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Research paper

Measure and Analysis of the Bullwhip Effect in Supply Chain When Demand Correlation Exists between Two Market Groups Under the First-Order Moving-Average Demand Processes

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Abstract

With supply chains becoming increasingly global, the issue of bullwhip effect, a phenomenon attributable to demand fluctuation in the upstream section of the supply chains, has received greater attention from many researchers. The phenomenon in which the variation of upstream members' orders is amplified than the variation of downstream members' demands in the supply chain is called the bullwhip effect (BWEF). Most of existing research studies did not realize the demand dependency of market demands. Thus, this research focused on the study of the influence of the demand correlation coefficient between two market groups on the BWEF. The incoming demand processes are assumed the separate first-order moving-average, [MA(1)] demand patterns. The scope of the supply chain structure used in this research is composed of one manufacturer and two distribution centers. The general result reveals that the coefficient of correlation is one of several factors affecting the BWEF.

Keywords: *supply chain; bullwhip effect; first-order moving-average model; demand correlation, order-up-to policy; decentralized warehouse*

1. Introduction

Supply chain management takes into consideration all tasks that could impact costs and customer requirements so as to maximize the overall value. Besides, the management of supply chain becomes increasingly challenging as a result of fluctuating demand and complex interactions among various organizations in the supply chain. Many researchers and suppliers have discovered that order quantities tend to exhibit greater fluctuation than do customer demands across the supply chain, the phenomenon of which is referred to as the bullwhip effect and invariably leads to inefficiencies in the supply chain [1]. The management of a supply chain requires understanding of multiple tasks and issues to make better and more informed decisions. In general, there are many performance indicators used to measure the efficient and responsive supply chain, e.g., on-time delivery, supplier lead times, and consistent quality [2]. Equally important in the measure of supply chain performance management is the magnitude of bullwhip effect (BWEF) in which the more upstream along the supply chain, the greater the fluctuations of orders become. Thus, the BWEF phenomenon invariably complicates the inventory and distribution management, often resulting in inferior customer service and unnecessary inventories.

Previous studies on BWEF attempted to lower or eliminate the demand variability using various different parameters. In addition, most existing research on the key influencing factors of the BWEF were focused on the forecasting techniques, information sharing, replenishment lead times, replenishment inventory policies, and demand processes. Readers thus should refer to a research study by Bhattacharya and Bandyopadhyay [3], who had carried out review of articles to determine possible causes of the BWEF.
Several research studies investigated the impacts of different forecasting methods and their parameters on the BWEF. It was reported that use of the moving-average (MoAv) forecasting method e.g., in the works of [4, 5, 6, 7] or the exponential smoothing (ExponSm) forecasting method e.g., in the works of [8, 9] always produced the BWEF. On the other hand, according to [10, 11, 12, 13], the minimum mean square error (MSE) forecasting method reduced some specific errors and thereby the BWEF. In addition, the authors reported of the non-existence of BWEF under certain conditions. The comparisons of the three forecasting techniques were carried out in [14, 15, 16] to identify the criteria for selecting a most suitable forecasting method.

Many research studies assumed that initial demand process at the end of the supply chain's stage was the Box-Jenkins models which propose an entire family of ARIMA models (autoregressive integrated moving average processes). The AR(1) model (the first-order autoregressive process) was used by Chen et al. [4] and Xu et al. [9] who compared the BWEF with and without the centralized demand information or the coordination of supply chains; by Sirikasemsuk [11] and Sirikasemsuk and Luong [12] who studied the impact of order splitting on the BWEF based on the minimum MSE forecasting method; by Kim et al. [5] and Duc et al. [17] who focused on how stochastic lead time affects the BWEF using the MoAv forecasting technique. Luong and Phien [18] applied the AR(2) and AR(p) models (the second and high order autoregressive processes) to derive the measure of the bullwhip phenomenon, noted that the BWEF did not necessarily increase with increase in lead time. In addition, the ARMA(1,1) model as the incoming customer demand was assumed to measure the BWEF by Duc et al. [19], including Wang et al. [16]. Moreover, there were a few research studies that considered the MA(1) model (the first-order moving-average process) especially, the work of Wang et al. [16] who provided the formulas under the above three forecasting methods for the ARMA(1,1), MA(1), and AR(1) models with the simple two-stage (one-retailer-and-one-supplier) supply chain.

The number of research studies was not concerned with Box-Jenkins models, e.g. those of [5, 20, 21]. In cases when the end customer demand was identified to follow the normal distribution, Chatfield et al. [5] simulated the phenomenon of the BWEF to study the influence of the quality and sharing of demand information, and the lead-time coefficient of variation; while Hassanzadeh et al. [20] compared the BWEF between centralized and decentralized demand information under the single retailer. In addition, Bayraktar et al. [21] generated the demands from the specific equation consisting of adjustable factors about slope, base, time, sin and seasonal parameters, to analyze the effect of the triple ExponSm forecasting method on the BWEF.

Sucky [7] reported of the correlation between customer demands as a factor that could affect the BWEF. Sucky [7] noted that the BWEF would be excessively amplified if a simple supply chain structure containing one supplier and one retailer was used without consideration of risk pooling. However, unlike Sucky's research which was focused on the centralized facility, this current research paper focuses on a decentralized facility. Zhang and Burke [22] studied the effects of the demand autocorrelation and joint price on the BWEF for the VAR(1) model (the first-order vector autoregression process), while Duc et al. [23] examined the impacts of a centralized warehouse on the cost of inventory and BWEF. Recently, Sirikasemsuk and Luong [24] studied the impact of coefficient of error correlation based on the VAR(1) model on the BWEF. It is noted that the effect of the coefficient of demand correlation under the separate two customer markets for the decentralized warehouses was nonetheless outside the research scopes of [7, 22, 23, 24].

With the existence of demand dependency, the measure of the BWEF for the separate MA(1) demand processes is developed in this research. Afterwards, the impact of the correlation coefficient between two market demands is examined herein.

2. Description of Supply Chain Model

This research aims at the decentralized-warehouse supply chain system with one manufacturer and two distribution centers (DCs). A manufacturer provides a single-type product for the two DCs, while each DC serves only one customer market. It is supposed that both two market groups possess the separate and stable MA(1) models by which the parameters of the demand manners are not necessarily equal.

Let's denote that $d_{1,t}^{MA}$ and $d_{2,t}^{MA}$ are demands faced by DC 1 and DC 2, respectively, in time period t; δ_1 and δ_2 are the constants of the MA(1) models; θ_1 and θ_2 are the parameters of the MA(1) models; $\varepsilon_{1,t}$ and $\varepsilon_{2,t}$ are the forecast errors in period t and are normally distributed random variables with mean 0 and variance σ_1^2 and σ_2^2, respectively. The MA(1) models can be given as

$$d_{1,t}^{MA} = \delta_1 + \varepsilon_{1,t} - \theta_1 \varepsilon_{1,t-1}, \qquad \text{for DC} \qquad (1)$$

And

$$d_{2,t}^{MA} = \delta_2 + \varepsilon_{2,t} - \theta_2 \varepsilon_{2,t-1}, \qquad (2)$$

The key assumption in this research is $COV(\varepsilon_{1,t}, \varepsilon_{2,t}) = \gamma_\varepsilon$, not necessarily equal to zero. However, $COV(\varepsilon_{1,t}, \varepsilon_{1,t-1}) = COV(\varepsilon_{2,t}, \varepsilon_{2,t-1}) = COV(\varepsilon_{1,t}, \varepsilon_{2,t-1}) = COV(\varepsilon_{1,t-1}, \varepsilon_{2,t}) = 0$. For a general MA(1) model, the stationary condition is none, while the invertibility condition must meet with the following restriction: $|\theta_1| < 1$ and $|\theta_2| < 1$. Their variance can be shown as

$$\sigma_{d_1^{MA}}^2 = (1+\theta_1^2)\sigma_1^2 \text{ and } \sigma_{d_2^{MA}}^2 = (1+\theta_2^2)\sigma_2^2. \qquad (3)$$

The demand information is not allowed to share across members. For example, at the end of every time period, only DC1 knows its demand that comes from the first market group, while the manufacturer obtains only the DC1's orders (including the DC2's orders) at the beginning of time periods.

After reviewing their own inventory levels, DC 1 and DC 2 in this supply chain model place the orders to the manufacturer at the beginning of period t. The following assumptions for the determination of the order quantities are shown below.

1) The replenishment lead times of the manufacturer are deterministic.
2) The minimum MSE forecasting method is used by all members for Box-Jenkins models
3) The base stock inventory policy (or the order-up-to inventory policy) is used to calculate the order quantities, which is conducted similar to many research studies, e.g., the works of [4, 10-13, 16-19].
4) The inventory costs (e.g., ordering cost and holding cost) are not respected here.

3. Determination of Bullwhip Effect

Let's denote that r_{MA} is the coefficient of correlation between the two demand markets and can be expressed as

$$r_{MA} = \frac{COV(d_{1,t}^{MA}, d_{2,t}^{MA})}{\sqrt{\sigma_{d_1^{MA}}^2 \sigma_{d_2^{MA}}^2}}, \text{ where } -1 \le r_{MA} \le 1; \qquad (4)$$

At DC1 and DC2, the orders will be placed to the manufacturer at the beginning of period t, denoted by $q_{1,t}^{MA}$ and $q_{2,t}^{MA}$, respectively. In addition, the sum of the orders received by the manufacturer and the sum of incoming demands can be denoted by Q_t^{MA} and D_t^{MA}, respectively. Hence, the expression of the BWEF can be identified as

$$BW^{MA} = \frac{VAR(Q_t^{MA})}{VAR(D_t^{MA})} = \frac{VAR(q_{1,t}^{MA} + q_{2,t}^{MA})}{VAR(d_{1,t}^{MA} + d_{2,t}^{MA})}. \qquad (5)$$

Proposition 1: For the decentralized-DC supply chain model under the separate MA(1) demand processes, the BWEF measure depends on θ_1, θ_2, σ_1^2, σ_2^2, and r_{MA}; but does not depend on the lead times and the time period t. It can be determined by the following expression:

$$BW^{MA} = \frac{(1-\theta_1)^2\sigma_1^2 + (1-\theta_2)^2\sigma_2^2 + 2(1-\theta_1)(1-\theta_2)\gamma_\varepsilon}{(1+\theta_1^2)\sigma_1^2 + (1+\theta_2^2)\sigma_2^2 + 2(1+\theta_1\theta_2)\gamma_\varepsilon} \quad (6)$$

where $\gamma_\varepsilon = \frac{r_{MA}\sqrt{\sigma^2_{d_1^{MA}}\sigma^2_{d_2^{MA}}}}{(1+\theta_1\theta_2)} = \frac{r_{MA}\sqrt{(1+\theta_1^2)(1+\theta_2^2)\sigma_1^2\sigma_2^2}}{(1+\theta_1\theta_2)}$. (7)

Proof. The following two main parts need to be derived: the variance of the total demand and the variance of the total order quantity. First, the variance of the total demand can be derived as

$$\begin{aligned} VAR(D_t^{MA}) &= VAR(d_{1,t}^{MA} + d_{2,t}^{MA}) \\ &= VAR(d_{1,t}^{MA}) + VAR(d_{2,t}^{MA}) + 2COV(d_{1,t}^{MA}, d_{2,t}^{MA}) \end{aligned} \quad (8)$$

Note that the covariance between $d_{1,t}^{MA}$ and $d_{2,t}^{MA}$ can be derived as

$$COV(d_{1,t}^{MA}, d_{2,t}^{MA}) = (1+\theta_1\theta_2)\gamma_\varepsilon. \quad (9)$$

Replacing (3) and (9) into (8), I have

$$\begin{aligned} VAR(D_t^{MA}) &= \sigma^2_{d_1^{MA}} + \sigma^2_{d_2^{MA}} + 2(1+\theta_1\theta_2)\gamma_\varepsilon \\ &= (1+\theta_1^2)\sigma_1^2 + (1+\theta_2^2)\sigma_2^2 + 2(1+\theta_1\theta_2)\gamma_\varepsilon. \end{aligned} \quad (10)$$

Second, the variance of the total order quantity can be calculated through the following procedure. According to Wang et al. [16] who considered the simple one-supplier-and-one-retailer supply chain structure, the order quantities issued by DC1 and DC2, i.e., $q_{1,t}^{MA}$ and $q_{2,t}^{MA}$, respectively, can be expressed as

$$q_{1,t}^{MA} = \delta_1 + (1-\theta_1)\varepsilon_{1,t-1} \quad (11)$$

And

$$q_{2,t}^{MA} = \delta_2 + (1-\theta_2)\varepsilon_{2,t-1}. \quad (12)$$

Hence, the total order quantity can be determined as

$$Q_t^{MA} = (\delta_1 + \delta_2) + (1-\theta_1)\varepsilon_{1,t-1} + (1-\theta_2)\varepsilon_{2,t-1}. \quad (13)$$

Taking the variance of (13), I get

$$\begin{aligned} VAR(Q_t^{MA}) &= (1-\theta_1)^2 VAR(\varepsilon_{1,t-1}) + (1-\theta_2)^2 VAR(\varepsilon_{2,t-1}) \\ &\quad + 2(1-\theta_1)(1-\theta_2)COV(\varepsilon_{1,t-1}, \varepsilon_{2,t-1}) \\ VAR(Q_t^{MA}) &= (1-\theta_1)^2\sigma_1^2 + (1-\theta_2)^2\sigma_2^2 \\ &\quad + 2(1-\theta_1)(1-\theta_2)\gamma_\varepsilon. \end{aligned} \quad (14)$$

Dividing (14) by (10) produces the BWEF measure as (6) in which the lead times and the time period *t* are not in the equation. This completes the proof.

4. Effect of Demand Correlation Coefficient

After reviewing the literatures, Wang et al. [16] could develop the BWEF measure on the basis of the two-stage supply chain with a retailer and a supplier. Their results could be extended to prove that 1) the BWEF exists for the negative values of θ; 2) the BWEF does not exist for the positive values of θ; and 3) the maximum size of the BWEF is the value of 2.

Based on (6), some specific findings should be addresses in the next proposition.

***Proposition 2*:** From (6), the following properties are satisfied.

2a) In the case of $\theta_1 = \theta_2 = \theta$, it is found that the variances of the errors (σ_1^2 and σ_2^2) and the coefficient of correlation (r_{MA}) do not affect the BWEF. The BWEF measure in this case is exactly the same as in the work of Wang et al. [16], i.e., $BW_{case2a)}^{MA} = 1 - \frac{2\theta}{1+\theta^2}$.

2b) In the case of $\sigma_1^2 = \sigma_2^2 = \sigma^2$, it is found that the BWEF does not depend on the variances of the errors (σ_1^2 and σ_2^2) and can be expressed as

$$BW_{case2b)}^{MA} = \frac{(1-\theta_1)^2 + (1-\theta_2)^2 + 2(1-\theta_1)(1-\theta_2)r_{MA}}{(1+\theta_1^2) + (1+\theta_2^2) + 2(1+\theta_1\theta_2)r_{MA}}. \quad (15)$$

Proof. The Propositions 2a) and 2b) are easily proved from replacing the specific condition in each case into (6). This completes the proof.

The numerical experiments are conducted. The symmetric case where $\sigma_1^2 = \sigma_2^2$ should be considered first. Hence, from (15) the important findings are that

1) for the positive θ_1 and θ_2 the BWEF may exist as illustrated in Fig. 1, when r_{MA} approaches or is equal to -1;

2) for the negative θ_1 and θ_2, the BWEF may not exist as illustrated in Fig. 2, if r_{MA} approaches or is equal to -1.

Later, let's study the cases when $\sigma_1^2 \neq \sigma_2^2$ and both θ_1 and θ_2 are negative. If I assume that $\sigma_1^2 = 5$ and $\sigma_1^2 = 3$, the demand correlation, r_{MA} that approaches minus one can lead to intensify or weaken the BWEF as shown in Fig. 3.

From Figs 1-3, the effect of r_{MA} does not limit the extent of the BWEF to a maximum of 2, when comparing with the work of Wang et al. [16] under the MA(1) demand process.

5. Summary and Discussion

In this research, the bullwhip measure for the existence of demand dependency is analytically derived for a two-stage supply chain with one manufacturer and two distribution centers (also called 'decentralized-warehouse system'). It is assumed that DC1 and DC2 face the two market demands with the separate MA(1) models. Furthermore, it is supposed that the minimum MSE forecasting plan and the base stock policy are used for all supply chain's members. The impacts of the coefficient of correlation between the two demand markets as well as the parameters θ_1 and θ_2 on the BWEF are investigated. The notable findings are that the BWEF with the positive coefficient of correlation may be far more robust than with the negative coefficients of correlation; and if the negative coefficient of correlation come up to the value of -1 (or equal to -1), the BWEF should be calculated, before making a planning application.

The MA(1) demand patterns used in this research study is a good starting point for exploring the BWEF in more complicated supply chain structures which entail multiple-interrelation of demands and multiple sourcing decisions. Future research should attempt to examine a case of the centralized distribution center under inventory pooling using a simple forecasting technique.

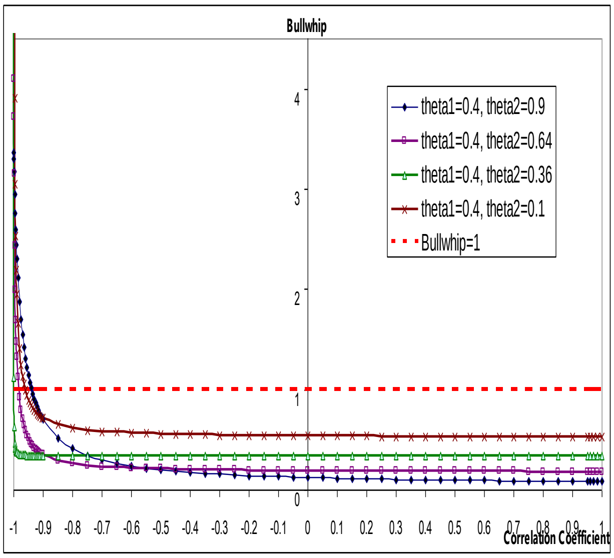

Fig. 1: Effects of r_{MA} on the BWEF for the positive values of θ_1 and θ_2 when $\sigma_1^2 = \sigma_2^2$

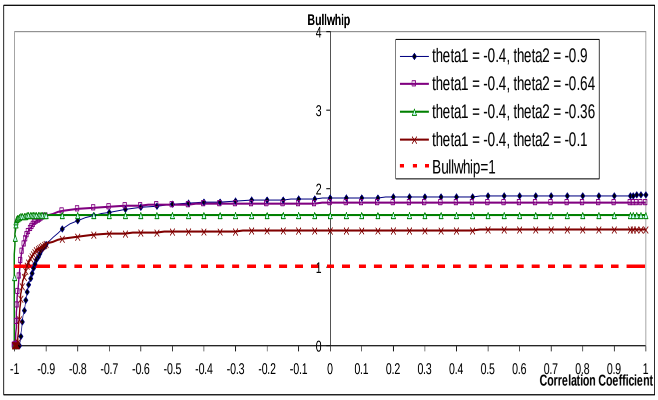

Fig. 2: Effects of r_{MA} on the BWEF for the negative values of θ_1 and θ_2 When $\sigma_1^2 = \sigma_2^2$

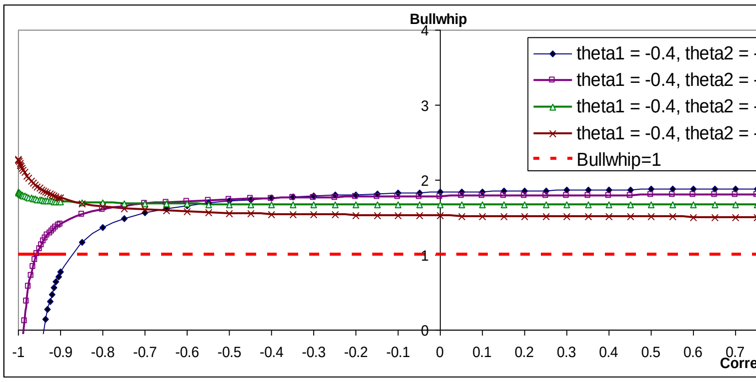

Fig. 3: Effects of r_{MA} on the BWEF for the negative values of θ_1 and θ_2 when $\sigma_1^2 = 5$ and $\sigma_1^2 = 3$

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