



Calculation methods for multicircuit transport network fluid dynamic processes

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Abstract

The aim of the work is to create an algorithm for a fast and sufficiently accurate calculation of the parameters of a hydraulic network through which fluid transport is carried out. This type of network is characterized by the presence of a large number of contours, elements for various purposes, and branching. The presence of many contours in the network increases the probability of losing the density of connections, the occurrence of leaks, and accordingly the emergency situation. In paper the construction of a finite-difference scheme and the corresponding calculation algorithm is considered, taking as a basis the model of heat and mass transfer by the flow of a single-phase compressible fluid. The proposed solution method is a generalization of the sweep method to the problems of calculating the heat and mass transfer circuit with an arbitrary number of branching and pipe joints.

Keywords: Heat and Mass Transfer; Mathematical Modeling; Network Simulation Model.

1. Introduction

The paper considers hydrodynamic processes in a multicircuit branched network in the presence of heat and mass transfer in the transported product. The mathematical and numerical models of the pipeline system also take into account heat exchange with the environment. The article proposes a generalization of the sweep method [1-3] in order to create an algorithm aimed at solving the practical problem of calculating the heat exchange network. Similar problems for network models on the basis of linearized equations on a graph were considered, for example, in [4]. In this case, separate elements contain a two-phase flow of fluid, whereas in many other elements a flow of a single-phase weakly compressible fluid can be considered as a perfectly good approximation. It is clear that for such a complex system it is practically impossible to observe the fulfillment of Courant's criterion at all stages of the calculation. At the same time, the questions of the stability of the scheme and the accuracy of the calculation while saving the total calculation time remain highly relevant. The newly developed algorithm should be adapted to a parallel computing system, like all modern competitive codes. Proposed in this paper, an approach is aimed at overcoming these problems; it is based on the development of the sweep algorithm, which is a well-known economical and robust method of computational mathematics.

2. Formulation of the problem

For simplicity of understanding, the further presentation is given in the approximation of an inviscid compressible fluid. However, dissipative factors (viscosity, thermal conductivity) can be taken into account in the balance equations of momentum and energy,

respectively. Thus, the basis of the proposed technique is the traditional system of hydrodynamics equations for an inviscid compressible fluid, including the equation of continuity (mass balance), the equation of momentum conservation, and the equation of energy conservation. Taking into account that the characteristic linear dimension in the cross-section of the pipeline (for example, its effective diameter) is much smaller than its characteristic dimension in the longitudinal direction, let's go on to a simplified quasi-one-dimensional model whose equations have the form:

$$\frac{\partial \rho}{\partial t} + \frac{\partial (s\rho w)}{s\partial x} = 0, \quad (1)$$

$$\frac{\partial (\rho w)}{\partial t} + \frac{\partial (s\rho w^2)}{s\partial x} = -\frac{\partial P}{\partial x}, \quad (2)$$

$$\frac{\partial E}{\partial t} + \frac{\partial (sEw)}{s\partial x} = -P \frac{\partial (sw)}{s\partial x}. \quad (3)$$

In equations (1)-(3) ρ means the fluid density, P is the internal pressure, E is the internal energy per unit volume, w is the velocity, t is the time, x is the space coordinate, s is the channel cross-sectional area.

On the basis of this physical model, the finite-difference equations are obtained. The applied approach consists in constructing a discrete model that satisfies the conservativeness conditions, i.e. constructing the mesh analogs of the basic conservation laws, which are valid for a physical problem.

The numerical technique is analyzed according to the criteria of stability and accuracy of the solution. A method for solving the developed finite-difference model, that is the symbol sweep method, is formulated. The advantage of the method is the natural parallelization of the corresponding computational work. The method

is promising for use on multiprocessor systems, including systems with hybrid architecture.

3. Features of the algorithm

Features of the proposed algorithm are as follows:

1. A version of the symbolic sweep is considered, allowing a logical parallelization of the calculation of fluid dynamics in a multicircuit system when the balance equations in the nodes of the branching of the circuit are executed.
2. There are no fundamental difficulties in terms of generalizing the algorithm to the model of two-phase fluid dynamics in a complex circuit. In this case, the presence of two phases in any part of the circuit is allowed, both in pipes and in the area of joints. The complexity of the corresponding finite-difference problem (in the sense of the dimension of the system of equations solved numerically) can turn out to be significantly smaller than, for example, in the well-known RELAP package which subject area is described in [5-7].
3. At the level of the finite-difference scheme, the balance of energy and mass continuity is fulfilled, while the divergent approximation of the flow terms of the equations in the grid nodes is performed on the upwind principle, which ensures the unconditional stability of the sweep coefficients.

4. Verification of the model

The problem of instantaneous opening of a bolt at the inlet of an unheated volume with a homogeneous medium at the zero time point and in the absence of friction losses in a hydraulic connection is considered. The nodalization scheme is shown in Figure 1.

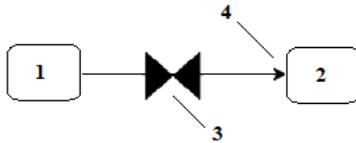


Fig. 1: Calculation scheme for verification test. Digit 1 means the time-dependent volume (setting the boundary pressure condition), digit 2 is the mixing chamber, digit 3 is the model of the bolt, digit 4 means model of the connecting pipeline.

In the modeling of the problem, the initial values and initial conditions for the parameters of the computational elements given in Table 1 were used.

Table 1: Initial values and initial conditions for the parameters of the computational elements

Parameter	Value
Volume of mixing chamber (V), m^3	10
The area of the cross-section of the connection with a fully open bolt (S), m^2	1
The length of the connection modeling the connecting pipeline (L), m	100
Pressure in a time-dependent volume (P_{IN}), MPa	16
Pressure in the mixing chamber at the initial time (P_0), MPa	15
Enthalpy in time-dependent volume (I), kJ/kg	100
Enthalpy in the mixing chamber at the initial time (I), kJ/kg	100

An analytical solution for the pressure in the element "mixture chamber" is the following:

$$P(t) = P_0 + [P_{IN} - P_0] \cdot [1 - \cos(\omega \cdot t)], \quad (4)$$

where $\omega = \sqrt{a \cdot b}$, $a = 1/V \cdot \left(\frac{\partial \gamma}{\partial P} + v \cdot \frac{\partial \gamma}{\partial I} \right)$, $b = \frac{S}{L} \cdot \frac{\partial \gamma}{\partial P} \cdot \frac{\partial \gamma}{\partial I}$

are the partial derivatives of the coolant density by pressure and enthalpy; v is the specific volume of heat-transfer agent.

The results of calculation analysis for the pressure are shown in Figure 2.

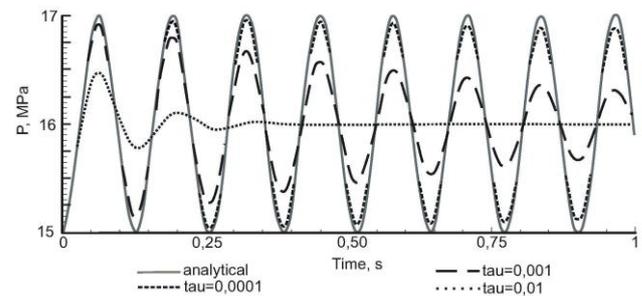


Fig. 2: Pressure in mixing chamber; tau is time step of finite-difference scheme; calculations were performed for space step of 1 m.

Comparison of the calculation results shows the good agreement between an analytical solution and the results for finite-difference scheme's time step 10^{-4} s with space step 1 m. Calculation analysis shows that for this physical model the best approximation of fluid density in nonstationary part of moment balance in divergence form is just an approximation with averaged density values of boundary cells.

Calculations of hydrodynamic fluid motion were also carried out taking into account dissipative factors (viscosity, thermal conductivity). In the Navier-Stokes equations system used in these calculations, the fluid parameters were calculated by the technique proposed in [8].

5. Conclusion

This paper presents the development of methodology for thermo-hydraulic calculation analysis of coolant circuit. Basic principles for this research are conservation of first principles equations both on differential equations level and on numerical scheme equation level. Also this methodology provides optimal structure of implicit and explicit time levels, and it has been analyzed both analytically and numerically. The results of verification show that balance relations for all step of analysis influent significantly into accuracy and sustainability of numerical scheme.

Acknowledgement

This work was supported by the Russian Science Foundation (project No. 17-71-10045).

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