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Research paper

Target heading estimation using Lagrange difference method through bearing only measurement

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Abstract

The bearing measurement in addition to the heading measurements increases the accuracy of the state estimate in bearing only tracking (BOT). Earlier, in [1] heading measurements are derived using set of three bearing measurements, mathematically known as centered difference method. In this paper we present the new approach using Lagrange three point difference method for deriving the heading measurements from set of bearing measurements. The performance analyses of the proposed approach is compared with the existing centered difference method using Root mean square error (RMSE), Root sum square error (RSSE) and maximum absolute error (MAE). The simulation results indicate that the Lagrange three point difference method performs comparatively better.

Keywords: *Lagrange Three Point Difference Method; Centered Difference Method; Derived Heading; Bearing Only Tracking; Maximum Absolute Error.*

1. Introduction

Target tracking using bearing only measurements is known as bearing only tracking (BOT) [4]. It is the widely used technique in many important applications like airborne radar, underwater sonar, military applications and air traffic control [12-15]. Generally used nonlinear filter for BOT is Extended Kalman filter (EKF), but this leads to poor estimates with improper initialization [10]. To improve the accuracy of target state estimate, many researchers tried to include target heading measurements through BOT [1, 2]. Lei et al in [6] have discussed the heading angle parameterized using Markov process to model the acceleration of 2D maneuvering target at each discrete time steps. Similarly in [7] Mallick et al. have explained heading parameterized multiple models to estimate the heading angle of the target. In both [6, 7], authors have considered range, range rate and azimuth angle as measurements. Xie et al. in [5] have discussed the multiple observers to estimate the heading angle of ship as a target in a slow varying sea state. In [5], the bank of nonlinear sub-observers used different band of frequencies to estimate the heading angle using position and angle measurements. All these techniques use the multiple models to obtain the accurate state estimate leads to time consuming process. To overcome this, Yang et al [2] proposed the methodology which is mathematically known as centered difference (CD) method to estimate the heading measurements from bearing measurements. It is the second order accurate difference method[16]. Initially, three bearing measurements were chosen to obtain the target heading with respect to the particular position. Using these bearing and heading measurements target states are estimated in Sonar. Similarly, Panakkal et.al [1] used the same methodology as in [2] and estimated the target state in radar application. Since heading measurements obtained by CD at a time uses only two bearings the overall error will be more [14,16]. Our work focuses on obtaining the heading measurements using Lagrange three point difference method from the set of consecutive three bearing measurements. It is also a second order difference formula, but it uses all the three bearing measurement at a time to obtain the target heading. Hence the heading information obtained will have a comparatively less error. Using measured bearing and estimated heading, target state is estimated through EKF and the results are compared for with and without heading measurements.

The outline of the paper is as follows. Section II describes derivation of heading calculation using Lagrange three point difference method. Section III describes the EKF with and without heading measurements using BOT. In section IV the comparative analysis of proposed technique with existing techniques [1-2] are given through results and discussion. Finally, section V describes the conclusion and future work.

2. Heading angle derivation using bearing measurements

The heading of the target was derived mathematically from set of three bearing measurements (b_{k-2}, b_{k-1}, b_k) as shown in Fig. 1. It is assumed that, target moves with constant velocity and heading remain constant. It is also assumed that bearing measurements are observed at equal intervals (i.e) the duration b_{k-2} to b_{k-1} and b_{k-1} to b_k are assumed to be c [1-2].

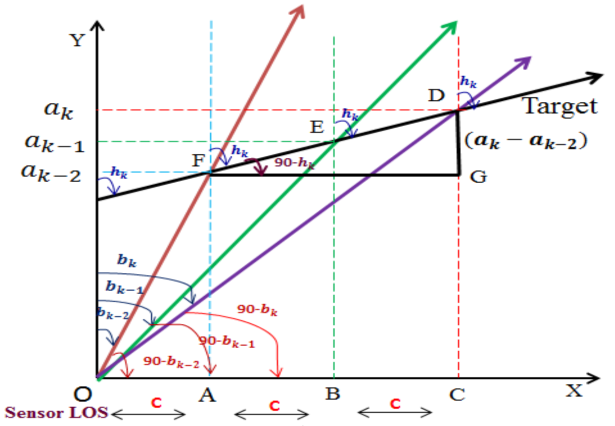

Fig. 1: Graphical Representation of Target Heading and Bearing Measurements.

Fig.1 shows target heading angle h_k and three bearing measurements. In all three bearing measurements it is assumed that the heading angle remains constant. In [2], CD formula is used to derive the heading angle using consecutive three bearing measurements. Authors in this paper attempt to use Lagrange three point difference method to derive the target heading angle.
In general for BOT, the state vector $[x_0 \quad y_0 \quad \dot{x}_0 \quad \dot{y}_0]'$ are derived using only bearing measurements [10-12]. Since bearing is the noisy measurement, it leads to error in target state. Hence along with bearing, derived heading measurement from bearing measurement are used to find the optimized target state as shown in Eq.(1).

$$x_0 = r_0 \sin b_0;\ y_0 = r_0 \cos b_0;\ \dot{x}_0 = s_0 \sin h_0;\ \dot{y}_0 = s_0 \cos h_0 \qquad (1)$$

where b_0 and h_0 are the initial bearing and heading measurement.
Here, an attempt is made to develop a mathematical model to find h_k using Lagrange difference method.

2.1. Target heading angle derivation using lagrange difference method

Let $f(x_0)$, $f(x_1)$, $f(x_2)$ are Lagrange Interpolating polynomial equations for the given three points (x_0, x_1, x_2) which are considered with an equal interval. The accuracy of $f(x_0)$ can be improved by considering the derivative of its polynomial function. Usually differentiation can be realized as a difference form numerically. Thus Lagrange three point difference method to improve the accuracy of Lagrange three point polynomial equation $f(x_0)$ is given by [14].

$$f'(x_0) = \frac{1}{2h}[-3f(x_0) + 4f(x_1) - f(x_2)] \qquad (2)$$

Since heading is the derivative of the bearing we would like to bring out the relationship between h_k and b_k using Eq. (2).
From Fig.1, let us consider $a_{k-2} = f(b_{k-2})$; $a_{k-1} = f(b_{k-1})$; $a_k = f(b_k)$; then according to Eq. (2)

$$f'(b_{k-2}) = \tan(90\text{-}h_k) = \cot(h_k) = \frac{1}{2c}[-3a_{k-2} + 4a_{k-1} - a_k] \qquad (3)$$

It is assumed that $h_{k-2} = h_{k-1} = h_k$
All a_{k-2}, a_{k-1}, a_k are derived geometrically from Fig.1 as follows
Consider the Δ^{le} OCD

$$\tan(90 - b_k) = \cot(b_k) = \frac{y_2}{x_2} = \frac{a_k}{3c}$$

Or

$$a_k = 3c \times \cot(b_k) \qquad (4)$$

Similarly from Δ^{le} OBE

$$\tan(90 - b_{k-1}) = \cot(b_{k-1}) = \frac{y_1}{x_1} = \frac{a_{k-1}}{2c}$$

Or

$$a_{k-1} = 2c \times \cot(b_{k-1}) \qquad (5)$$

From Δ^{le} OAF

$$\tan(90 - b_{k-2}) = \cot(b_{k-2}) = \frac{y_0}{x_0} = \frac{a_{k-2}}{c}$$

Or

$$a_{k-2} = c \times \cot(b_{k-2}) \qquad (6)$$

Substitute Eq. (4), (5) and (6) in Eq. (3), we obtain

$$\cot(h_k) = \frac{1}{2c}[-3c\cot(b_{k-2}) + 4(2c)\cot(b_{k-1}) - (3c)\cot(b_k)] \qquad (7)$$

After algebraic simplification,

$$\cot(h_k) = [-\tfrac{3}{2}\cot(b_{k-2}) + 4\cot(b_{k-1}) - \tfrac{3}{2}\cot(b_k)] \qquad (8)$$

Eq. (8) can be written as,

$$h_k = \cot^{-1}[-\tfrac{3}{2}\cot(b_{k-2}) + 4\cot(b_{k-1}) - \tfrac{3}{2}\cot(b_k)] \qquad (9)$$

represents the target heading h_k obtained using Lagrange three point difference method using three bearing measurements.

2.2. Target heading angle using centered difference method

The heading h_k obtained using centered difference method from Fig.1, is given as

$$\cot(h_k) = \frac{a_k - a_{k-2}}{2c} \qquad (10)$$

$$\cot(h_k) = \frac{(3c) \times \cot(b_k) - c \times \cot(b_{k-2})}{2c} \qquad (11)$$

$$h_k = \cot^{-1}[\tfrac{3}{2}\cot(b_k) - \tfrac{1}{2}\cot(b_{k-2})] \qquad (12)$$

While comparing Eq. (9) and Eq. (12), Eq. (9) involves only two bearing measurements b_k and b_{k-2}. Whereas Eq. (12) includes all the three points b_{k-2}, b_{k-1} and b_k and hence the information obtained about h_k will be accurate.

3. Target state estimation using computed heading

This section explains the estimation of target position using EKF with and without heading measurements along with the bearing measurements. The nonlinear filter EKF is used to track the position and velocity of the target [11]. The target state is initialized using Eq. (1) through computed heading for the known initial target range [1]. The computed heading obtained by CD and Lagrange difference method using Eq. (9) and Eq. (12) are used separately in EKF to obtain the better state estimate.
In general EKF state prediction and its covariance is given as [13], [8],

$$\hat{X}_{k|k-1} = F\hat{X}_{k-1|k-1} \qquad (13)$$

$$P_{k|k-1} = FP_{k-1|k-1}F' + Q \qquad (14)$$

Where Q is the process noise covariance and F is the transition matrix and is given as [13],

$$F = \begin{bmatrix} 1 & 0 & T & 0 \\ 0 & 1 & 0 & T \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (15)$$

The nonlinear predicted measurement is given by [3],

$$\hat{Z}_{k|k-1} = h(\hat{X}_{k|k-1}) \quad (16)$$

The Jacobian (H) of the nonlinear measurement function h without heading is given as [13],

$$h(\hat{X}_{k|k-1}) = \frac{\partial}{\partial X}[\hat{X}_{k|k-1}] = \frac{\partial}{\partial X}\left[arctan\left(\frac{\hat{x}_{k|k-1}}{\hat{y}_{k|k-1}}\right)\right] \quad (17)$$

with heading is defined as [1],

$$h(\hat{X}_{k|k-1}) = \frac{\partial}{\partial X}[\hat{X}_{k|k-1}] = \frac{\partial}{\partial X}\begin{bmatrix} arctan\left(\frac{\hat{x}_{k|k-1}}{\hat{y}_{k|k-1}}\right) \\ arctan\left(\frac{\dot{\hat{x}}_{k|k-1}}{\dot{\hat{y}}_{k|k-1}}\right) \end{bmatrix}$$

The innovation and its covariance is given as [12],

$$\vartheta_k = Z_k - \hat{Z}_{k|k-1} \quad (18)$$

$$S_k = HP_{k|k-1}H' + R \quad (19)$$

Where R is the measurement noise covariance and for without heading it is given as,

$$R = \sigma_\theta^2$$

With heading it is defined as,

$$R = \begin{bmatrix} \sigma_\theta^2 & 0 \\ 0 & \sigma_h^2 \end{bmatrix}$$

Where σ_θ^2 and σ_h^2 are the variances for bearing and heading measurement.
The filter gain is defined as [3],

$$W_k = P_{k|k-1}H'S_k^{-1} \quad (20)$$

The updated state and its covariance is given as [13],

$$\hat{X}_{k|k} = \hat{X}_{k|k-1} + W_k\vartheta_k \quad (21)$$

$$P_{k|k} = P_{k|k-1} - W_kS_kW_k' \quad (22)$$

4. Results and discussion

This section validates the performance of the target state estimation with heading measurements. Also analyses the performance of the proposed Lagrange three point difference method with CD method. The results obtained by calculating the heading measurement using CD and Lagrange difference method are compared to find the best estimate of target state.
The MATLAB simulation is made by using following assumption. Let the target moves with the constant velocity of 20 (m/s). The coherent integrate time pertaining to a single position measurement is assumed to be T = 1sec. The bearing angle of target was assumed to be 275°. The bearing and heading measurement noise variance was assumed to be 1° and 0.5°. The initial range was assumed to be 5km with the variance of 100m. Considering three bearing measurements at a time to calculate the single heading. The total number of heading measurements estimated is 200 from the set of 203 bearing measurements and the graphs are plotted for 200 bearing and heading measurements. Simulations are repeated for 100 Monte Carlo runs to compare the performance analyses.
The performances are analyzed using root mean square error (RMSE), root sum square error (RSSE), maximum absolute error (MAE).
The RMS error is given by [9], [13],

$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left\|X_k^i - \hat{X}_{k|k}^i\right\|^2} \quad (23)$$

Where X_k^i and $\hat{X}_{k|k}^i$ are the true target state and estimated target state at k^{th} time instant in i^{th} Monte Carlo run.

(A)

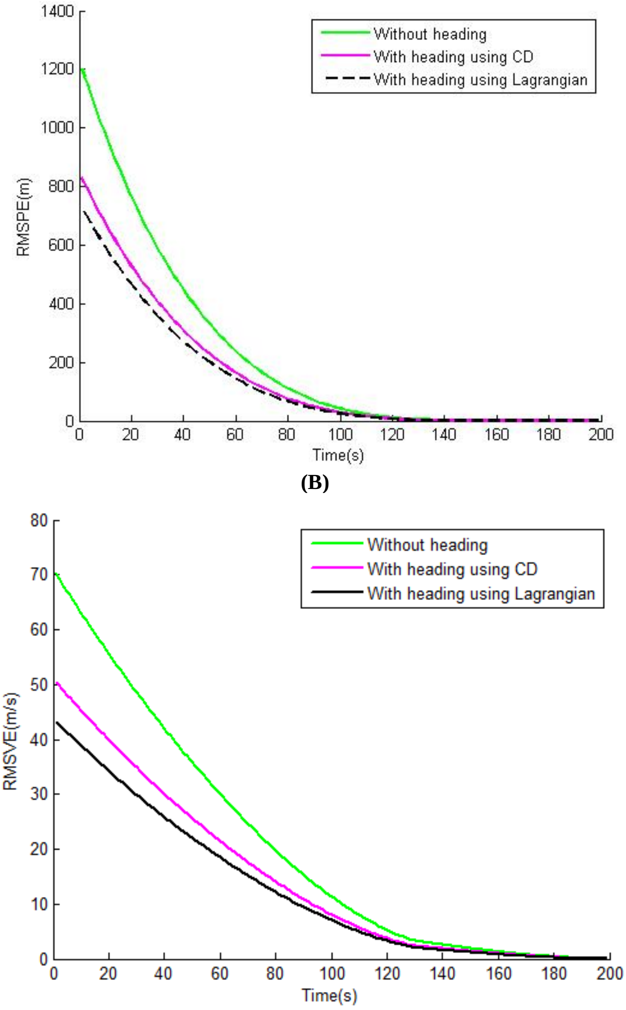

Fig. 2: Root Mean Square Position and Velocity Error.

The RMS position and velocity error for the simulation results involving with and without heading measurement is shown in Fig. 2 (a) and (b). The results shows error got minimized with heading measurement. Further error got reduced using our proposed method both in position and velocity errors.
The second parameter which is used to analyze the performance is RSSE and is defined as [8-9],

$$RSSE = \sqrt{\left\|X_k^i - \hat{X}_{k|k}^i\right\|} \quad (24)$$

Where X_k^i and $\hat{X}_{k|k}^i$ are the true and estimated states as defined in Eq. (23).
Fig. 3(a) and 3(b) indicates the root sum square position and velocity error. From the plots, estimation process without heading shows larger error compared with the estimation process using heading measurements. From Fig. 3(a), heading calculation using

Lagrange difference method shows less error compared with heading using CD. From Fig. 3(b), the heading measurement using Lagrange and CD shows almost similar error with some slight difference in error.

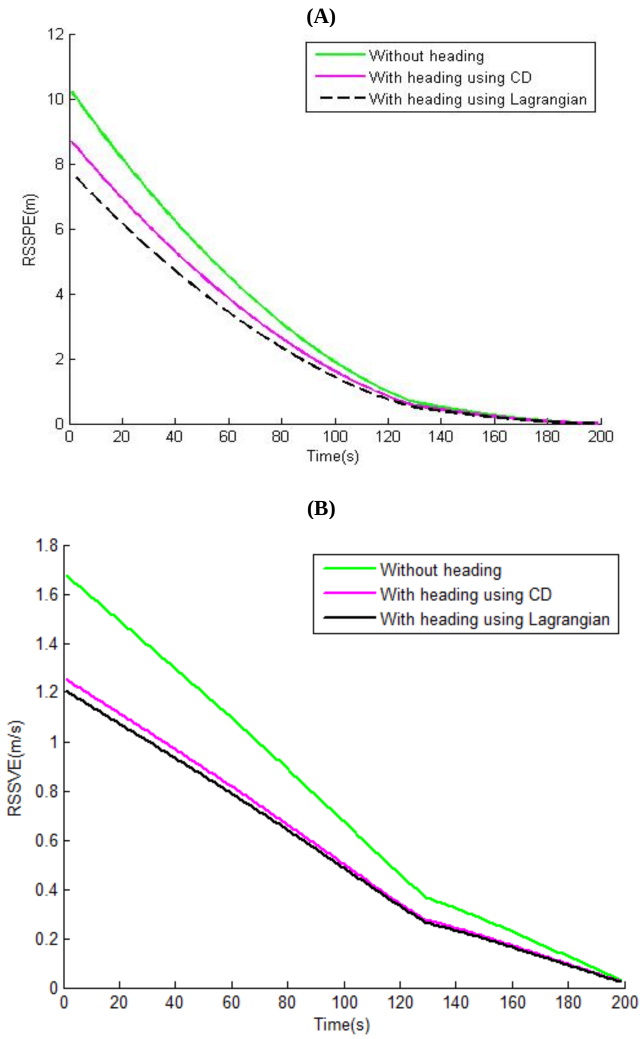

Fig. 3: Root Sum Square Position and Velocity Error.

The other performance measure is the mean absolute error (MAE) and is given as [8],

$$MAE = \frac{1}{N}\sum_{i=1}^{N}\left\|X_k^i - \hat{X}_{k|k}^i\right\| \quad (25)$$

The mean values of the position and velocity error for MAE are given in Table 1. It is observed that, heading using Lagrange three point difference method has less error compared with heading using CD. The MAE error for without heading measurements are higher compared to the other two methods.

Table 1: Maximum Absolute Position and Velocity Error

Methods	Position error $[x\ y]$	Velocity error $[\dot{x}\ \dot{y}]$
Without heading	[2.3799 2.0616]	[0.2188 0.1238]
Heading using CD	[1.9410 1.8513]	[0.1825 0.1367]
Heading using Lagrange method	[0.9608 1.0369]	[0.1191 0.1029]

5. Conclusion

This paper once again proved that with heading measurements BOT gives better state estimation in comparison with bearing only measurements. We also found that our proposed Lagrange three point difference method gives better estimate compared to the existing centered difference method to estimate the heading measurement. Here we have used only three point difference method. This can also be extended to multi point Lagrange difference method to find the tradeoff between performance and computational time. Usually tedious process like Taylor series method is used to approximate the nonlinear measurements. In the place of Taylor series approximation, simple Lagrange difference method can also be used.

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