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Research Paper

Free vibration analysis of isotopic rectangular plate with one edge free of support (CSCF and SCFC plate)

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Abstract

The paper presents a theoretical formulation based on Ibearugbulem's shape function and application of Ritz method. In this study, the free vibration of simply supported plate with one free edge was analyzed. The Ibearugbulem's shape function derived was substituted into the potential energy functional, which was minimized to obtain the fundamental natural frequency. Aspect ratios from 0.1 to 2.0 with 0.1 increments were considered. The values of fundamental natural frequencies of the first mode were determined for different aspect ratio. For aspect ratio of 1.0, the value of non-dimensional parameter of fundamental natural frequency obtained was 23.86. Comparison was made for values of non-dimensional parameter of fundamental natural frequencies obtained in this study with those of previous research works. It was seen that there is no significant difference between values obtained in this study with those of previous studies.

Keywords: *fundamental natural frequency; Ibearugbulem's shape function; CSCF plate; Ritz method; SCFC plate*.

1. Introduction

It had been herculean task getting the exact solutions of rectangular plates under lateral vibration. Over the years, problems had been treated by mainly the use of trigonometric series as the shape function of the deformed plate or by using method of superposition. Researches had been carried out on the problems from equilibrium approach and others solved the problems from energy and numerical approaches. However, no matter the approach used, trigonometric series had been the most widely used shape functions. The analysis of the free vibration of plates was well documented by Leissa [1] and includes a variety of boundary conditions and aspect ratios using trigonometric series. Gorman [2] solved problem on free vibration of rectangular plate with different boundary conditions, aspect ratio and Poisson ratios using method of superposition. Both the approaches of Leissa and Gorman pose serious demanding computations to accomplish. Their results though vary from each other, have been well accepted to be close to exact results. The limitations of both the works of Leissa and Gorman gave rise to this present study. To come up with an approach that applies Ibearugbulem's [3] shape function in Ritz method to determine the fundamental natural frequencies of rectangular plates is the main objective of this study.

2. Formulation of fundamental natural frequency

Chakraverty [4] gave the maximum kinetic energy functional as

$$K_{max} = \frac{\lambda^2}{2} \iint \rho h W^2(x,y) \partial x \partial y \quad (1)$$

Making use of the non-dimensional parameters, R and Q, equation (1) becomes

$$= \frac{ab\lambda^2 \rho h}{2} \iint w^2 \partial R \partial Q \quad (2)$$

Where ρ is the weight per unit area of the plate, h is the plate thickness and λ is the frequency of plate vibration. The maximum strain energy functional for a thin rectangular isotropic plate under vibration was given by Ibearugbulem [3] as follow:

$$\text{Umax} = \frac{D}{2}\iint [(W''^{x})^2 + 2(W''^{xy})^2 + (W''^{y})^2]\partial x\partial y \quad (3)$$

Adding equations (2) and (3) gave the total potential energy functional of rectangular plate under lateral vibration as:

$$\prod max = \frac{aDb}{2}\iint \left[\frac{1}{a^4}(W''^{R})^2 + \frac{2}{a^2b^2}(W''^{RQ})^2 + \frac{1}{b^4}(W''^{Q})^2\right]\partial R\partial Q - \frac{ab\lambda^2\rho h}{2}\iint w^2\partial R\partial Q \quad (4)$$

Factorizing out b/a^3 gave:

$$\prod max = \frac{Db}{2a^3}\iint \left[(W''^{R})^2 + \frac{2a^2}{b^2}(W''^{RQ})^2 + \frac{a^4}{b^4}(W''^{Q})^2\right]\partial R\partial Q - \frac{ab\lambda^2\rho h}{2}\iint w^2\partial R\partial Q \quad (5)$$

If the aspect ratio, P = a/b, then:

$$\prod max = \frac{Db}{2a^3}\iint [(W''^{R})^2 + 2P^2(W''^{RQ})^2 + P^4(W''^{Q})^2]\partial R\partial Q - \frac{ab\lambda^2\rho h}{2}\iint w^2\partial R\partial Q \quad (6)$$

If the aspect ratio, p = b/a, then:

$$\prod max = \frac{Db}{2a^3}\iint \left[(W''^{R})^2 + \frac{2}{P^2}(W''^{RQ})^2 + \frac{1}{P^4}(W''^{Q})^2\right]\partial R\partial Q - \frac{ab\lambda^2\rho h}{2}\iint w^2\partial R\partial Q \quad (7)$$

From equation (4), if a/b^3 is factorized out then:

$$\prod max = \frac{Da}{2b^3}\iint \left[\frac{b^4}{a^4}(W''^{R})^2 + \frac{2b^2}{a^2}(W''^{RQ})^2 + (W''^{Q})^2\right]\partial R\partial Q - \frac{ab\lambda^2\rho h}{2}\iint w^2\partial R\partial Q \quad (8)$$

If the aspect ratio, P = a/b, then:

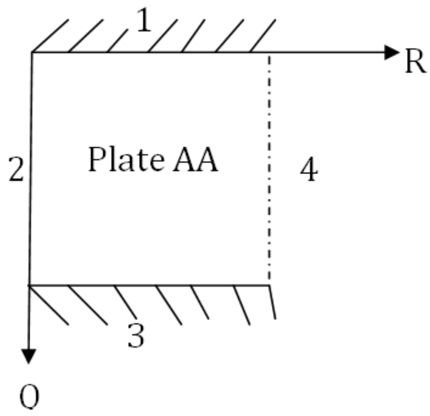
$$\prod max = \frac{Da}{2b^3}\iint \left[\frac{1}{P^4}(W''^{R})^2 + \frac{2}{P^2}(W''^{RQ})^2 + (W''^{Q})^2\right]\partial R\partial Q - \frac{ab\lambda^2\rho h}{2}\iint w^2\partial R\partial Q \quad (9)$$

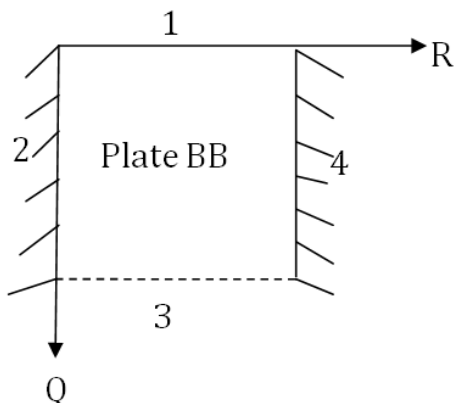
If the aspect ratio, P = b/a, then:

$$\prod max = \frac{Da}{2b^3}\iint [P^4(W''^{R})^2 + 2P^2(W''^{RQ})^2 + (W''^{Q})^2]\partial R\partial Q - \frac{ab\lambda^2\rho h}{2}\iint w^2\partial R\partial Q \quad (10)$$

Taylor-mcLaren's series' shape function

Displacement function for CSCF and SCFC plate





Deflection equation

The deflection equation, according to Ibearugbulem's [3] is:

$$W = (a_0 + a_1R + a_2R^2 + a_3R^3 + a_4R^4)(b_0 + b_1Q + b_2Q^2 + b_3Q^3 + b_4Q^4) \quad (11)$$

That is to say $W = W^R . W^Q$ (12)

Boundary Condition	Boundary condition
$W\ (R=0)=0, W\ (Q=0)=0$	$W\ (R=0)=0, W\ (Q=0)=0$
$W''(R=0)=0, W'(Q=0)=0$	$W'(R=0)=0, W''(Q=0)=0$
$V^R(R=1)=0\ , W'(Q=1)=0$	$W'(R=1)=0, V^Q(Q=1)=0$

W, W^R, W^Q, V^R, V^Q are general deflection shape function, partial shape functions in x and y axes, and partial shear force functions in x and y axes respectively. For simply supported edges, both kinematic and dynamic boundary conditions were satisfied. At the free edge, the kinematics boundary conditions and one of the dynamic boundary conditions were satisfied. The dynamic boundary conditions are the shear force and bending moment. Only the shear force condition was satisfied at the free edge. Using plate AA, the final particular deflection (shape) function, in line with Ibearugbulem's [3] is:

$$W = A\,(8R - 4R^3 + R^4)\,(Q^2 - 2Q^3 + Q^4) \tag{13}$$

Total potential energy for CSCF and SCFC plate

Integrating the squares of different derivatives of deflection function, W with respect to R and Q gave the following:

$$\int_0^1\int_0^1 (W''^R)^2 \partial R \partial Q = A^2(76.8)(1.587301587 \times 10^{-3}) = 0.121904761619\, A^2 \tag{14}$$

$$\int_0^1\int_0^1 (W'^{R/Q})^2 \partial R \partial Q = A^2(31.08571429)(0.019047619051) = 0.5921088436\, A^2 \tag{15}$$

$$\int_0^1\int_0^1 (W''^Q)^2 \partial R \partial Q = A^2(12.5968254)(0.8) = 10.07746032\, A^2 \tag{16}$$

$$\int_0^1\int_0^1 (W)^2 \partial R \partial Q = A^2(12.5968254)(1.587301587 \times 10^{-3}) = 0.0199946095\, A^2 \tag{17}$$

Substituting equations (14), (15), (16), and (17) into equations (6), (7), (9) and (10) gave respectively:

For aspect ratio, P = a/b,

$$\prod \max = \frac{DbA^2}{2a^3}[0.12190476 + 1.18421769P^2 + 10.077460P^4] - \frac{ab\lambda^2\rho hA^2}{2}[0.01999496] \tag{18}$$

For aspect ratio, p = b/a,

$$\prod \max = \frac{DbA^2}{2a^3}\left[0.12190476 + \frac{1.18421769}{P^2} + \frac{10.077460}{P^4}\right] - \frac{ab\lambda^2\rho hA^2}{2}[0.01999496] \tag{19}$$

For aspect ratio, P = a/b,

$$\prod \max = \frac{DaA^2}{2b^3}\left[\frac{0.12190476}{P^4} + \frac{1.18421769}{P^2} + 10.077460\right] - \frac{ab\lambda^2\rho hA^2}{2}[0.01999496] \tag{20}$$

For aspect ratio, P=b/a,

$$\prod \max = \frac{DaA^2}{2b^3}[0.12190476P^4 + 1.18421769P^2 + 10.077460] - \frac{ab\lambda^2\rho hA^2}{2}[0.01999496] \tag{21}$$

Equations (18), (19), (20) and (21) were minimized by $\frac{\partial \prod max}{\partial A} = 0$. After minimization, the values of the fundamental natural frequencies, λ were determined and used in plotting the graph of fundamental natural frequencies parameter, k against the aspect ratios. These graphs were shown on figures 1 to 8, for both X-X axis and Y-Y axis

Fig.1 presents the result from equation (18) for CSCF plate with aspect ratio ranging from 0.1 to 2.0 with P = a/b. The free edge of the plate is on X – X axis. The natural frequency increases as the aspect ratio increases. The line of best fit of the curve of fig.1 is forth order polynomial equation of y = $0.320x^4 - 1.829x^3 + 26.31x^2 - 3.667x + 2.714$.

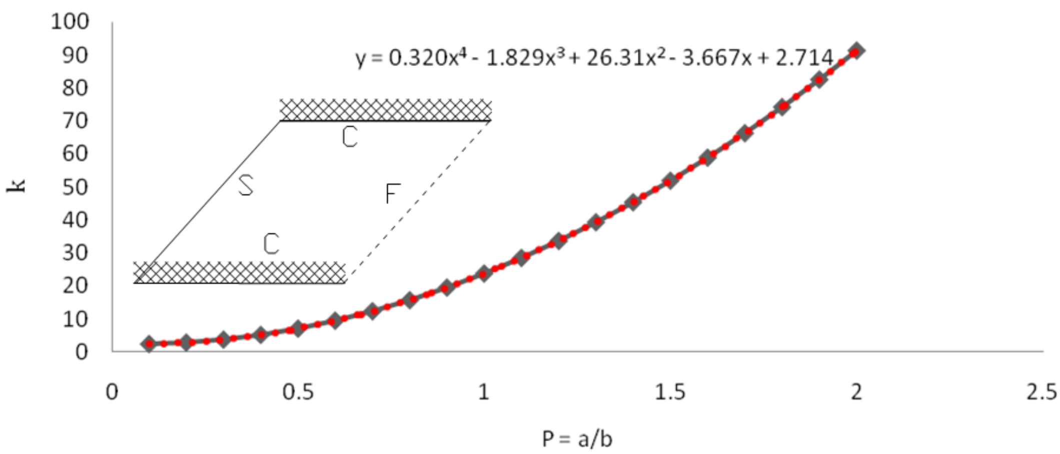


Fig. 1: Graph of CSCF plate P = a/b (note: y = k and x = aspect ratio, p)

Fig.2 presents the result from equation (19) for CSCF plate with aspect ratio ranging from 0.1 to 2.0 with P = b/a. The free edge of the plate is on X – X axis. The natural frequency decreases as the aspect ratio increases. The line of best fit of the curve is power curve of $y = 24.93x^{-1.91}$.

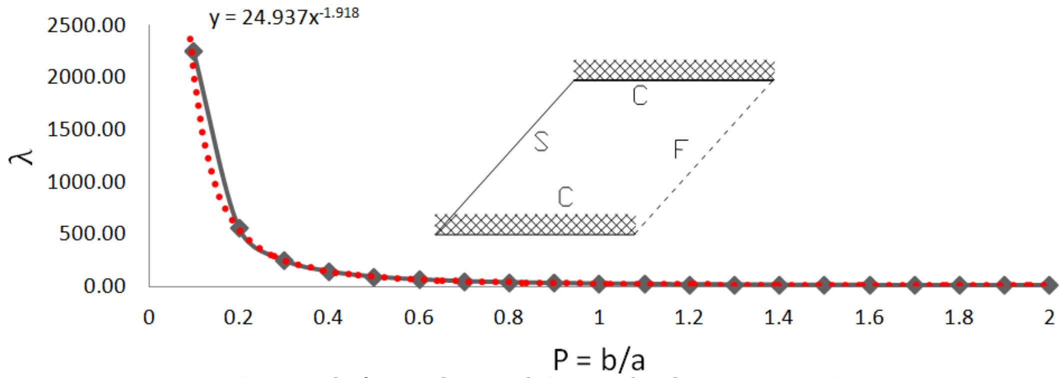


Fig. 2: Graph of CSCF plate P = a/b (note: y = k and x = aspect ratio, p)

Fig.3 presents the result from equation (20) for CSCF plate with aspect ratio ranging from 0.1 to 2.0 with P = a/b. The free edge of the plate is on X – X axis. The natural frequency decreases as the aspect ratio increases. With aspect ratios from 0.1 to 1, the line of best fit of the curve is a polynomial of sixth order $y = 23538x^6 - 88013x^5 + 13227x^4 - 10202x^3 + 42577x^2 - 9173.x + 840.5$. With aspect ratios from 1 to 2, the line of best fit of the curve is a straight line of $y = -0.862x + 24.42$.

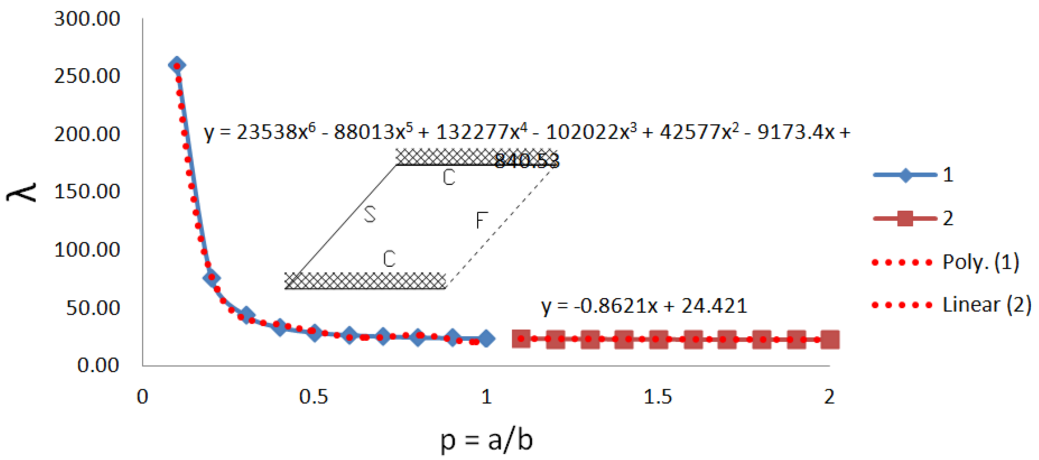


Fig. 3: Graph of CSCF plate P = a/b (note: y = k and x = aspect ratio, p)

Fig.4 presents the result from equation (21) for CSCF plate with aspect ratio ranging from 0.1 to 2.0 with P = b/a. The free edge of the plate is on X – X axis. The natural frequency increases as the aspect ratio increases. The line of best fit of the curve of fig.4 is forth order equation $y = 0.017x^4 + 0.189x^3 + 1.151x^2 + 0.057x + 22.44$.

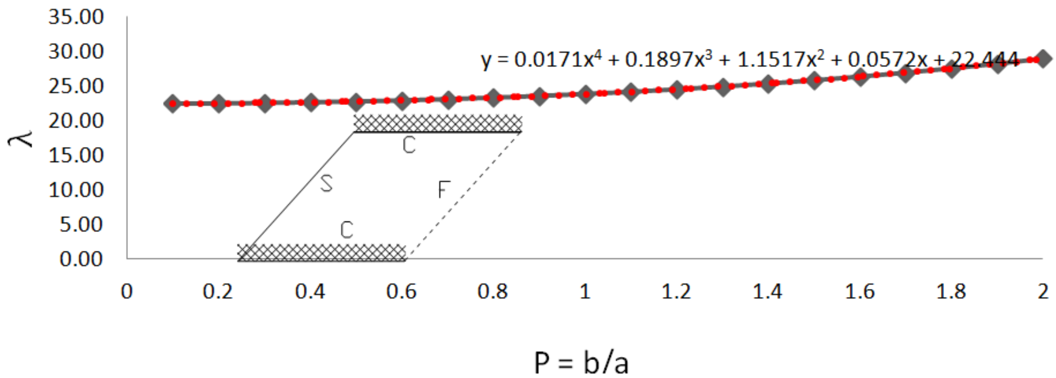


Fig. 4: Graph of CSCF plate P = a/b (note: y = k and x = aspect ratio, p)

Result of SCFC plate

Fig.5 presents the result from equation (18) for SCFC Plate with aspect ratio ranging from 0.1 to 2.0 with P = a/b. The free edge of the plate is on Y – Y axis. The natural frequency increases as the aspect ratio increases. The line of best fit of the curve of fig.5 is forth order equation of $y = 0.017x^4 + 0.189x^3 + 1.151x^2 + 0.057x + 22.44$.

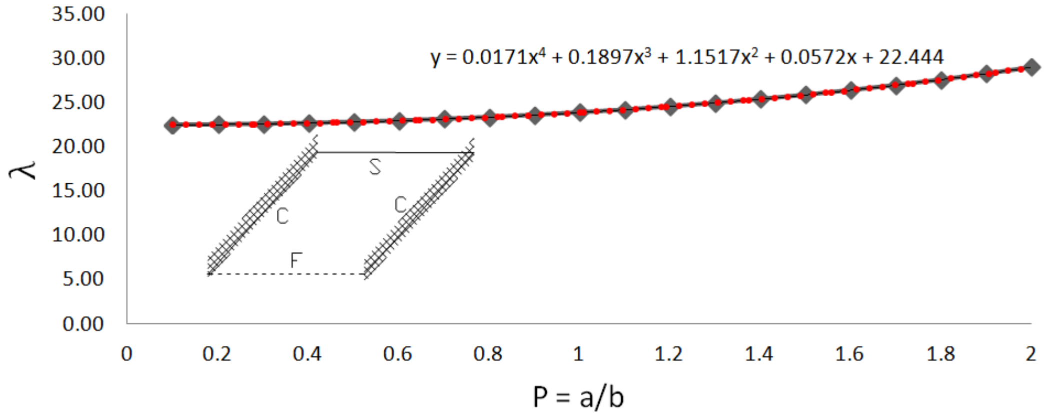


Fig. 5: Graph of SCFC plate p = a/b (note: y = k and x = aspect ratio, p)

Fig.6 presents the result from equation (19) for SCFC Plate with aspect ratio ranging from 0.1 to 2.0 with P = b/a. The free edge of the plate is on Y – Y axis. The natural frequency decreases as the aspect ratio increases. With aspect ratios from 0.1 to 1, the line of best fit of the curve of figure 3 is a polynomial of sixth order $y = 23538x^6 - 88013x^5 + 13227x^4 - 10202x^3 + 42577x^2 - 9173.x + 840.5$. With aspect ratios from 1 to 2, the line of best fit of the curve is a straight line of $y = -0.862x + 24.42$.

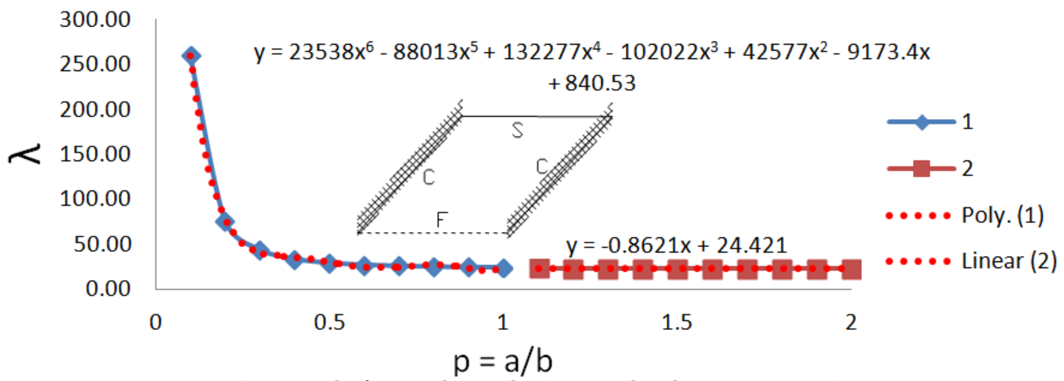


Fig. 6: Graph of SCFC plate p = b/a (note: y = k and x = aspect ratio, p)

Fig.7 presents the result from equation (20) for SCFC Plate with aspect ratio ranging from 0.1 to 2.0 with P = a/b. The free edge of the plate is on Y – Y axis. Also, the natural frequency decreases as the aspect ratio increases. The line of best fit of the curve of fig.7 is power equation of $y = 24.93x^{-1.91}$.

Fig.8 presents the result from equation (21) for SCFC Plate with aspect ratio ranging from 0.1 to 2.0 with P = b/a. The free edge of the plate is on Y – Y axis. The natural frequency increases as the aspect ratio increases. The line of best fit of the curve is power equation y = 0.320x4 - 1.829x3 + 26.31x2 - 3.667x + 2.714.

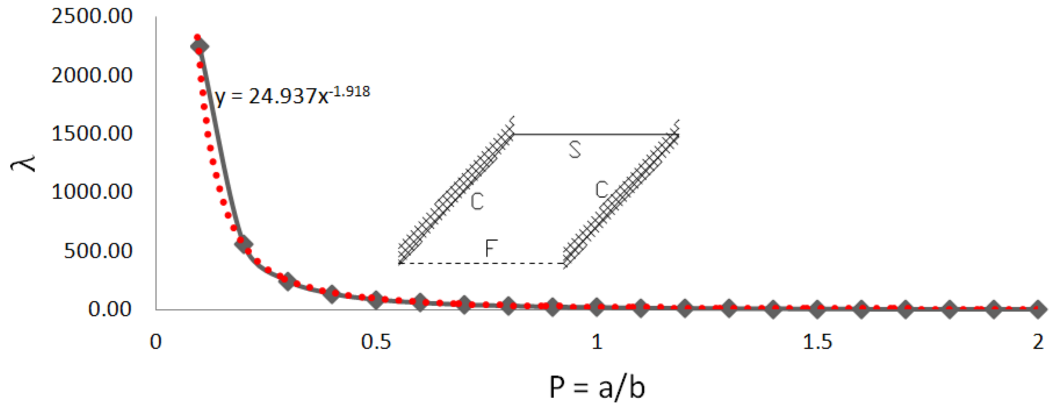


Fig. 7: Graph of SCFC plate p = a/b (note: y = k and x = aspect ratio, p)

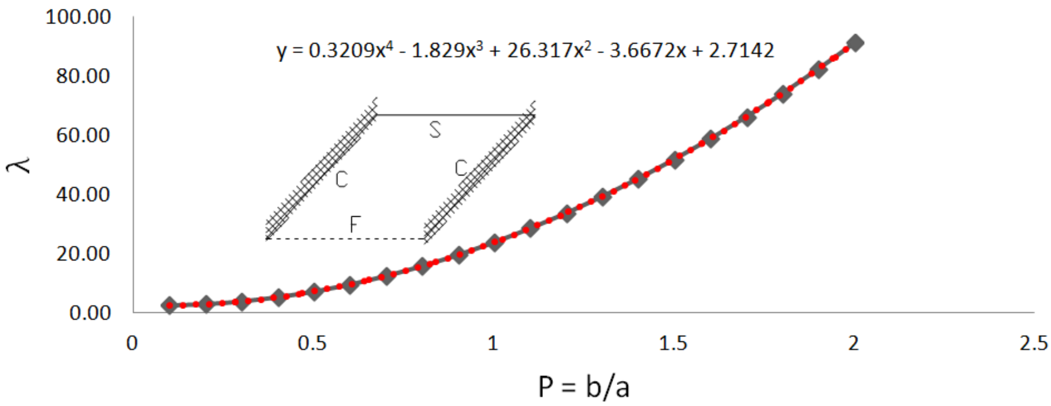


Fig. 8: Graph of SCFC plate p = b/a (note: y = k and x = aspect ratio, p)

Comparison of results for SCFC plate

The comparison of the values of fundamental natural frequency from the present study and the work of Leiss [1] was presented on table 1. The difference between Lessia [1] and the present result ranges from 0.53% to 4.33% given an average difference of 1.60%.

Table 1: Percentage difference of SCFC Plate, P =a/b

$\lambda = \frac{K}{a^2}\sqrt{\frac{D}{\rho h}}$			
$P = \frac{a}{b}$	Present	Lessia (1973)	% diff
0.4	22.6635	22.544	0.53
0.6	22.9371	22.855	0.36
1.0	23.8605	23.460	1.17
1.5	25.8481	24.775	4.33

S. Chakraverty worked on aspect ratio P = 1, with λ = 23.411, giving a difference of 1.92% to the present value.

3. Conclusion

The study obtained new energy functional based on Ritz's total potential energy and Taylor series deflection equation for CSCF and SCFC panels. The study came up with a new relationship between fundamental natural frequency and aspect ratios. The study came up with graphical models which can be used in place of the primary equations. Finally the study has created a new data base of fundamental natural frequencies for different panel and aspect ratio for designers.

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