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A New Approach To Solving The Time-Cost Trade-Off Problem Based on The Genetic Algorithm and Fuzzy Theory

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Abstract

Time and cost are two important and controllable objectives in project structures which are considerably dependent on each other. Recently, beneficiaries' demands for cost and time reduction in completing a project have been increased. This study proposes a new method for time-cost trade off problem (TCTP) in uncertainty condition. To solve the model a multi objective genetic algorithm has been integrated with fuzzy theory. Efficiency of this algorithm is demonstrated through an existing case example from the literature. Finally to make the algorithm more efficient the existing parameters in the model have been set through Taguchi method.

***Keywords**: time-cost trade-off problem, scheduling, multi objective genetic algorithm, fuzzy theory.*

1 introduction

Time and cost are two main concerns of project managers, especially when they try to minimize the costs without violating contract obligations and complete the project by the deadline or sooner. Time and cost are two important and controllable objectives in project structures which are considerably dependent on each other. Generally the cheaper the resources used, the longer it takes to complete a project. Projects are often done in a dynamic and uncertain environment. A common assumption in time-cost trade off problem (TCPT) problems is that an activity can only be defined in one way and with certainty. However cost and time are not certain in reality and reasons such as designing changes or mistakes, social and economic tensions, inflation and natural disasters lead to uncertainty in completing the project. Regarding the uncertainty in the projects, it is sometimes necessary to do some activities sooner than normal to ensure completing the project on time. To overcome the problem, the activities are considered as several performing methods. Since the project environment is dynamic and uncertain, time and cost are considered as fuzzy, so that TCTP becomes more realistic. Different techniques are common to control time and cost of a project, including: heuristic [1-2], metaheuristic and mathematical programming. Mathematical programming has been widely used in linear programming [3-4], integer programming [5] and dynamic programming [6], regardless of uncertainty. TCTP is actually known as an NP-hard problem [7]. Although metaheuristic algorithms present under optimal answers and convergence is not ensured in them, different version of genetic algorithms was used successfully for TCTP in certainty mode [8-12]. Feng, Liu and Liu, Eshtehardian, and Jean [13-15] used fuzzy set of theory for completion time of the project. Those approaches do not use fuzzy set theory directly.

Table1: Defects of proposed techniques

Author	**year**	**Method**	**Input data**	**solution**	**output**	**defect**
Feng et al.	1997	GA	crisp	crisp	crisp	Method, Input data, output
Hegazy	1999	GA	crisp	crisp	crisp	Method, Input data, output
Feng et al.	2000	GA and simulation techniques	uncertain	crisp	crisp	Method, output
Zheng et al.	2004	GA	crisp	crisp	crisp	Method, Input data, output
Zheng et al.	2005	GA and Fuzzy Theory	uncertain	crisp	crisp	Method, output
Eshtehardian et al.	2008	GA and Fuzzy Theory	Crisp (time)	Crisp (time)	Crisp (time)	Method, output

In this study an efficient algorithm is proposed to solve time-cost trade-off problem in uncertain environment. In this method completion time and cost of the activities are considered as fuzzy. The suggested fuzzy logic is integrated with genetic algorithm and input and output are also fuzzy in this approach. This algorithm is suitable for solving problems with relatively large space. To prove it an applied example is solved. To make the algorithm more efficient its parameters are set. To fulfill the purpose of this study the following sections will provide characteristic features of the study namely, section 2 which examines TCTP problem, section 3 involves formulation of the problem, section 4 presents a new approach, section 5 describes the results from the applied example and parameters setting and section 6 includes conclusion and further suggestion.

2 Time-Cost Trade-Off Problem

Structure of time-cost trade-off problem is the most important aspect of decision-making in a project [8].in a project, time and cost of an activity depends on the execution methods and the applied equipments. The discrete relationship between time and cost is more common in literature. The aim of TCTP model is looking for the best possible combination of execution methods and equipments that minimizes project time and cost. Project completion time equals sum of execution time of all activities in the project which is called project critical path. Traditionally for different modes of an activity, time and cost are assumed crisp [9]. But in fact, because of some new activities in the project, lack of detailed information before task completion and some unclear internal and external events, project time and cost are not identified certainly and precisely. Although statistic was used for project scheduling, most of activity's time and cost cannot be defined as a random number with density function. For example, in a project with a new activity, there is no statistical data or the probability distribution is not identified. Therefore, using statistics in this project would not be a suitable method. In recent years Fuzzy theory widely used to model uncertain environment. Fuzzy numbers theory is a suitable option in dealing with environments that do not sufficient statistical data. This study tries to make the model more realistic through using fuzzy numbers theory and considering several modes for each activity. One of the advantages of this algorithm is the use of fuzzy numbers theory. According to table 1, previous algorithms had three kinds of structural problems. We have solved two kinds of those problems in this algorithm. Moreover, input/output data are presented as fuzzy numbers. Finally we present an algorithm to solve TCTP problems which is efficient and can solve big problem in reasonable time.

3 Problem formulation

In this model, time and cost of performing the activities are considered as fuzzy numbers. Costs include direct and indirect costs. Fuzzy number of $\tilde{B}$ with membership function $\mu_B(x), x \in R$ can be defined as:

$$B = \{(x, \mu_B(x)) | x \in X\} \tag{1}$$

In which $\mu_B(x)$ takes values between 0 and 1 shows importance of different values of x. In this study trapezoidal fuzzy numbers are used. $\tilde{A} = (a_1, a_2, a_A, \beta_A)$

and $\tilde{B}=(b_1,b_2,a_B,\beta_B)$ are two fuzzy numbers, summation and multiplying is computed by:

$$\tilde{A}+\tilde{B}=(a_1+b_1,a_2+b_2,a_A+a_B,\beta_A+\beta_B) \tag{2}$$

$$\tilde{A}-\tilde{B}=(a_1-b_2,a_2-b_1,a_A+\beta_B,a_B+\beta_A) \tag{3}$$

$$If \quad k \succ 0 \quad k\,(.)\,\tilde{A}=(Ka_1,Ka_2,Ka_A,K\beta_A) \tag{4}$$

$$If \quad k \prec 0 \quad k\,(.)\,\tilde{A}=(Ka_2,Ka_1,-K\beta_A,-Ka_A) \tag{5}$$

Which k is a crisp number.

To compare two fuzzy numbers of $\tilde{B}_1,\tilde{B}_1$,we used the distance method [17]. To compare these numbers it is necessary to find the central point $(\tilde{x}_0+\tilde{y}_0)$ of them from equation (5) and (6):

$$\tilde{x}_0(\tilde{B})=\frac{\int_a^b (xf_{\tilde{B}}^l)+\int_b^c xdx+\int_c^d (xf_{\tilde{B}}^r)}{\int_a^b f_{\tilde{B}}^l+\int_b^c dx+\int_c^d f_{\tilde{B}}^r} \tag{6}$$

$$\tilde{y}_0(\tilde{B})=\frac{\int_0^1 (yg_{\tilde{B}}^l)dx+\int_0^1 (yg_{\tilde{B}}^r)dx}{\int_0^1 (g_{\tilde{B}}^l)dx+\int_0^1 (g_{\tilde{B}}^r)dx} \tag{7}$$

$$f_{\tilde{B}}=\begin{cases}\frac{x-a}{b-a}, a\le x\le b\\ 1, b\le x\le c\\ \frac{x-d}{c-d}, c\le x\le d\\ 0, otherwise\end{cases} \tag{8}$$

$$R(\tilde{B})=\sqrt{(\tilde{x}_0)^2+(\tilde{y}_0)^2} \tag{9}$$

$g_{\tilde{B}}^r$ And $g_{\tilde{B}}^l$ are inverse function of $f_{\tilde{B}}^r$, $f_{\tilde{B}}^l$. These two variables are calculated through equation (9) and the one which is bigger; its corresponding fuzzy number will be bigger.

In this paper, the project is defined by a direct acyclic graph *G*= (*V*, *E*), in which *V* (i.e., vertex) and *E* (i.e., edge) are the sets of nodes (or activities) and arcs (events), respectively. *G* (*V*, *E*) is demonstrated as a matrix $A_{m\times n}$, in which *m* and *n* denote the number of nodes and arcs. This matrix is called the node arc incidence matrix for graph *G* (*V*, *E*). Matrix *A* has one row for each node of the network and one column for each arc. Each column of *A* contains exactly two nonzero coefficients: namely "+1" and "-1". The column corresponding to arc *j* contains "+1" if *i* is the node starting arc *j*, "-1" if *i* is the node ending arc *j*, and "0" otherwise [18].

$$A=\left[a_{ij}\right], \qquad i=1,\ldots,\ldots,m\ , \quad j=1,\ldots,\ldots n \tag{10}$$

$$a_{ie}=\begin{cases}1 & \text{if node i starts arc } e\\ -1 & \text{if node i ends arc } e\\ 0 & \text{otherwise}\end{cases} \tag{11}$$

Each project activities (e_i) has different execution methods (m_j) in which k$\in$j.

Each activity j includes fuzzy time $\tilde{T}_{jk}$ and fuzzy cost $\tilde{C}_{jk}$.

By solving this model, we can calculate the shortest time of the performing (execution) the project.

x_j : is a binary variable. If activity j is on the path, $x_j=1$, otherwise $x_j=0$.

y_{ij} : is a binary variable. If mode k is assigned to activity j, $y_{ij}=1$, otherwise $y_{ij}=0$.

b_i : Available source of node i.

$\tilde{T}^{\bar{k}}_{CPM}$: Fuzzy time of critical path

$$\tilde{T}^{\bar{k}}_{CPM}=\max\sum_{j=1}^{n}x_j\sum_{k\in m_j}\tilde{T}_{jk}y_{ij}$$

St:

$$\sum_{j=1}^{n}a_{ij}x_j=b_i \qquad , \text{i=1, 2, …, m} \tag{12}$$

$x_j\epsilon\{0,1\}\ \forall j\quad ,\quad \bar{k}=\{k_1,k_2,\ldots,k_n\}$

$$b_i=\begin{cases}1 & if\ \ i=1\\ -1 & if\ \ i=m\\ 0 & otherwise\end{cases} \qquad)$$

Time-cost trade-off problem is formulated as follows:

$\tilde{C}_t$ = fuzzy cost of the whole project (direct and indirect)

$\tilde{T}_t$ = fuzzy duration of the whole project.

$\tilde{C}_{id}$ = fuzzy indirect cost for each time unit.

$$\min\tilde{C}_t=\sum\sum\tilde{C}_{jk}y_{jk}+\tilde{C}_{id}\tilde{T}_{CPM}$$

$$\sum_{k\ \epsilon m_j}y_{jk}=1 \tag{14}$$

$$y_{ij}\ \in\{0,1\},\ \forall \text{j,k} \tag{15}$$

The first objective function of the model is minimizing the project cost and the second one is minimizing the project time. Limitation of model (14) guarantees the execution of only one mode from different modes of each activity.

4 Methodology

Since time-cost trade-off problem is known as NP-hard, using evolutionary algorithm is necessary for solving it. One of the most popular evolutionary algorithms is genetic algorithm which has been widely used in optimizing the problems. This algorithm was proposed first by Goldberg.
In this algorithm at first an allowable initial random population is generated. Then fitness of each chromosome is calculated. The best chromosomes of this generation are combines together, the best offspring and parent are selected and they all transferred into next generation. This repetition continues until termination condition has been reached.

4.1 Encoding the chromosome

This problems chromosome is a set of integer values. Gene *I* shows the selected mode for execution of activity *i*. each chromosome contains n genes (n is the number of projects activities.)Each gene can take a number between 1 and number of defined modes for execution of the activity. Each chromosome chooses only one mode for each activity.

$[m_1, m_2, ..., m_n] : m_1 \in [1, M_j]$

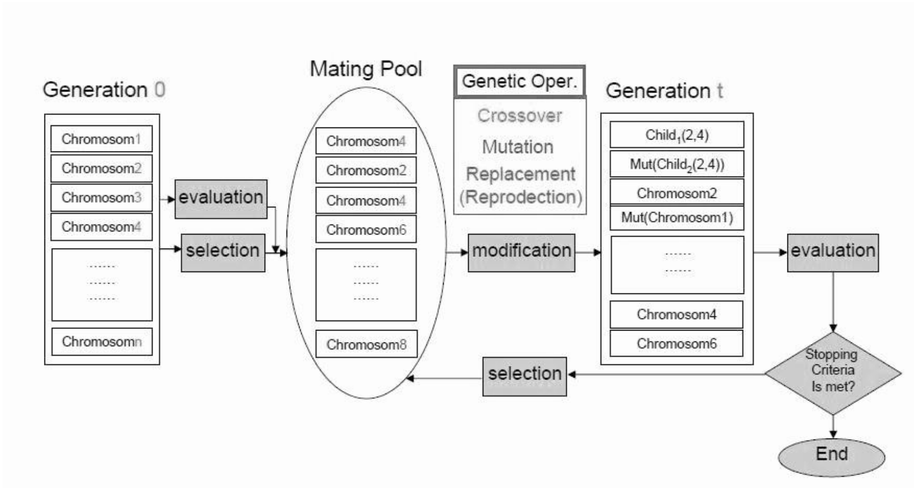


Figure 1. The Genetic Algorithm cycle

4.2 Initial population and genetic operators

Initial population contains N chromosomes. Each chromosomes gene is a random number between 1 and number of activity’s execution modes. In our suggested genetic algorithm, three kinds of operators, namely crossover, mutation and replacement are used. This algorithm uses uniform crossover. Performance of crossover is shown as follow:

Parent1:[4,3,2,5,4,1,3,2,5]
Parent2:[2,1,5,3,2,4,1,5,3],

Let the problem has 9 activities, if the generated chromosome is [1,1,0,1,0,0,0,1,1], the generated offspring are:

Offspring1:[2,1,2,3,4,1,3,5,3]
Offspring2:[4,3,5,5,2,4,1,2,5]

Single point mutation is used in this algorithm. Mutation is done with probability *pm*. In each repetition *pm* is less than the next repetition. To reduce the mutation rate, the following formula is used:

Pop= size of the population

pc_{start} = probability of doing the mutation at the beginning of the algorithm.

$$pc(i) = 1/2(pc_{start})(1 - \tanh(\frac{10i}{pop} - 5)) \quad (16)$$

This algorithm transfers some percentage of best answers of previous generation into the next.

4.3 NHGA1 algorithm

NHGA1 algorithms steps can be seen as follows:

Step1: enter input data of TCTP model; enter the initial parameters of the problem. Chromosomal N with acceptance genes is generated randomly for the initial population. Parameters of NHGA1 algorithm are:

- Number of generation (*pop*)
- Size of the population (*N*)
- Crossover rate (*pc*)
- Mutation rate (*pm*)
- Replacement rate (*pr*)

Step2: direct fuzzy time and fuzzy cost of the whole project are computed foe each generated chromosome.

$$\tilde{C}_{d(s)} = \sum_{j=1}^{n} \tilde{C}_{sj} \quad s = 1, 2, \ldots, N \quad (17)$$

Fuzzy time of whole project: summation of activities' time that placed on the critical path.

Step 3: calculating feasibility of each generated chromosome.

f_s : Fitness of each chromosome

$\tilde{C}_{\max}$: Maximum cost of project within the existing population.

$\tilde{C}_{\min}$: Minimum cost of project within the existing population.

$\tilde{T}_{\max}$: Maximum time of execution of the project within the existing population.

$\tilde{T}_{\min}$: Minimum time of execution of the project within the existing population.

$$f_s = \frac{\tilde{C}_{d(s)} - \tilde{C}_{\min}}{\tilde{C}_{\max} - \tilde{C}_{\min}} + \frac{\tilde{T}_s - \tilde{T}_{\min}}{\tilde{T}_{\max} - \tilde{T}_{\min}} \quad , \; s = 1, 2, \ldots, N \tag{18}$$

In formula (18) numerator and denominator are calculated through relation (2) and the resultant answers are converted into crisp number through (6, 7, and 8) relations. Fitness of each chromosome is computed.
Step 4= updating pm through formula (19).
Step 5: repeating step 2 to 4 until a termination condition has been reached.

5 A Case example

To demonstrate fundamental concepts and efficiency of the suggested algorithm, an applied example of time-cost trade-off problem in a certain environment is presented. This example which has 9 activities is adapted from [18].
(Figure 2) Each activities has 4 to 6 execution modes and each mode involves time and cost (table 3). A trapezoidal number is defined for each mode. The network matrix is shown in table 2.

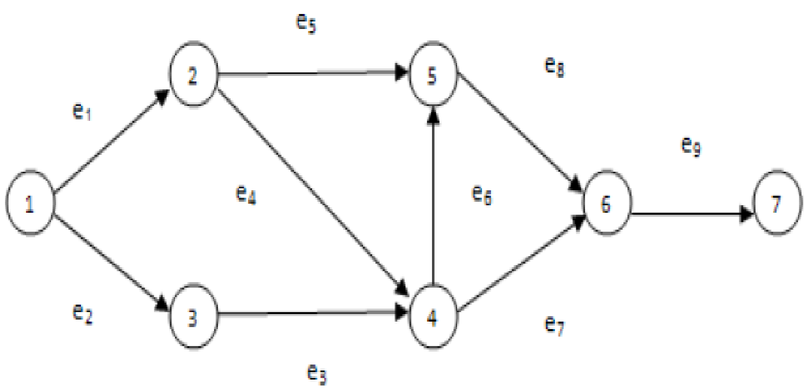


Figure 2. The Project Network

Table 2: Matrix A

	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9
1	1	1	0	0	0	0	0	0	0
2	-1	0	0	1	1	0	0	0	0
3	0	-1	1	0	0	0	0	0	0
4	0	0	-1	-1	0	1	1	0	0
5	0	0	0	0	-1	-1	0	1	0
6	0	0	0	0	0	0	-1	-1	1
7	0	0	0	0	0	0	0	0	-1

Table 3. The fuzzy time and cost of the project activities

Activity	Mode	Time	cost	activity	mode	Time	Cost
1	1 2 3 4 5	5 6 8 9 4 5 7 8 3 4 6 7 2 3 5 6 1 2 4 5	140 150 170 180 160 170 190 200 170 180 200 210 180 190 210 230 210 220 240 250	6	1 2 3 4 5	6 7 9 10 5 6 8 9 4 5 7 8 3 4 6 7 2 3 5 6	110 120 140 150 120 130 150 160 130 140 160 170 150 160 180 190 150 180 200 210
2	1 2 3 4 5	6 7 9 10 5 6 8 9 4 5 7 8 3 4 6 7 2 3 5 6	120 130 150 160 130 140 160 170 150 160 180 190 160 170 190 200 180 190 210 220	7	1 2 3 4	9 10 12 13 8 9 11 12 7 8 10 11 6 7 9 10	130 140 160 170 160 170 190 200 180 190 200 210 180 190 210 220
3	1 2 3 4 5	6 7 9 10 5 6 8 9 4 5 7 8 3 4 6 7 2 3 5 6	90 100 120 130 100 110 130 140 120 130 150 160 130 140 160 170 150 160 180 190	8	1 2 3 4 5	9 10 12 13 8 9 11 12 7 8 10 11 6 7 9 10 5 6 8 9	120 130 150 160 130 140 160 170 140 150 170 180 160 170 180 190 245 255 275 285
4	1 2 3 4 5	8 9 11 12 7 8 10 11 6 7 9 10 5 6 8 9 4 5 7 8	80 90 110 120 110 120 140 150 120 130 150 160 130 140 160 170 145 155 175 185	9	1 2 3 4	9 10 12 13 8 9 11 12 7 8 10 11 6 7 9 10	130 140 160 170 150 160 180 190 160 170 190 200 180 190 210 220
5	1 2 3 4 5 6	13 15 16 12 12 14 15 11 11 13 14 10 10 12 13 9 9 11 12 8 7 8 10 11	140 150 170 180 150 160 180 190 160 170 190 200 180 190 210 220 200 210 230 240 220 230 250 260				

This model is programmed in VBA in excel on a 2.4 GH CPU and 4G RAM computer. We investigated the effect of for factors on the efficiency of NHGA1 algorithm through Taguchi method. We have defined four levels for each factor as shown in table 5.

Taguchi method is one of the methods for designing experiments which regulate

output answers and efficiency of the algorithm and also save time. The first factor is the ordered pair of generation number and population size. Number of chromosomes that give the optimal answer is considered as desirably index and effectiveness of all parameters is determined through variance analysis table.
According to mean of parameters effectiveness diagram, levels 3 and 4 of the first factor have more desirable effect on algorithm efficiency and mean effect of S/N. we can conclude that level 3 in factor one is more suitable in term of efficiency and strength of output answers. With the same reasoning for factors 2, 3, 4, and5 level 3(86%), level 3(6%), level 1(15%) and level 4 (14) are the best regulations for this algorithm among the defined level.

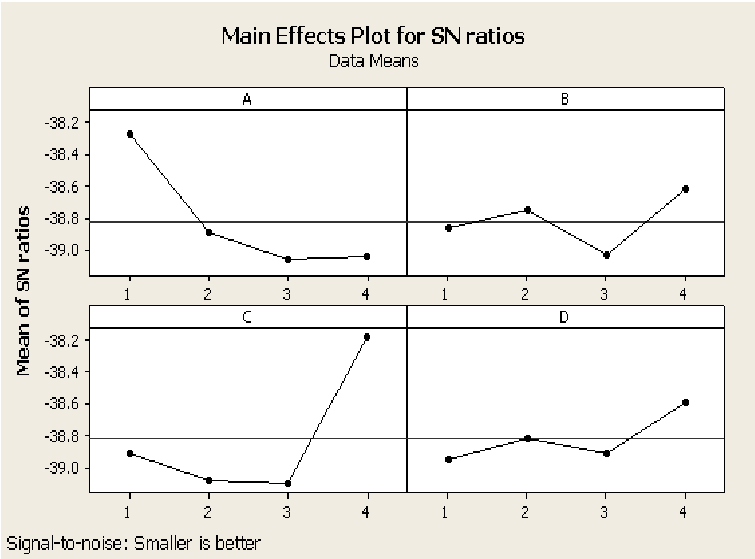
$$s/n = -10\log(\frac{1}{n}\sum_{i=1}^{n} y_i^2) \qquad (18)$$

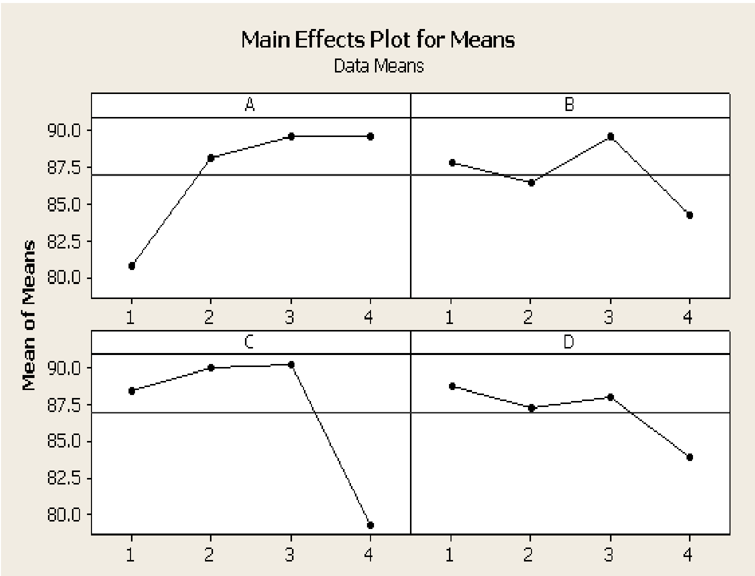
Table 4: ANOVA

Source	DF	Seq. SS	Adj. SS	Adj. MS	F	P
(pop, N)	3	117	117	39	2.78	0.104
Pc	3	100.89	100.89	33.63	2.4	0.12
Pm	3	535.97	535.97	178.66	12.73	0.005
Pr	3	145	145	48.88	3.48	0.043
Residual	3	42.11	42.11	14.03		
total	16	940.50				

Table 5: The Taguchi method's factor levels

Factor	Level
Population size an iteration number	(400, 1250), (200, 2500), (100, 5000), (50 , 10000)
Cross over rate	0.76, 0.8, ,0.86, 0.9,
Mutation rate	0.09, 0.08,0.06, 0.04
Replacement rate	0.15, 0.12,0.08,0.06





This example has 15,000,000 feasible solutions. To achieve optimal solutions NGA1 algorithm was used and it took 104 seconds to calculate optimal chromosome [4,2,2,1,1,5,1,4,4]. Regarding the increase in computation volume, this algorithm could reduce computation time for 54% in fuzzy mode. Shahsavari Pour et al. solved the problem in 227 seconds [18].

6 Conclusion

In real world, the aim of many project's managers is to reduce execution time and cost of the project. This method has a more realistic look on solving the problem and its final answer contains fuzzy time and cost. In this study, NHGA1 algorithm is presented to solve time-cost trade-off problem which is more effective than previous methods and compute the optimal answer in a shorter time the results show that the proposed algorithm is effective in solving relatively large problem in uncertain environments.

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