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Research paper

Discrete modeling of building structures geometric images

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Abstract

For discrete modeling of geometric images, it is possible to use the numerical method of finite differences, the static-geometric method, the mathematical apparatus of numerical sequences. Each of them has certain advantages and disadvantages, which depend on the solution of specific practical problems.
This article proposes to use the geometric apparatus of superpositions together with the above-mentioned methods. It allows significantly to improve the efficiency and expanding capacities of the process of geometric images discrete modeling. In particular, it makes it possible to investigate an opportunity of interpolating as parabolic functions as well as other elementary functional dependencies.
The purpose of this article is to expand capacities of the classical finite difference method and static-geometric method by using the geometric apparatus of superpositions. It allows using hyperbolic functions as interpolators for the geometric images discrete modeling. The result of this study is the obtained interpolating and extrapolating templates, which allow modeling geometric images of architectural and building constructions in a form of chain lines discrete frames.

Keywords: *Discrete Modeling; Geometric Images; Finite Difference Method; Static-Geometric Method; Geometric Apparatus Of Superpositions.*

1. Introduction

In the process of creating discrete modeling methods of architectural and building constructions geometric images of different problems must be solved. The most common problem is the problem of changing the discrete information about a geometric image into continuous and an inverse one. The first problem can be solved by interpolation methods. One of the most perspective directions of solving these problems is a wide usage of discrete geometric modeling methods [1, 2]. They allow significantly simplify algorithms and programs and to ensure the economy of computing resources.
It is proved that the theoretical axis of an arch corresponds as in the mirror to the shape of a thread, fixed in the bases of the arch [3].
This property allows using the shape of a sagging thread for building structures, which work both on compression and on stretching. Taking into consideration design static features give an opportunity to search for a form that corresponds to a minimum of unwanted stresses arising in it, and their concentrations. All this leads to the saving of building materials, which are used to provide additional rigidity and resistance to various loads.
A curve model is much easier to research than a surface model. It should be expected that a set of properties of a discrete line model can be transferred to a surface model, which is formed according to the same laws, if this line is considered as a component of the surface frame. Other properties of a discrete surface model can be obtained as a result of generalization of line model corresponding properties.
A sagging thread, evenly loaded in length, takes the form of a chain line. When an even load distributes along the horizontal axe, then that thread takes the shape of a parabola. Changing a loading schedule of a thread, it becomes possible to manage its shape. This corresponds to one of the static-geometric method principles of constructing curve lines and contours [4]. Analytically, curve equations are obtained by double integration of a differential equation of the inextensible flexible thread equilibrium.
Another idealized interpretation of a thread as an absolutely extensible one allows to describe its shape in a discrete form analytically simply. The shape of a chain line resembles a parabola (Fig. 1, 2), but it is described by a hyperbolic cosine [5]:

$$y = a \cdot ch\frac{x}{a}. \tag{1}$$

Fig. 1. A chain that sags under its own weight

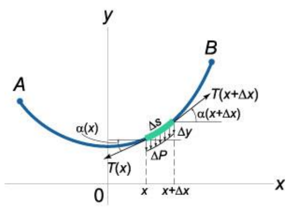


Fig. 2. A chain line with different parameter values

Its form is uniquely determined by parameter a, the dependence of which is shown in Figure 3. Chain lines are often found in nature and technology (Fig. 4). In architecture and construction, arches in the form of an inverted chain (Figs. 5, 6) have high stability due to the fact that internal compression forces are perfectly compensated and do not cause any deflection [6].

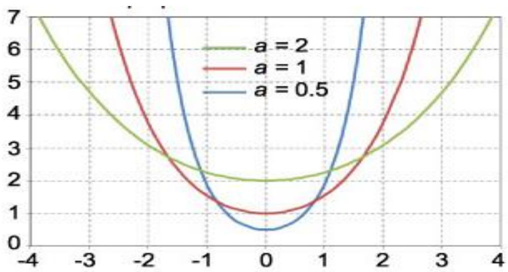


Fig. 3. A chain line with different parameter values

Fig.4. Golden Gate Bridge

Fig. 5. Arch «Gate to the West»

Fig. 6. Train Station Budapest-Keleti

When the chain is rotated around *Ox* axis, a surface called a catenoid is formed (Fig. 7). The catenoid is a minimal surface, each part of which will be smaller than any other surface, bordered by the same contour. [7].

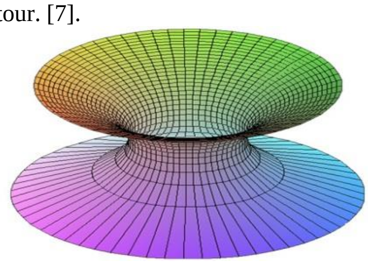


Fig. 7. The discrete frame of a catenoid surface

Taking into account the foregoing, it is considered expedient for a discrete determination of building structures geometric images to use hyperbolic functions in form (1) as interpolants.

The finite difference method allows analytical description of discrete geometric images.

For the discrete representation of a line curve with points of a certain step h along Ox axis:

$x_{i+1} = x_i + h$.

A right finite difference of the first order has the form:

$\Delta y_i = y_{i+1} - y_i$.

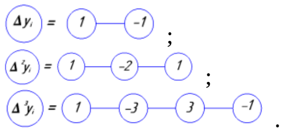
A right finite difference of the second order is a difference between two finite differences of the first order:

$$\Delta^2 y_i = \Delta y_{i+1} - \Delta y_i = (y_{i+2} - y_{i+1}) - (y_{i+1} - y_i) = y_{i+2} - 2y_{i+1} + y_i$$

Correspondingly a central difference of the second order has the form:

$$\Delta^2 y_i = y_{i-1} - 2y_i + y_{i+1}$$

For clarity, finite differences are often represented as "computational templates" or "difference operators". Such computational templates for central differences are as follows:



These templates allow to receive discrete analogs of polynomial curves of corresponding degrees.

2. Main body

The authors of this article have shown in papers [8-13] some approaches to the definition of discrete analogues of geometric images, which were based on the geometric apparatus of point sets superpositions. This allows forming discrete images without compiling and solving cumbersome systems of equations. The management of discretely represented forms of images is carried out by varying magnitudes of superposition coefficients. It was considered a possibility of generating computational templates for a discrete definition of hyperbolic function (1), which is depicted in Fig. (5), similar to the templates, which can be formed for polynomial curves by applying the geometric superposition apparatus.

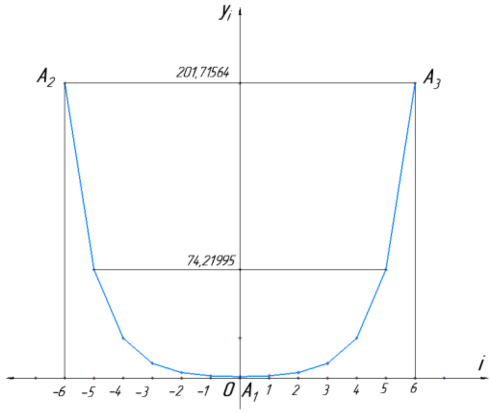


Fig. 8. The graph of numerical sequence $y_i = ch\ i$

As in applied geometry [14], superpositions are sums of multiplied functions and weight coefficients. More exactly, superpositions of the corresponding points of n sets in m-space in Cartesian coordinate system are determined by equations (2):

$$\begin{aligned} u_1 &= k_{1,1}u_{1,1} + k_{1,2}u_{1,2} + \cdots + k_{1,j}u_{1,j} + \cdots + k_{1,n}u_{1,n} \\ &\ldots \\ u_i &= k_{i,1}u_{i,1} + k_{i,2}u_{i,2} + \cdots + k_{i,j}u_{i,j} + \cdots + k_{i,n}u_{i,n}, \\ &\ldots \\ u_m &= k_{m,1}u_{m,1} + k_{m,2}u_{m,2} + \cdots + k_{m,j}u_{m,j} + \cdots + k_{m,n}u_{m,n}, \end{aligned} \quad (2)$$

where: *i* – the number of an axis;

j – the number of an initial set of superposition;

$k_{i,j}$ – superposition coefficient.

Without loss of generality, let us take hyperbolic cosine equation (1) *a=1*. Then the closed form of this transcendental function numerical sequence will have the form:

$$y_i = ch\ i\ . \quad (3)$$

In [15], it was proved that the coordinate of any point of a one-dimensional set of points is the superposition (4) of coordinates of three arbitrary points of this set:

$$\begin{aligned} x_0 &= k_1x_1 + k_2x_2 + k_3x_3 \\ y_0 &= k_1y_1 + k_2y_2 + k_3y_3 \end{aligned}, \quad (4)$$

where: $k_3 = 1 - k_1 - k_2$.

Formulae (5) are obtained for calculating the superposition coefficients k_1, k_2 values:

$$k_1 = \frac{(x_0-x_3)(y_2-y_3)-(x_2-x_3)(y_0-y_3)}{(x_1-x_3)(y_2-y_3)-(x_2-x_3)(y_1-y_3)}\ ;$$

$$k_2 = \frac{(x_1-x_3)(y_0-y_3)-(x_0-x_3)(y_1-y_3)}{(x_1-x_3)(y_2-y_3)-(x_2-x_3)(y_1-y_3)}. \qquad (5)$$

Table 1 shows a set of numerical sequence values (3) under condition *a=1* .

Table 1: The values of numerical sequence $y_i = ch\ i$

i	-7	-6	-5	-4	-3	-2	-1	0
y_i	548,31704	201,71564	74,20995	27,31823	10,06766	3,76220	1,54308	1
i	1	2	3	4	5	6	7	
y_i	1,54308	3,76220	10,06766	27,31823	74,20995	201,71564	548,31704	

By analogy with initial data for calculating right finite differences, we take the following coordinate of points:

1. $x_0 = 2; y_0 = 3{,}76220$; $x_1 = 3; y_1 = 10{,}06766$, $x_2 = 4; y_2 = 27{,}31823$; $x_3 = 5; y_3 = 74{,}20995$.

The values of superposition coefficients for calculating the ordinates of a desired point according to the ordinates of three adjacent points are determined by formulas (5):

$k_1 = 2{,}368667648918668$; $k_2 = -1{,}737335297837336$.

2. $x_0 = 3; y_0 = 10{,}06766$; $x_1 = 4; y_1 = 27{,}31823$, $x_2 = 5; y_2 = 74{,}20995$; $x_3 = 6; y_3 = 201{,}71564$.

The values of superposition coefficients are calculated by formulas (5):

$k_1 = 2{,}367986144430418$; $k_2 = -1{,}735972288860837$.

3. $x_0 = 4; y_0 = 27{,}31823$; $x_1 = 5; y_1 = 74{,}20995$; $x_2 = 6; y_2 = 201{,}71564$; $x_3 = 7; y_3 = 548{,}31704$;

the values of superposition coefficients are calculated by formulas (5):

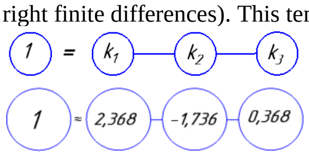
$k_1 = 2{,}367893882454095$; $k_2 = -1{,}735787764908191$.

4. $x_0 = 5; y_0 = 74{,}20995$; $x_1 = 6; y_1 = 201{,}71564$, $x_2 = 7; y_2 = 548{,}31704$; $x_3 = 8; y_3 = 1490{,}47916$;

the values of superposition coefficients are calculated by formulas (5):

$k_1 = 2{,}367881395596904$; $k_2 = -1{,}735762791193807$.

As can be seen from the above four examples, the superposition coefficients of the three adjacent points will be the same up to the third decimal place. Therefore, as for polynomial curves, a computational template can be created for a discrete modeling of one-dimensional geometric images by interpolating the given nodal points by hyperbolic functions (similar to the templates of right finite differences). This template has a form:



(6)

By analogy with initial data for calculating central finite differences, we take the following coordinate of points:

1. $x_0 = 2; y_0 = 3{,}76220$; $x_1 = 1; y_1 = 1{,}54308$, $x_2 = 3; y_2 = 10{,}06766$; $x_3 = 4; y_3 = 27{,}31823$.

The values of superposition coefficients for calculating the ordinates of a desired point according to the ordinates of three adjacent points are determined from formulas (5):

$k_1 = 4{,}212848081925265 \cdot 10^{-1}$;
$k_2 = 7{,}361455754224206 \cdot 10^{-1}$.

2. $x_0 = 3; y_0 = 10{,}06766$; $x_1 = 2; y_1 = 3{,}76220$, $x_2 = 4; y_2 = 27{,}31823$; $x_3 = 5; y_3 = 74{,}20995$;

the values of superposition coefficients are calculated by formulas (5):

$k_1 = 4{,}22178265683037 \cdot 10^{-1}$;
$k_2 = 7{,}33465202950889 \cdot 10^{-1}$.

3. $x_0 = 4; y_0 = 27{,}31823$; $x_1 = 3; y_1 = 10{,}06766$, $x_2 = 5; y_2 = 74{,}20995$; $x_3 = 6; y_3 = 201{,}71564$;

the values of superposition coefficients are calculated by formulas (5):

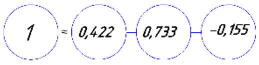
$k_1 = 4{,}222997682448578 \cdot 10^{-1}$;
$k_2 = 7{,}331006952654266 \cdot 10^{-1}$.

4. $x_0 = 5; y_0 = 74{,}20995$; $x_1 = 4; y_1 = 27{,}31823$, $x_2 = 6; y_2 = 201{,}71564$; $x_3 = 7; y_3 = 548{,}31704$;

the values of superposition coefficients are calculated by formulas (5):

$k_1 = 4{,}223162226187247 \cdot 10^{-1}$;
$k_2 = 7{,}33051332143826 \cdot 10^{-1}$.

As can be seen from the above given examples, the superposition coefficients of the three adjacent points will be the same up to the third decimal place after the decimal point. Therefore, as for polynomial curves, a computational template can be created for discrete modeling of one-dimensional geometric images by interpolation of the given nodal points by hyperbolic functions (in analogy to the templates of central finite differences. This template has a form:



. (7)

The system of equations, which determinates the ordinates of sequence (3) nodal points by analogy with equations (4), has a form:

$$\begin{cases} y_i - y_{i+2} = k_1(y_{i-1} - y_{i+2}) + k_2(y_{i+1} - y_{i+2}) \\ y_{i+1} - y_{i+3} = k_1(y_i - y_{i+3}) + k_2(y_{i+2} - y_{i+3}) \end{cases} \qquad (8)$$

From (8) expressions for calculating the values of superposition coefficients similar to formulas (5) are found:

$$k_1 = \frac{(y_i-y_{i+2})(y_{i+2}-y_{i+3})-(y_{i+1}-y_{i+2})(y_{i+1}-y_{i+3})}{(y_{i-1}-y_{i+2})(y_{i+2}-y_{i+3})-(y_{i+1}-y_{i+2})(y_i-y_{i+3})};$$

$$k_2 = \frac{(y_{i-1}-y_{i+2})(y_{i+1}-y_{i+3})-(y_i-y_{i+2})(y_i-y_{i+3})}{(y_{i-1}-y_{i+2})(y_{i+2}-y_{i+3})-(y_{i+1}-y_{i+2})(y_i-y_{i+3})}. \qquad (9)$$

The values of superposition coefficients by formulas (9) were also calculated.

By analogy with initial data for calculating central finite differences, we take the following ordinates of points from Table 1:

1. $y_{i-1} = 3{,}762195691$, $y_i = 10{,}067662$, $y_{i+1} = 27{,}30823284$; $y_{i+2} = 74{,}20994852$.

$k_1 = 2{,}44728471054793 \cdot 10^{-1}$;
$k_2 = 1{,}000000000000016$;

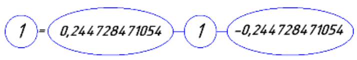
the values of superposition coefficients are calculated by formulas (9).

2. $y_{i-1} = 27{,}30823284$, $y_i = 74{,}20994852$, $y_{i+1} = 201{,}7156361$; $y_{i+2} = 548{,}3170352$,

the values of superposition coefficients are calculated by formulas (9):

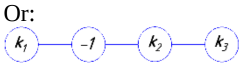
$k_1 = 2{,}4472847105373753 \cdot 10^{-1}$;
$k_2 = 1{,}000000000002819$.

As can be seen from the above examples, the superposition coefficients of the three adjacent points will be the same up to the twelfth decimal place (we can assume that they are equal). Therefore, as for polynomial curves, a computational template for a discrete modeling of one-dimensional geometric images can be created by interpolating given nodal points by hyperbolic functions. This template has a form:



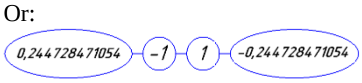
. (10)

Or:



,

Or:



. (11)

Example. We construct discrete models of curves with the following initial data:

1. *A*(*x_A=0, y_A=3*); *B*(*x_B=3, y_B=1*); *C*(*x_C=6, y_C=5*);
2. *A*(*x_A=0, y_A=3*); *B*(*x_B=3, y_B=0*); *C*(*x_C=6, y_C=5*);
3. *A*(*x_A=0, y_A=3*); *B*(*x_B=3, y_B=-1*); *C*(*x_C=6, y_C=5*).

Taking into consideration a unit step along Ox axis, we compose the system of equations for all unknown nodes *(x_i = 1, 2, 4, 5)* of a numerical sequence model $y_i = ch\ i$:

$$\begin{cases} y_1 = 0{,}244728471054 y_A + y_2 - 0{,}244728471054 y_B \\ y_2 = 0{,}244728471054 y_1 + y_B - 0{,}244728471054 y_4 \\ y_B = 0{,}244728471054 y_2 + y_4 - 0{,}244728471054 y_5 \\ y_4 = 0{,}244728471054 y_B + y_5 - 0{,}244728471054 y_C \end{cases}.$$

Substituting initial data 1, 2, 3 to that system, we have got the next results:

1. y_1 = *1.551822587916218* ; y_2 = *1.062365645808218* ; y_4 = *1.296986501458353* ;
y_5 = *2.275900385674355* ;
2. y_1 = *0.85644308351465* ; y_2 = *0.12225767035265* ; y_4 = *0.35687852600278* ;
y_5 = *1.580520881272782* ;
3. y_1 = *0.16106357911308* ; y_2 = *-0.81785030510292* ; y_4 = *-0.58322944945279* ; y_5 = *0.88514137687121*.

The obtained discrete models of the curves according to the initial data are presented in Figure 6.

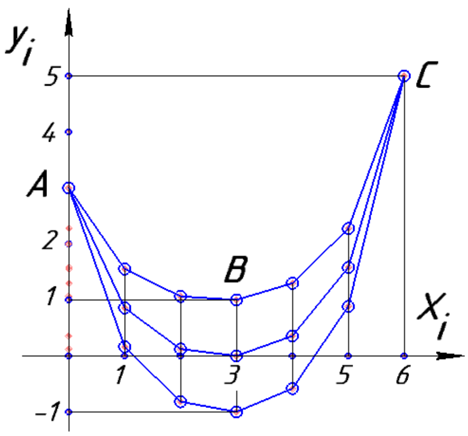


Fig. 9. Discrete models of the curves based on numerical sequence $y_i = chi$.

Different problems of forecasting processes in various fields of science can be solved by extrapolation methods. For similar extrapolation problems it is better to use asymmetric computational templates. Templates (6), (7), (11) are asymmetric dependencies, which connect the ordinates of four adjacent nodes. The given node points are located on both sides from a desired point (on the right and on the left).

Let's input in formulae (9) the ordinates of four adjacent points, which are all located on one side from a desire point (for example, to the right from it):

$$k_1 = \frac{(y_i - y_{i+3})(y_{i+3} - y_{i+4}) - (y_{i+2} - y_{i+3})(y_{i+1} - y_{i+4})}{(y_{i+1} - y_{i+3})(y_{i+3} - y_{i+4}) - (y_{i+2} - y_{i+3})(y_{i+2} - y_{i+4})} ;$$
$$k_2 = \frac{(y_{i+1} - y_{i+3})(y_{i+1} - y_{i+4}) - (y_i - y_{i+3})(y_{i+2} - y_{i+4})}{(y_{i+1} - y_{i+3})(y_{i+3} - y_{i+4}) - (y_{i+2} - y_{i+3})(y_{i+2} - y_{i+4})} . \quad (12)$$

The values of the adjacent points ordinates are taken from Table 1:

1. $y_i = 1{,}543080634815$; $y_{i+1} = 3{,}762195691084$;
$y_{i+2} = 10{,}067661995778$;
$y_{i+3} = 27{,}308232836016$; $y_{i+4} = 74{,}209948524788$.

The values of superposition coefficients for calculating the ordinates of a desired point according to the ordinates of three adjacent points will be determined by formulas (12):

$k_1 = 4{,}086161269652398$; $k_2 = -4{,}086161269660338$.

2. $y_i = 3{,}762195691084$; $y_{i+1} = 10{,}067661995778$;
$y_{i+2} = 27{,}308232836016$;
$y_{i+3} = 74{,}209948524788$; $y_{i+4} = 201{,}715636122456$.

The values of superposition coefficients are calculated by formulas (12):

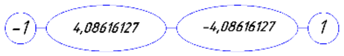
$k_1 = 4{,}086161269312475$; $k_2 = -4{,}086161269195054$.

3. $y_i = 10{,}067661995778$; $y_{i+1} = 27{,}308232836016$;
$y_{i+2} = 74{,}209948524788$;
$y_{i+3} = 201{,}715636122456$; $y_{i+4} = 548{,}317035155212$.

The values of superposition coefficients are calculated by formulas (12):

$k_1 = 4{,}08611270328305$; $k_2 = -4{,}086161270579006$.

As can be seen from the above given examples, the superposition coefficients of three adjacent points will be the same up to the eighth decimal place (they can be considered equal). Therefore a computational template for a discrete modeling of one-dimensional geometric images can be created by extrapolating the given nodal points to hyperbolic functions. This template has the form (13):



. (13)

For interpolation problems it is better to have symmetric templates. The symmetric template can be formed in the next way.

The ordinates of five adjacent curve nodes will be connected by dependencies:

$$k_1 y_{i-2} - y_{i-1} + k_2 y_i + k_3 y_{i+1} = 0 \quad (14)$$

$$k_1 y_{i-1} - y_i + k_2 y_{i+1} + k_3 y_{i+2} = 0 \quad (15)$$

By analogy with the finite difference method, equations (14) and (15) can be reduced to one equation: $k_1 y_{i-2} - y_{i-1} - k_1 y_{i-1} + k_2 y_i + y_i + k_3 y_{i+1} - k_2 y_{i+1} - k_3 y_{i+2} = 0 \Rightarrow$
$k_1 y_{i-2} - (1 + k_1) y_{i-1} + (1 + k_2) y_i + +(k_3 - k_2) y_{i+1} - k_3 y_{i+2} = 0$ (16)

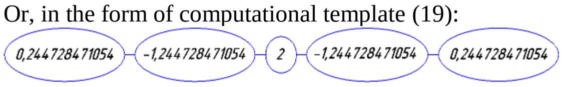
Considering that $-(1 + k_1) = (k_3 - k_2)$, $(1 + k_2) = 2$, equation (16) can be represented in the form of computational template (17):

. (17)

From (14) and (15) we have got the following:

$$0{,}244728471054 \cdot y_{i-2} - 1{,}244728471054 \cdot y_{i-1} + 2y_i -$$
$$-1{,}244728471054 \cdot y_{i+1} + 0{,}244728471054 \cdot y_{i+2} = 0 . (18)$$

Or, in the form of computational template (19):



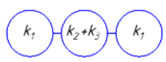
. (19)

Similarly, for example, equation (15), which connects the ordinates of four nodal points, can be formed from two equations (20) and (21), which link the ordinates of those points:

$$k_1 y_{i-1} - (k_2 + k_3) y_i + k_1 y_{i+1} = 0 , \quad (20)$$

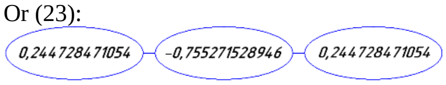
$$k_1 y_i - (k_2 + k_3) y_{i+1} + k_1 y_{i+2} = 0 . \quad (21)$$

They can be represented in the form of symmetric computational template (22):



. (22)

Or (23):



. (23)

Or, in the form of the recurrent formula:

$$0{,}244728471054 \cdot y_{i-1} - 0{,}755271528946 \cdot y_i +$$
$$+0{,}244728471054 \cdot y_{i+1} = 0 . \quad (24)$$

Recurrent formula (24) allows discret determination of the central nodal point ordinate (for two given adjacent points) of a modeling curve in the form of chain line sections. It also can interpolate by the function in form (3).

3. Conclusion

Based on the geometric apparatus of superpositions, computational templates for a discrete formation of geometric images by numerical sequences of hyperbolic functional dependences have been obtained. This extends capacities of discrete geometric modeling. Traditional methods of interpolation do not allow using transcendental functions as interpolants, because when substituting initial data we obtain the system of transcendental equations, which can not be solved in the general case.

The developed method allows conducting hyperbolic curves through the given points, which is impossible in most cases when using traditional interpolation methods.

The results of this work can be a base for further research on discrete formation of geometric images by one-dimensional numerical sequences of not only parabolic, hyperbolic, but also other elementary functional dependences. They also can be used for the formation of two-dimensional geometric images.

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