The proving for the above segment following conditions. Let L_i be the number of customer in the model instantly after the exit of i^{th} customer.
Then $\{L_i : i\varepsilon L\}$ providing the equation $L_i=\begin{Bmatrix}L_{i-1}-1+w_i, if\ L_{i-1}\ge 1\\ w_i\ if\ L_{i-1}=0\end{Bmatrix}$ where w_i is the number of occurrences over the service of i^{th} customer. evidently $\{L_i : i\varepsilon L\}$ is an irreducible a periodic Markov chain. Thus if, $\frac{\lambda}{\mu}<1$ the markov chain $\{L_i : i\varepsilon L\}$ is ergodic and hence the condition is sufficient.

Now we obtain the position for the system to extent a steady state. Let $A_1=\{a_0,a_1,a_2...\}$ be the steady probability vector Q.If a_0 is a scalar, $A_1=\{a_{10},a_{11},a_{12}\}$ and $A_i=\{a_{i1},a_{i2}\}\ for\ i\ge 2$ .The vector A satisfies the condition AQ=0and Ae=1.where e is a column vector of order i+1 with all its elements equal to one. When the stableness position is fulfilled, the sub vector of A, equivalent to the different steps are given by the equations $A_j=A_2\Delta^{l-2}, l\ge 3, where\ \Delta$ is the minimal nonnegative resolution of the second order equation

$$\Delta^2F_2+\Delta F_1+F_0=0 \qquad (1)$$

If Δ, a_0, a_1 *and* a_2 *are obtained by solving the equation*

$$a_0J_{00}+a_1J_{10}=0\ \to(2)$$

$$a_0J_{01}+a_1J_{11}+a_2J_{21}=0\ \to(3)$$

$$a_1J_{12}+a_2F_1+a_2\Delta F_2=0\ \to(4)$$

and the normalizing equation

$$a_0+a_1e+a_2(I-\Delta)^{-1}e=1\to(5)$$

The matrix Δ *of equation*(1) *is given by* $\Delta=\begin{bmatrix}\Delta_{11} & \Delta_{12}\\ 0 & \Delta_{22}\end{bmatrix}$

$$where\ \Delta_n=\lambda+\mu_1+\xi\mu_2+\beta-\frac{\sqrt{(\lambda+\mu_1+\xi\mu_2+\beta)^2-4\lambda\mu_1-4\lambda\xi\mu_2}}{2\mu_1+2\xi\mu_2}$$

$$\Delta_{12}=\frac{\frac{\lambda}{\mu}-(\mu_1+\xi\mu_2)\Delta_{11}}{\mu},\Delta_{22}=\frac{\lambda}{\mu}$$

The proving for the above results as $F_0, F_1\ and\ F_2$ are upper triangular matrix, Δ is necessarily an upper triangular matrix. The estimation of Δ_{11}, are comes from the report that Δ is minimal nonnegative solution of equation(1).
Remark: Through Δ has a nice format which permit as to construct the possibilities are

$$\Delta^k = \begin{bmatrix} \Delta_n^{\ k} & \Delta_{12}\sum_{l=0}^{g-1}\Delta_{11}^{\ l}\Delta_{22}^{\ g-l-1} \\ 0 & \Delta_{22}^{\ g} \end{bmatrix}$$

for $g \geq 1$*.The expression* Δ_{11} *for may not be easy to carry out the computations.*

5. Analysis of busy period

For the functioning vacation system, busy for the system is the time distance between two consecutive departures, that leaves the system empty. Let $R_{ll}^{\ 1}(g,x)$ the conditional probability that a Quasi birth death process starting in the state(i,l) at time t=0 attain the level i-1 for the starting time t=0 extent the level i-1 for the first time no following time after exactly 'g' transitions to the left and does soby entering the state(i-1, l').

For our contribution we initiate the joint transform

$$\overline{R}_{ll}^{\ '} = \sum_{g=1}^{\infty} z^g \int_0^{\infty} e^{-qx} dR_{ll}^{\ '}(g,x); |Z| \leq 1, \operatorname{Re} s(q) \geq 0 \qquad (6)$$

and the matrix $\overline{R}(z,q) = \left(R_{ll'}(z,q)\right) \rightarrow (7)$ .The matrix $\overline{G}(z,q)$ is the unique solution of the equation [see ref(5)]. $\overline{R}(z,q) = z(qI-F_1)^{-1}F_2 + (qI-F_1)^{-1}F_0\overline{R}^2(z,q) \rightarrow (8)$

The matrix $R = \overline{R}(1,0)$ do the first sentence, except for the boundary conditions. If we know Δ matrix then R matrix can be evaluated. $R = -\left(F_1 + \Delta F_2\right)^{-1} F_2 \rightarrow (9)$ otherwise, to apply reduction method to determine R. For the extremity level conditions 2,1,0. *Let* $R_{ll'}^{\ (2,1)}(g,x), R_{ll'}^{\ (1,0)}(g,x), R_{ll'}^{\ (0,0)}(g,x)$ be the conditional probability for the first paragraph t from level 2 to level1,level 1 to level 0 and the first return time to the level 0 respectively. Then the equation(8) we get,

$$\overline{R}^{(2,1)}(z,q) = z\left(qI - F_1\right)^{-1} J_{21} + \left(qI - F_1\right)^{-1} F_0 \overline{R}(z,q)\overline{R}^{(2,1)}(z,q) \rightarrow (10)$$

$$\overline{R}^{(1,0)}(z,q) = z\left(qI - J_{11}\right)^{-1} J_{10} + \left(qI - J_{11}\right)^{-1} J_{12}\overline{R}^{(2,1)}(z,q)\overline{R}^{(1,0)}(z,q) \rightarrow (11)$$

$$\overline{R}^{(0,0)}(z,q) = \begin{bmatrix} \frac{\lambda}{q+\lambda} & 0 & 0 \end{bmatrix} \overline{R}^{(1,0)}(z,q) \rightarrow (12)$$

Then the Laplace stieltjes transform(LST) of the busy period is the first element $\overline{R}^{(1,0)}(z,q)$,Where $\overline{R}^{(1,0)}(z,q)$ is a 3×1 matrix .For our advantage

$$R_{21} = \overline{R}^{(2,1)}(1,0), R_{10} = \overline{R}^{(1,0)}(1,0), R_{00} = \overline{R}^{(0,0)}(1,0) \rightarrow (13)$$

The positive recurrence of the quasi birth death process, matrices $R, R_{21}, R_{10}, R_{00}$ are all stochastic.

If we let $D_0 = \left(-F_1\right)^{-1} F_2 \ ; D_2 = \left(-F_1\right)^{-1} F_0 \rightarrow (14)$ .Then R is the minimal nonnegative solution [ref [9]] to the matrix equation $R = D_0 + D_2 R^2 \rightarrow (15)$ from (10),(11)&(12) becomes and we get

$$R_{21} = -\left(F_1 + F_0 R\right)^{-1} J_{21} \rightarrow (16)$$

$$R_{10} = -\left(J_{11} + J_{12} R_{21}\right)^{-1} J_{10} \rightarrow (17)$$

$$R_{00} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} R_{10} \rightarrow (18)$$

The equation (8)is equivalent to

$$zF_2 - \left(qI - F_1\right)\overline{R}(z,q) + F_0\overline{R}^2(z,q) = 0 \qquad \rightarrow (19)$$

$$W = -\left.\frac{\partial \overline{R}(z,q)}{\partial q}\right]_{z=1,q=0} \quad and \quad \overline{W} = \left.\frac{\partial \overline{R}(z,q)}{\partial q}\right]_{z=1,q=0} \qquad \rightarrow (20)$$

The derivative of equation (19) [ref [9]]

$$W = -F_1^{-1}R + D_2\left(RW + WR\right) \rightarrow (21)$$

$$\overline{W} = D_0 + D_2 R\overline{W} + D_2\overline{W}R \quad \rightarrow (22)$$

With zero as beginning value for W and $\overline{W}$,the consecutive substitutions in the above equations to produce the values of W and $\overline{W}$ .Applying an absolutely similar arguments (10),(11)&(12) we get

$$W_{21} = -\left(F_1 + F_0 R\right)^{-1}\left(I + F_0 W\right) R_{21} \rightarrow (23)$$

$$W_{10} = \left(J_{11} + J_{12}R_{21}\right)^{-1}\left(I + J_{12}W_{21}\right) R_{10} \rightarrow (24)$$

$$W_{00} = \begin{bmatrix} \frac{1}{\lambda} & 0 & 0 \end{bmatrix} R_{10} + \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} W_{10} \rightarrow (25)$$

$$where W_{21} = \frac{-\partial \overline{R}^{(2,1)}(z,q)}{\partial q}, W_{10} = \frac{-\partial \overline{R}^{(1,0)}(z,q)}{\partial q}$$

$$W_{00} = \frac{-\partial \overline{R}^{(0,0)}(z,q)}{\partial q} \ and\ W_{10}\ is\ a\ 3\times 1\ matrix$$

W_{00} *is a scalar*.

The first element of the matrix W_{10} & W_{00} are average lengths of a busy period and a busy cycle respectively. It follows that equation (10)&(11) that

$$\overline{W_{21}} = -\left(F_1 + F_0 R\right)^{-1}\left(J_{21} + F_0 W R_{21}\right) \rightarrow (26)$$

$$\overline{W_{10}} = -\left(J_{11} + J_{12}R_{21}\right)^{-1}\left(J_{10} + J_{12}W_{21}R_{10}\right) \rightarrow (27)$$

The first elemental of the vector $\overline{W_{10}}$ is the average number of service finishing in a busy period.

6. Waiting time in the queue

Let H(t) be the waiting in the queue of an arriving customer. There is no customer in the system, the arrival receives service instantly. If the either of the two servers is not busy then also there would be no delay in getting service. The customer gets his service without waiting for probability is $a_0 + a_{10} + a_{11} + a_{12}$ .The customer has no wait before getting the service for the probability is $1 - a_0 - a_{10} - a_{11} - a_{12}$ .The waiting time may be consider as the time until absorption in a markov chain with position space $S_1 = \{*\} \cup \{2,3.....\} \rightarrow (28)$ .Here * is the absorbing state, which corresponds to taking the tagged customer in to service and is obtained by lumping together the level states $0 = \{(0,0)\}$ & $1 = \{(1,0),(1,1),(1,2)\}$ *for* $i \geq 2$, the level I is given by $i = \{(i,j), i = 1 or 2\}$. the tagged customer joins the queue, the following arrivals will not affect the waiting time in the queue. $\overline{Q}$ of the Markov process is specified by

$$\overline{Q} = \begin{pmatrix} F_2 e & T & \\ & F_2 & T \end{pmatrix} \rightarrow (29)$$

$$Where T = \begin{pmatrix} -\mu_1 - \xi\mu_2 - \beta & \beta \\ 0 & -\mu_1 - \mu_2 \end{pmatrix}$$

Define vector

$$Z(t) = \left(z_*(t), z_2(t), z_3(t)....\right) \rightarrow (30)$$

$$where Z_i(t) = \left(z_{ii}(t), z_{i2}(t)\right), whereas\ i \geq 2, \rightarrow (31)$$

The elements of $Z_i(t)$ are the probabilities that at time t, the CTMC with generator $\overline{Q}$ is in the respective conditions of level i. By the Poisson arrivals see time averages $Z(0) = \left(a_0 + a_{11} + a_{10} + a_{12}, a_2, a_3......\right) \rightarrow (32)$, obviously $W(t) = Z_*(t)$ whereas $t \geq 0$ .The LST of W(t) is specified by

$$\overline{W} = \sum_{i=2}^{\infty} Z_i(0)\left[\left(qI - T\right)^{-1} F_2\right]^{i-2}\left(qI - T\right)^{-1} F_2 e \rightarrow (33)$$

The average waiting time can be determined from $\overline{W}(S)$ as,

$$E(w) = -\overline{w}'(0) = \sum_{i=1}^{\infty} a_{2+i} \sum_{j=0}^{i-1} A^j (-T)^{-1} A^{i-j} Ae + \sum_{i=0}^{\infty} a_{2+i} A^i (-T)^{-2} F_2 e \rightarrow (34)$$

Where $(-T)^{-1}\ F_2$ is a stochastic matrix, hence (33)can be clarified as

$$E(w) = -\overline{w}'(0) = \sum_{i=1}^{\infty} a_{2+i} \sum_{j=0}^{i-1} A^j (-T)^{-1} e + \sum_{i=0}^{\infty} a_{2+i} A^i (-T)^{-1} e \ \rightarrow (35)$$

Let $B = \sum_{i=0}^{\infty} a_{2+i} A^i \rightarrow (36)$ since A is stochastic, we get $Be = a_2 (I-\Delta)^{-1} e = 1 - a_0 - a_{10} - a_{11} - a_{12} \rightarrow (37)$ The outcome can be used to obtain a near to the value of B and so that the 2nd term in (35) to any likely degree of accuracy. Now consider the matrix $A_2 = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} \rightarrow (38)$ which has the property, $AA_2 = A_2 A = A_2 \rightarrow (39)$ Then we get, $\sum_{j=0}^{i-1} A^j (I - A + A_2) = I - A^i + iA_2,\ for\ i \geq 1 \rightarrow (40)$.

The matrix $(I - A + A_2)$ is nonsingular, then

$$\left[\sum_{i=1}^{\infty} a_{2+i} (I - A^i + iA_2) \right] (I - A + A_2)^{-1} (-T)^{-1} = e \rightarrow (41)$$

with this simplification,

$$E(w) = \left[a_2 \left(\Delta (I-\Delta)^{-1} + I + \Delta (I-\Delta)^{-2} A_2 \right) - B \right] \times (I - A + A_2)^{-1} (-T)^{-1} e + A(-T)^{-1} e \rightarrow (42)$$

7. Queue length

Let Q_v be the queue length of the vacation, based on the condition that the servers1 and 2 are busy. If $\frac{\lambda}{\mu} < 1$,then $Q_v = Q_0 + Q_d$,where Q_0 & Q_d are two independent stochastic variables. Q_0 is the queue length of the heterogeneous servers beyond the vacation and Q_d can be explained as the additional queue length due to vacation and slow service, based on the condition that the servers1 and 2 are busy. The above segment proving for as,

Let P_b indicate the probability that the servers 1and 2 are busy, then

$$P_b = \sum_{n=2}^{\infty} a_{n2} = \sum_{n=2}^{\infty} a_{22} \left(\frac{\lambda}{\mu} \right)^{n-2} + \sum_{n=3}^{\infty} a_{21} \Delta_{12} \sum_{j=0}^{n-3} \Delta^j_{11} \left(\frac{\lambda}{\mu} \right)^{n-j-3}$$

$$= a_{22} \left(\frac{\lambda}{\mu} \right) \sum_{c=0}^{\infty} \left(\frac{\lambda}{\mu} \right)^c + a_{21} \Delta_{12} \sum_{c=0}^{\infty} \Delta^c_{11} \sum_{c=0}^{\infty} \left(\frac{\lambda}{\mu} \right)^c;$$

Where c=n-3

$$P_b = \left(1 - \frac{\lambda}{\mu} \right)^{-1} \left[a_{22} \left(\frac{\lambda}{\mu} \right) + a_{21} \Delta_{12} (1 - \Delta_{11})^{-1} \right] \text{ so that}$$

$Q_v(z)$ be the generating function of the queue length based on the condition that the servers1and 2 are busy as follows

$$Q_v(z) = \frac{1}{P_b} \sum_{n=1}^{\infty} a_{n1} z^{n-2}$$

$$Q_v(z) = \frac{1}{P_b} \sum_{n=2}^{\infty} a_{22} \left(\frac{\lambda}{\mu} \right)^{n-2} z^{n-2} + \frac{1}{P_b} \sum_{n=3}^{\infty} \left(a_{21} \Delta_{12} \sum_{j=0}^{n-3} \Delta^j \left(\frac{\lambda}{\mu} \right)^{n-j-3} \right) z^{n-3} \rightarrow (45)$$

specified by the estimating step similar to that of P_b ,then

$$Q_v(z) = \frac{1 - \frac{\lambda}{\mu}}{1 - \left(\frac{\lambda}{\mu} \right) z} \left\{ \tau(a_{22}) \left(\frac{\lambda}{\mu} \right) z + \frac{a_{21} \Delta_{12}}{1 - \Delta_{11} z} \right\}$$

$$Q_v(z) = Q_0(z) Q_d(z) \rightarrow (46)$$

$$where\ Q_0(z) = \frac{1 - \frac{\lambda}{\mu}}{1 - \left(\frac{\lambda}{\mu} \right) z} \ and\ Q_d(z) = \tau \left(a_{22} \left(\frac{\lambda}{\mu} \right) z + \frac{a_{21} \Delta_{12}}{1 - \Delta_{11} z} \right)$$

8. Performance Measures

1. The expectation that the system is empty: $P_0 = a_0$
2. The expectation that the server1 is idle: $P_{idle} = a_0 + a_{11} + a_{12}$
3. The expectation that the server2 is on vacation: $P_{vac} = a_0 + a_{10}$
4. The expectation that the server2 is working in vacation mode:
   $P_s = \sum_{j=1}^{\infty} a_{ji} = \frac{a_{11} + a_{21}}{1 - \Delta_{11}}$
5. The expectation that the server2 is usual mode: $P_N = 1 - a_0 - P_S$
6. The average number of customer in the model:
   $\mu_{N1} = \sum_{j=1}^{\infty} j a_j e$
   $\mu_{N1} = a_{10} + a_{11} + a_{12} + a_2 (1-\Delta)^{-2} \Delta^{-1} e - a_2 \Delta^{-1} e$

9. Numerical results

Investigate the outcome of parameters λ, β, ξ on the basic performance measures.

μ_w : Average waiting time in the queue

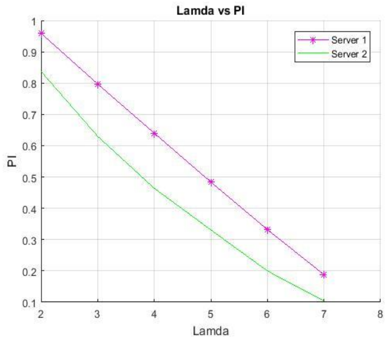
μ_{L1} :Average length of a busy period

μ_{L2} : Average length of a busy cycle

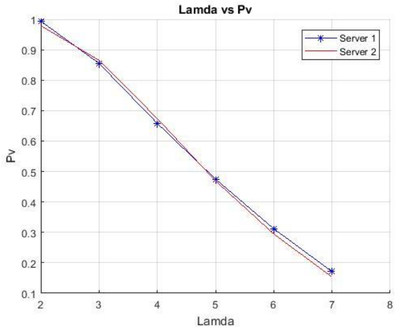
Table

Case 1: $\mu_1 = 10, \mu_2 = 8, \beta = 1, \xi = 0.6$ Case2: $\mu_1 = 8, \mu_2 = 1, \beta = 1, \xi = 0.6$

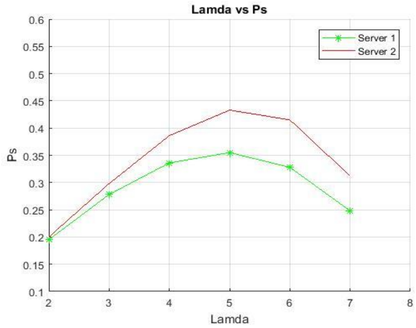
λ	1/2	P_I	P_V	P_S	P_N	μ_{N1}	μ_W	μ_{L1}	μ_{L2}
2	1 2	0.9590 0.83700	0.9938 0.9789	0.1950 0.2000	0.1520 0.1440	1.7460 2.2460	0.0720 0.0750	0.2102 0.2550	0.6260 0.6120
3	1 2	0.7970 0.6300	0.8550 0.8650	0.2780 0.2980	0.1970 0.1670	2.0040 2.3610	0.0790 0.0860	0.1770 0.2070	0.3560 0.3460
4	1 2	0.6400 0.4640	0.6580 0.6740	0.3360 0.3860	0.2760 0.2200	2.2490 2.6600	0.0870 0.1010	0.1520 0.17870	0.2560 0.2540
5	1 2	0.4840 0.3310	0.4750 0.4690	0.3550 0.4330	0.3810 0.3080	2.6170 3.2480	0.0970 0.1170	0.1320 0.1640	0.2020 0.2130
6	1 2	0.3320 0.2000	0.3120 0.2950	0.3280 0.4150	0.5100 0.4400	3.2630 4.3320	0.1060 0.1320	0.1220 0.1582	0.1720 0.2030
7	1 2	0.1880 0.1040	0.1710 0.1530	0.2480 0.3120	0.6710 0.6150	4.7720 6.6960	0.1130 0.1390	0.1153 0.1423	0.1683 0.1987



Picture 1



Picture 2



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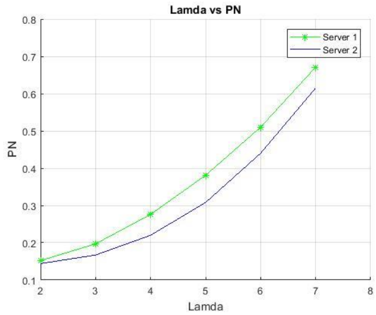
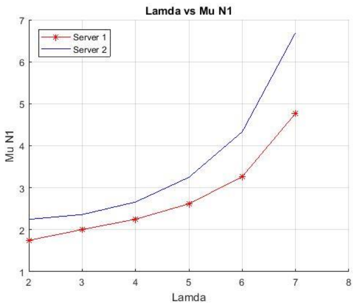
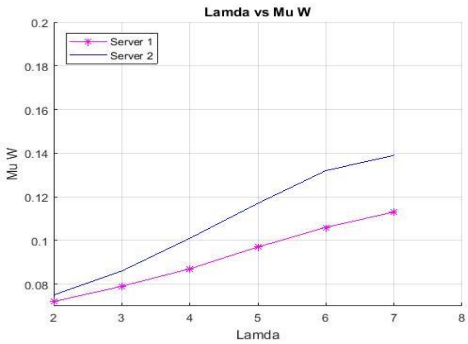


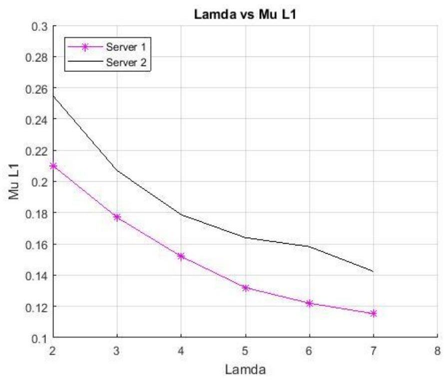
Picture 4



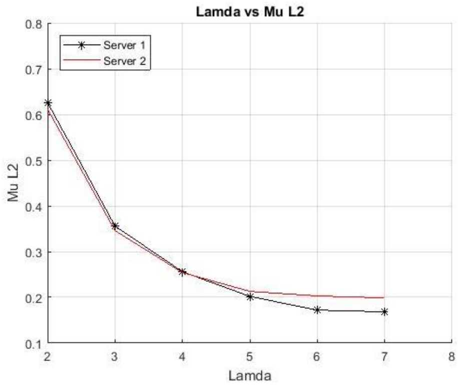
Picture 5



Picture 6



Picture 7



Picture 8

Since $\mu_1 \& \mu_2$ are fixed, then the traffic intensity $\frac{\lambda}{\mu}$ increases with λ. Due to the values of P_N, μ_{N1}, μ_w raised and $P_V \& P_{idle}$ reduced as λ increases. Therefore, initially the value of P_S becoming larger with P_S. For this reason μ_{L1}, μ_{L2} display an early descending direction. But as λ in addition that raised P_S reduction as established due to the high utilization factor. Therefore $\mu_{N1}, \mu_w, \mu_{L1}$ reverse the way of change. For the reason that the effect of P_V and P_{idle}, this change in direction occurs only at a next stage for μ_{L2}. The effect of the vacation parameter β becomes more predominant when $\mu_1 < \mu_2$. Due to this the measures $P_V \& P_{idle}$ take smaller values and the measures $\mu_{N1}, \mu_w, \mu_{L1}$ take larger values in case2 compared to their values in case1.

10. Conclusion

In this article, we investigated as multiserver queueing model with heterogeneous service. One server take the place of multiple vacation. But this server provides the service at a lesser rate during vacation. The busy period analysis are discussed. Mean waiting time of a customer has been computed, and also, we derive the queue length. The numerical examples are given to test the feasibility. The graphs show that the correctness of the result.

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