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Research paper

Monophonic wirelength on Circulant Networks

K. Uma Samundesvari[1]*, Sindhuja G. Michael [2]

[1]*Assistant Professor, Department of Mathematics, Noorul Islam Center for Higher Education, Thuckalay-629 180, Kanyakumari,India*
[2]*Assistant Professor, Department of Mathematics, Ponjesly College of Engineering, Nagercoil, 629003, India*
**Corresponding author E-mail:kuskrishna@gmail.com*

Abstract

In this paper, we define the monophonic embedding of graph *G* into graph *H* and we present an algorithm for finding the monophonic wirelength of circulant networks into the family of grids $M(n\times 2)$, $n\geq 2$.The monophonic embedding of a graph *G* into a graph *H* is an embedding denoted by f_m is a bijective map from the vertex set of *G* into the vertex set of *H* and f_m is a one-one mapping from the edge set (x, y) of *G* into $P_m(H)$ where $P_m(H)$ is the set of monophonic paths between $f_m(x)$ and $f_m(y)$ for every $f_m(x), f_m(y)\in H$. The monophonic wirelength of f_m of *G* into *H* is the sum of distances of monophonic paths between two vertices $f_m(x)$ and $f_m(y)$ in *H* such that $(x, y)\in E(G)$. This paper presents a monophonic algorithm to find the monophonic wirelength of circulant networks $G(2n, \pm S)$, where $S\subseteq \{1,2,3,\dots,n\}$ into the family of grids $M[n\times 2]$,$n\geq 2$. We also derived a Lemma to get the monophonic edge congestion $MEC(G,H)$.

Keywords: *Circulant Networks; Edge Congestion; Grid; Monophonic Distance; Wirelength.*

1. Introduction

The distance d(x, y) between two vertices x and y in a graph G is the length of the shortest path from x to y in G . An edge x_ix_j is a chord of a path $x_0,x_1,x_2,\dots,x_n$ if $j \geq i+2$. A monophonic path is a path if it contains no chord. The length of the longest x-y monophonic path of a graph G is called the monophonic distance $d_m(x,y)$ for every vertices x,y in G. A monophonic path from x to y with length $d_m(x,y)$ is called an x - y monophonic "as stated in [1,2,3,4]". Consider a graph H, since other graphs or networks are embedded into it , as host graph and graphs or networks which are embedded in H are called guest graph "as given in [5,6,7]". Let G(V,E) and H(V,E) be finite graphs with n vertices. An embedding f of G into H is defined as follows:

1) f is a bijective map from V (G) → V (H)
2) f is a one-to-one map from E(G) to $\{P_f(f(u), f(v)) : P_f(f(u), f(v))$ is a path in H between $f(u)$ and $f(v)$ for $(u, v) \in E(G)\}$ "as defined in [8,9]. i.e., The embedding f of G to H is a bijective mapping from the vertex set of G to the vertex set of H and every edge (u, v) ∈ E(G) is mapped to a path between f (u) and f (v) in H. The edge congestion of an embedding f of G into H is the maximum number of edges of the graph G that are embedded on any single edge of H. The wirelength of an embedding f of G into H is given by

$$WL_f(G,H)=\sum_{(u,v)\in E(G)} d_H\left(f(u),f(v)\right)=\sum_{e\in E(H)} EC_f\left(G,H(e)\right) \tag{1}$$

If we find an embedding of G into H which produces the minimum wirelength WL(G, H), such problem is called the wirelength problem "as stated in [9]". We use definitions, Lemmas and Theorems from [1], [2], [5], [6], [7], [8] and [9] for this work.

2. Monophonic wirelength problem

Definition 2.1: *Let G(V, E) and H (V, E) be finite graphs with n vertices. An embedding f_m: G → H is called a monophonic embedding if f_m maps each vertex of G into a vertex of H and each edge (x, y) of G is mapped to a monophonic path between $f_m(x)$ and $f_m(y)$ in H.*

Definition 2.2: *Let f_m: G → H be a monophonic embedding. The monophonic edge congestion of f_m of G into H is the maximum number of edges of the graph G that are embedded on an edge e∈H and is given by*

$$MEC_{f_m}(G,H)=\max MEC_{f_m}(G,H(e)) \tag{2}$$

The monophonic wirelength problem of a graph G into H is the problem of finding a monophonic embedding f_m : G→ H that produces the monophonic wire length MW L(G, H).

Definition 2.3: *Let f_m: G → H be a monophonic embedding. The monophonic wirelength MWL (G, H) of f_m is given as*

$$MWL_{f_m}(G,H)=\sum_{(x,y)\in E(G)} d_m(f_m(x), f_m(y)) \tag{3}$$

Proposition 2.4: *For embeddings f : G → H and the monophonic embeddings f_m: G → H, $WL_f(G, H) \leq MWL_{fm}(G,H)$*

Proof: From Lemma 2.3 "as proved in [2]", we have

$$\sum_{(x,y)\in E(G)} d_H\left(f(x),f(y)\right)\leq \sum_{(x,y)\in E(G)} d_m\left(f_m(x),f_m(y)\right) \tag{4}$$

And using definitions "as defined in [9]" we write,

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$$
\begin{aligned}
WL_f(G,H) &= \sum_{(x,y)\in E(G)} d_H(f(x), f(y)) \\
&= \sum_{e\in E(H)} EC_f(G, H(e)) \\
&\le \sum_{(x,y)\in E(G)} d_m(f_m(x), f_m(y)) \\
&= MWL_{f_m}(G,H) \qquad (5)
\end{aligned}
$$

Therefore,

$$WL_f(G,H) \le MWL_{f_m}(G,H) \qquad (6)$$

Lemma 2.5: *(Monophonic congestion Lemma)Let G be an r-regular graph with n vertices. Let H be a finite graph with n vertices. Let f_m: G → H be a monophonic embedding of G into H. Let the graph H \ E_j, j = 1,2,…,p; 0<p≤|E(G)| , have the components H_i ,i =1,2 and G_i = f_m $^{-1}$(H_i),where E_j's are the edge cuts of H, form a partition in H and have the following properties:*

i) For m≥0, there are m edges (x, y) ∈G_i, i = 1, 2; such that the monophonic path $P_{f_m}(f_m(x), f_m(y))$ has exactly two edges in E_j.
ii) The monophonic path $P_{f_m}(f_m(x), f_m(y))$ has exactly one edge in E_j for every (x,y)∈ G with x∈ G_1& y∈G_2 where G_1 is the maximum subgraph in G. Then $MEC_{f_m}(E_j)$ is monophonic and the monophonic wirelength of f_m of G into H is given by

$$MWL_{fm}(G,H) = \sum_{j=1}^{p} MEC_{fm}(E_j) \qquad (7)$$

Where, *$MEC_{fm}(E_j)$= r |V (G1)| −2 |E (G1)|+2, m ≥ 0.*

Proof: As E_j , j = 1,2,…,p are edge cuts of H, $E_j = \{(u,v)\in E(H); u\in H_1, v\in H_2\}$.
Let $T = \{(x,y)\in E(G); x\in G_1, y\in G_2\}$. Since there are 'm' edges (x,y) ∈ G_i, i = 1,2; the monophonic path $P_{f_m}(f_m(x), f_m(y))$ in H has exactly two edges in E_j. Therefore, the monophonic edge congestion is increased by 2m from the edge congestion of E_j. Also the monophonic path $P_{f_m}(f_m(x), f_m(y))$ in H has exactly one edge in E_j for every (x,y)∈G with x∈G_1 and y∈G_2. Hence $MEC_{f_m}(E_j) = |T+2m|\cdot$
Where |T| = r |V (G_1)| −2 |E (G_1)| by Lemma 2 in [9].
Therefore, EC_{fm} is monophonic as G_1 is maximum in G. The edge cuts E_j, j = 1, 2, p form a partition in H. Thus, we write

$$MWL_{fm}(G,H) = \sum_{j=1}^{p} MEC_{fm}(E_j) \qquad (8)$$

3. Monophonic wirelength on circulant networks

Definition 3.1:*A circulant undirected graph denoted by G(n, ±S) where S⊆ {1,2,3,…,[n/2]} ,n ≥ 3 is defined as a graph consisting of the vertex set V ={0,1,2,…,n-1} and the edge set E={ (i,j) : |i-j|≡s(mod n), s∈S} " as defined in [5]".*
To present the monophonic wirelength on circulant networks, we consider the monophonic embedding f_m from the circulant graph G[2n, ±S], S⊆ {1, 2, 3… n}, into the grid M (n×2), n ≥2.

3.1. Monophonic algorithm

Consider the monophonic embedding f_m : G[2n, ±S]⟶M [n×2]. Let V(G [2n, ±S]) = {0, 1, 2… 2n-1} and these vertices are labeled as the vertices of a cycle in clockwise.Let V(M [n×2]) = {0, 1, 2… 2n-1}, these vertices are named as follows

- In Column 1 of M [n×2] the vertices {0, 1, 2… n-1} are named in an ascending order from the top.
- In Column 2 of M [n×2] the vertices {n, n+1…, 2n-1} are named in an ascending order from the top.

Lemma 3.2:*For n ≥2, the rows of the grid M[n×2] is defined as R_i = {i-1, n+i-1} ; i = 1,2,…,n are maximum subgraphs in G[2n;{1,2,…,n}].*
The proof holds by Theorems 3.3 and 3.4 "as proved in [6]".

Lemma 3.3:*For j = 1,and n ≥ 2, the column of the grid M[n×2] is given by C_j ={0,1,2,…,n-1}, which is maximum in G[2n;{1,2,…,n}].*
The proof follows from Theorem 3.3 and 3.4 "as proved in [6]".

Theorem 3.4:*Let f_m : G[2n, ±S] ⟶M [n×2] be a monophonic embedding. For n ≥ 2, the wirelength of G[2n,±S], S⊆ { 1, 2…, n} into M [n×2] induced by f_m is monophonic.*

Proof: Let A_{i1} and A_{i2} be the components of M[n×2] \ H_i , H_i be the horizontal edge cut of the grid M[n×2] .Then the vertex set of A_{i1} is the rows of the component A_{i1}. (ie) V(A_{i1}) is R_i ,i =1,2,…,2n-1. Refer Figure 1(a). Let B_{j1} and B_{j2} be the components of M [n×2] \ W_j,W_j be the vertical edge cut of the grid M [n×2]. Then the vertex set of B_{j1} is C_j, j = 1. Under f_m let G_{i1} = $f_m^{-1}(A_{i1})$ and G_{i2}= $f_m^{-1}(A_{i2})$. Since the horizontal edge cuts satisfy the properties stated in Lemma 2.5 and are maximum in G, $MEC_{f_m}(H_i)$ is monophonic for i=1,2,…2n-1. For this see Figure 1(b). Let G_{j1} = $f_m^{-1}(B_{j1})$ and G_{j2}= $f_m^{-1}(B_{j2})$. Moreover the vertical edge cut W_j satisfies the properties stated in Lemma 2.5. Also by Lemma 3.3, G_{j1} is a maximum sub graph induced by the vertices of C_j, j = 1. Hence by Lemma 2.5, $MEC_{f_m}(W_j)$ is monophonic for j=1.Thus $MEC_{f_m}(G,H)$ is monophonic.

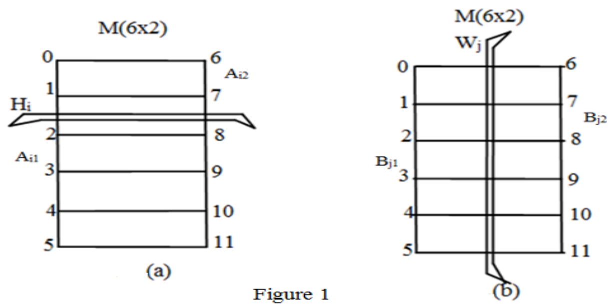

Fig. 1:(a) Each $H_i$is an edge cut of the grid on M [6×2] which disconnects M [6×2] into two components $A_{i1}$and A_{i2} where V (A_{i1}) Is R_i. (b) Each $W_j$is an edge cut of the grid on M[6×2] which disconnects M [6×2] into two components $B_{j1}$and B_{j2} where V (B_{j2}) is C_j.

Theorem 3.5:*The monophonic wirelength of an r-regular graph G with 2n vertices into the grid M[n×2], n≥2 is given by MWL(G, M[n×2]) = WL (G, M [n×2]) +2m*

Proof: Let f_m : G → M [n×2] be a monophonic embedding. Since each edge (x, y) of G is mapped to a monophonic path between f_m(x) and f_m(y) in M[n×2], the edges of G are transformed into edges vertically or horizontally in M[n×2].Therefore there exists horizontal edge cuts H_i , i= 1,2,…,2n-1 in M[n×2]. As there are no monophonic paths $P_{f_m}(f_m(x), f_m(y))$, for every (x, y)∈G_i, i = 1, 2; having edges in H_i , the monophonic edge congestion of H_i is equivalent to the edge congestion of the edges of H_i .Also there exists a vertical edge cut W_j , j = 1. For m ≥ 0, there are m edges

(x,y)∈G_i , i = 1, 2; the monophonic paths $P_{f_m}(f_m(x), f_m(y))$ have exactly two edges in W_j and so the monophonic edge congestion of the edges of $W_j$is increased by 2m from the edge congestion of the edges of W_j. Hence the monophonic wirelength of each row equals the wirelength of each row of M[n×2] and the monophonic wirelength of each column differs by 2m from the wirelength of columns of M[n×2].
Therefore MWL(G,M[n×2]) = WL(G,M[n×2])+2m

Theorem 3.6:*MWL(G[2n,±1] , M[n×2])= WL(G[2n,±1], M[n×2])= 2(2n-1).*
Proof: As there is no edge (a ,b) ∈ G_i, i = 1, 2; the monophonic path $P_{f_m}(f_m(a), f_m(b))$ *∈M[n×2] has exactly two edges in E_j , m=0. Therefore, the result follows from Theorem 3.5.*

4. Monophonic embedding algorithm

4.1. Aim

To find a monophonic embedding f_m : G→ H that produces the monophonic wirelength $MWL_{f_m}(G,H)$, where G is the family of circulant graph with 2n vertices of r-regular and H is the family of grid M[n×2],n ≥ 2.

4.2. Monophonic algorithm

i) Label the vertices of G[2n,{1,2,3…,n-1}] as a cycle from 0,1,2,…,2n-1
ii) Label the vertices of M[n×2] as follows:
- In Column 1 of M [n×2] the vertices {0, 1, 2, n-1} are labeled in an ascending order from the top.
- In Column 2 of M [n×2] the vertices {n, n+1…, 2n-1} are labeled in an ascending order from the top.

Case (i): Input:
Pre-image: The family of circulant graph G[2n,{1, 2, n-1}], n ≥2.
Image: The family of grids M[n×2] n ≥ 2.
Output: A monophonic embedding f_m of G[2n,{1,2,3…,n-1}] into M(n×2) given by f_m(x) = x with monophonic wire length MWL(G[2n,{1,2,3,…,n-1}] , M[n×2]) = WL+2(n-1)(n-2) ,n≥2.
Case (ii): Input:
Pre-image: The family of circulant graphs G[2n,{1, 2, n-2}], n ≥ 3.
Image: The family of grids M[n×2], n≥ 3.
Output: A monophonic embedding f_m of G[2n,{1,2,3…,n-2}] into M[n×2] given by f_m(x) = x with monophonic wire length MWL(G[2n,{1,2,3,…,n-2}], M[n×2]) = WL+2n(n-3) ,n≥3.
Case (iii): Input:
Pre-image: The family of circulant graphs G[2n,{1, 2,…, n-4}], n ≥ 5.
Image: The family of grids M[n×2],n ≥ 5.
Output: A monophonic embedding f_m of G[2n,{1,2,3…,n-4}] into M[n×2] given by f_m(x) = x with monophonic wire length MWL(G[2n,{1,2,3,…,n-4}], M[n×2]) = WL+2(n+2)(n-5) ,n≥5.
Case (iv): Input:
Pre-image: The family of circulant graphs G [2n, {1, 2, …,n}], n ≥2.
Image: The family of grids M [n×2], n ≥ 2.
Output: A monophonic embedding f_m of G[2n,{1,2,3…,n}] into M[n×2] given by f_m(x) = x with monophonic wire length MWL(G[2n,{1,2,3,…,n}], M[n×2]) = WL+2(n-1)(n-2) ,n≥2.
Proof: For all the above Cases (i) to (iv), using Theorem 3.4, the mapping f_m is monophonic and by Theorem 3.5, the results follows.

5. Conclusion

In this paper, we applied the monophonic idea on graph embedding *f* of two graphs from G into H. Using this concept; we have obtained a modified result in the wirelengthproblem, which does not exist. A new technique was found from the existing one. We have taken all possible family of circulant networks under study and applied the monophonic algorithm on *f* and based on the statistical data we obtained we came to the conclusion which yields the above findings.

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