International Journal of Engineering & Technology, 7 (3.19) (2018) 92-96

International Journal of Engineering & Technology

Website: www.sciencepubco.com/index.php/IJET

Research paper

Calculation of Resonant Radiation Power of a Direct Sodium Lamp With Low Pressure

Sveshnikov V.K [1]*, Bazarkin A.F [2], Kurenschikov A.V [3]

[1] *Professor of Technical Sciences Department of Physics and the Methods of Teaching Physics*
[2] *Associate Professor of Technical Sciences Department of Physics and the Methods of Teaching Physics*
[3] *Associate Professor of Technical Sciences Department of Physics and the Methods of Teaching Physics*
**Corresponding author E-mail: sveshnikovmgpi@mail.ru*

Abstract

They consider the calculation technique for the resonance radiation power of a direct low-pressure sodium lamp with a sickle-shaped discharge tube cross-section. The calculation of the sodium discharge radiation power is difficult in such a tube, as compared with the calculation in a cylindrical shape tube. The dependence of the relative output of sodium discharge resonant radiation occurring in equivalent discharge tubes in the form of a parallelepiped and a cylinder is based on the involvement of the theory by M. Cayless. The obtained results were used to determine the power of DNaO-85M lamp resonant radiation. They presented the scheme of the device to study the influence of the tube geometry on sodium discharge power. It is shown that the power of the resonance radiation generated by an equivalent discharge will be the greater, the larger the cross section of the discharge tube. The discrepancy between the calculated and the experimental data for resonant radiation power determination in a crescent-shaped tube is less than 4%.

Keywords: *resonant radiation, equivalent discharge, crescent shaped discharge tube, diffusion.*

1. Introduction

Low-pressure sodium lamps are the most energy-efficient light sources. Up to 80% of the electric energy for a plasma column can be converted into visible radiation with the wavelength of 589.0-589.6 nm. The light output of low-pressure sodium lamps commercially available abroad makes 160-200 lm/W, which is 1.5-2 times higher than that of LED lamps [1].

The design of a direct sodium lamp DNaO-85M has been developed and improved in Russia [2, 6, 7, 8]. The discharge tube (DT) of DNaO-85M lamp has a sickle-shaped cross-section. The calculation of sodium discharge radiation power in such a DT is difficult as compared with the calculation in a DT of a cylindrical shape [4].

Below we consider the relative yield of equivalent discharge resonance radiation, occurring in a parallelepiped and a cylinder, respectively. The obtained results were used to determine the radiation power of a direct sodium lamp with a crescent-shaped cross-section of DT.

2. Influence of Discharge Tube Geometry on Sodium Discharge Radiation Power

Let us determine the dependence of sodium discharge resonance radiation relative output occurring in DTs of a parallelepiped and a cylinder form on geometric factors, using the theory by M. Cayless [5]. In contrast to [5], which compares the powers of equivalent mercury discharges for plane sections, we have considered the case for low-pressure sodium discharges.

For a low-pressure sodium discharge in inert gas-sodium vapor mixture, the assumptions [3] are valid. The mean free path of electrons and ions at the neon pressure of 133 Pa-1.33 kPa is much smaller than the radius of a discharge tube. The inert gas of the discharge is almost not excited and is not ionized, due to which diffuse discharge conditions are created. The main processes under the diffusion discharge regime are the ionization of neutral sodium atoms and the recombination of electrons and ions on the walls of a tube dielectric shell due to ambipolar diffusion. The recombination of charged particles in the plasma volume is small. The tubes with a crescent-shaped cross-section provide almost a uniform distribution of sodium during the lamp operation, since it is concentrated in the capillary cavities of the DT. Taking these assumptions, it is possible to calculate the radiation power of equivalent discharges.

A discharge tube consists of the sections of various lengths (Fig. 1). Let us establish the dependence of the sodium discharge resonance radiation relative output occurring in an equivalent discharge tube of rectangular cross section for the section (A, A) on geometrical factors. Let's take a tube of circular cross-section as an equivalent discharge tube.

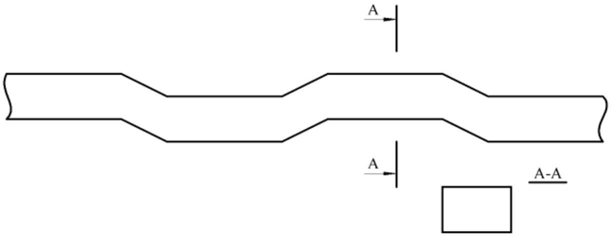

Figure 1: Discharge tube

The condition of a discharge existence in a DT is the following one:

$$D_a \Delta n_e + v_i\, n_a = 0\,, \tag{1}$$

where D_a is the ambipolar diffusion coefficient; n_e – electron concentration; v_i – the number of ionizations per atom and one electron in a time unit; n_a – the concentration of neutral atoms.

Denoting

$$\Lambda^2 = D_a / v_i\,, \tag{2}$$

we get the following:

$$\Lambda^2 \Delta n_e + n_a = 0\,. \tag{3}$$

The equation (3) determines the characteristic diffusion length Λ.

Let's find the solution of equation (3) for the section (A, A), DT (Fig. 1) representing a straight parallelepiped. In the Cartesian coordinate system, the equation (3) has the following form:

$$\frac{\partial^2 n_e}{\partial x^2} + \frac{\partial^2 n_e}{\partial y^2} - \frac{\partial^2 n_e}{\partial z^2} + \frac{1}{\Lambda_{\text{п}}^2} n_e = 0. \tag{4}$$

The solution of this equation satisfying the following boundary conditions is the following one:

$n_e = 0,$ at $\{ y = \pm m/2$ $\{ y = 0$ $n_e = n_0$, at

is the following:

$$n_e = n_0 \cos\frac{}{}x\cos\cos\frac{}{}y\cos\frac{}{}z,$$

or

$$N_n = \frac{}{} = \cos\frac{}{}x\cos\cos\frac{}{}y\cos\frac{}{}z.$$

The characteristic diffusion length $\Lambda_{\text{п}}$ is determined from (4) by inserting $\partial^2 N_n/\partial x^2$, $\partial^2 N_n/\partial y^2$, $\partial^2 N_n/\partial z^2$ into it.

Hence,

$$\Lambda_{\text{п}} = lmh_n / \sqrt{m^2 h_n^2 + l^2 h_n^2 + l^2 m^2} \tag{5}$$

Then we find the solution of the equation (3) for a cylinder with an axial symmetry. The equation (3) in the cylindrical coordinate system has the following form:

$$\frac{}{} + \frac{}{} + \frac{}{} + \frac{}{} n_e = 0, \tag{6}$$

where r is the current radius.

The solution of the equation satisfying the given boundary conditions:

$n_e = n_0$, при $\{$ [illegible]; $n_e = 0$, при

is the following equation:

$$n_e = J_0(2{,}4r/R)\, n_0 \cos\frac{}{}z,$$

or

$$N = J_0(2{,}4r/R) \cos\frac{}{}z. \tag{7}$$

The characteristic diffusion length $\Lambda_{\text{п}}$ will be the following:

$$\Lambda_{\text{п}} = \sqrt{D_a}/\sqrt{v_i} = h_{\text{п}} 2/\sqrt{(\pi R)^2 + (2{,}4h_{\text{п}})^2} \tag{8}$$

Now let's establish the dependence of the discharge resonance radiation relative power in an equivalent discharge tube of a rectangular shape on geometric factors. Let's take a tube of a circular cross-section as an equivalent discharge tube. The output of the resonance radiation is determined by the following expression [5]:

$$W = Z_e U_a n_a \int n_e ds\,, \tag{9}$$

where Z is the number of atom transitions from a normal state to an excited one due to the collision with electrons; e is the electron charge; U_a - the potential for resonance level excitation.

The author of [5], introducing dimensionless quantities:

$$J = \frac{}{}\int N ds, \quad N = \frac{}{}, \quad K = \frac{}{}, \tag{10}$$

Puts down the equation (4) in the following form:

$$W = JK\theta, \tag{11}$$

where $\theta = eU_a n n_0 Z D_a / v_i$ is the maximum value n_e.

The ratio W to the output of resonant radiation in an equivalent discharge of an equivalent cylindrical tube $W_{\text{п}}$ is the following one:

$$W/W_{\text{п}} = JK\theta / J_{\text{п}} K_{\text{п}} \theta_{\text{п}}. \tag{12}$$

According to the definition of equivalent discharges in equivalent tubes $\Lambda^2 = D_a/v_i = const.$

This gives the expression for the relative power of equivalent discharge resonance radiation:

$$H = W/W_{\text{п}} = JK/J_{\text{п}}K_{\text{п}}\,, \tag{13}$$

or

$$J_1 = J/J_{\text{п}}, \quad K_1 = K/K_{\text{п}},$$

where K_1 – is the quantity proportional to the relative area of an equivalent tube cross section, J_1 – the value that determines the degree of area filling by plasma.

Let us determine the degree of a discharge tube considered volume filling $J = J_{\text{п}}/J_{\text{ц}}$.

According to (10), the degree of a direct parallelepiped filling by plasma is equal to the following:

$$J_{\text{п}} = \frac{1}{mlh}\int_{-h/2}^{h/2} dz \int_{-l/2}^{l/2} dx \int_{-m/2}^{m/2} N_n dy = \frac{8}{\pi^3} = 0{,}258\,. \tag{14}$$

Now, let's determine the degree of the cylinder volume V filling by plasma:

$$J_{\text{ц}} = \frac{1}{\pi R^2 h_{\text{ц}}}\int_{-h_{\text{ц}}/2}^{h_{\text{ц}}/2} dz \int_0^R \cos\frac{\pi}{h_{\text{ц}}} z J_0(2{,}4r/R) 2\pi r dr = 0{,}276\,. \tag{15}$$

Thus, the required value J is equal to the following:

$$J = J_{\text{п}}/J_{\text{ц}} = \frac{0{,}258}{0{,}276} = 0{,}934. \tag{16}$$

The condition for the equivalence of discharges is $\Lambda_{\text{п}} = \Lambda_{\text{ц}} = const$ or taking (10) and (13):

$$\frac{lmh_{\text{п}}}{\pi\sqrt{(mh_{\text{п}})^2 + (lh_{\text{п}})^2 + (lm)^2}} = \frac{h_{\text{ц}} R}{\sqrt{(\pi R)^2 + (2{,}4h_{\text{ц}})^2}} \tag{17}$$

For the case when the lengths $h'_ц$, $h'_п$ of a cylinder and a straight parallelepiped are equal, the condition (17) will take the following form:

$$R/2{,}4 = lm/\pi\sqrt{l^2+m^2}\,, \tag{18}$$

where $\Lambda_ц = R/2{,}4$; $\Lambda_п = l_m/\pi\sqrt{l^2+m^2}$.

Taking $R = 1$, we obtain that

$$1/_{2,4} = \frac{lm}{\pi\sqrt{l^2+m^2}}\,. \tag{19}$$

Taking $m_1 = 2{,}000$; $m_2 = 2{,}570$; $m_3 = 4{,}000$ from the equation (19) we get the definite values of the equivalent discharge height l in a straight parallelepiped (Figure 2), i.e. $l_1 = 1{,}727$; $l_2 = 1{,}517$; $l_3 = 1{,}3820$.

The relative power of resonant radiation generated by an equivalent discharge in a cylindrical tube is the following one:

$H = JK$; но $J = J_П/J_Ц = 0{,}934$; $K = K_П/K_Ц = ml/\pi R^2$,

hence,

$$H = 0{,}934ml/\pi R_0^2\,. \tag{20}$$

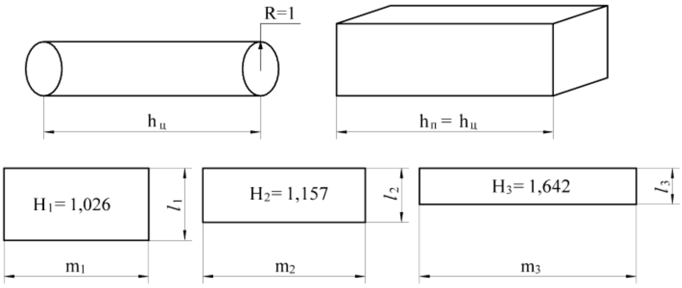

Figure 2: The geometry of equivalent tubes.

Then, for the values of m_1, m_2, m_3 given earlier, we define H_1, H_2, H_3 according to the formula (20):

$H_1=0{,}934m_1l_1/\pi R^2=0{,}934\cdot 2\cdot 1{,}727/3{,}144\cdot 1=1{,}026$,

$H_2=0{,}934m_2l_2/\pi R^2=0{,}934\cdot 2{,}57\cdot 1{,}517/3{,}144\cdot 1=1{,}157$,

$H_3=0{,}934m_3l_3/\pi R^2=0{,}934\cdot 4\cdot 1{,}382/3{,}144\cdot 1=1{,}642$.

From the obtained results it follows that at identical lengths $h_п$ of the cylinder and $h_п$ of a direct parallelepiped (Figure 2) the power of the resonance radiation generated by an equivalent discharge per length unit is greater, the larger the ratio m/l. The degree of a discharge tube (with a rectangular cross-section) filling with plasma remains constant.

Thus, the power $W_п$ of the resonance radiation for a tube with a rectangular cross section, expressed via the radiation power generated by an equivalent discharge in the tube $W_ц$ of cylindrical shape, for the case when $h_ц$=$h_п$, has the following expression:

$$W'_п = W_ц 0{,}934ml/\pi R^2. \tag{21}$$

The power of the resonance radiation of a generated equivalent discharge will be the greater, the more flattened the cross section is, which agrees with [5].

Having calculated $W_ц$ for a discharge tube of a circular cross section of unit length, we can calculate the resonance radiation power for a crescent-shaped tube using the formula (21). The value of the discharge current density is proportional to the geometrical factor m/l.

2.1 Determination of Resonance Radiation Power for A Crescent-Shaped Discharge Tube

Fig. 3 shows a cross-sectional view of a discharge tube of a direct sodium lamp. During the lamp operation, sodium is concentrated in the capillary cavities to limit its migration along the DT. Taking into account the specific features of the DT, its sickle shape is replaced by a rectangular shape, equal to it by area with the height of the rectangle $l = R$ and the length m (fig. 3)

$$m = S_C/R, \tag{22}$$

where S_C – is the crescent section area.

From the condition (18) of discharge equivalence, in the case of cylinder length equality $h_ц$ and $h_п$ of a straight parallelepiped, we determine the radius of an equivalent discharge tube.

$$R_э = 2{,}4\,\frac{lm}{\pi\sqrt{l^2+m^2}}\,\pi\,. \tag{23}$$

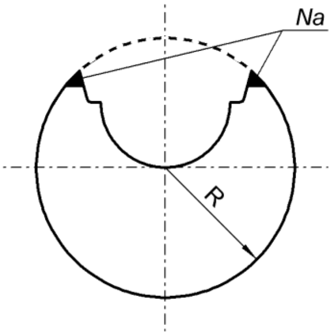

Figure 3: The cross section of a crescent discharge tube.

The radiation power of a discharge tube with a crescent-shaped cross-section is determined by the formula (21).

The gradient of the equivalent discharge potential in this case is identical, and the power of the discharge current in the tube of a circular cross section should be reduced by the amount of

$\pi R^2{}_п/ml$.

Light flux measurement in equivalent discharges.

3. Discussion of Results

The scheme of DT geometry influence study on the power of the sodium discharge resonance radiation is proportional to the light flux F is shown on Fig. 4. The discharge tube 1 of the grooved type of the variable cross-section is placed inside the thermostat 2. The tube with the length of 5 x 10-1 m contains the areas with a round rectangular and sickle cross sections. It is filled with neon up to the pressure of 1.3 kPa. A smoothly adjustable voltage of 50 Hz is applied to the tube electrodes. The temperature of the tube is controlled by the thermocouple 3.

The luminous flux is recorded using epy photocell 4 of SF-25 type. The photocell is provided with a tube for light prevention in which the light filter is located. The device design provides a photocell movement along a longitudinal axis of a discharge tube, which makes it possible to determine the discharge radiation power of its various sections.

In order to perform the measurements, a thermostat heater is turned on. When the tube reaches the temperature of 160 °C, a discharge is excited in it. The measurements of the light flux in relative units are carried out in a steady state at the tube temperature of 260 °C.

The time of a steady-state sodium discharge establishment makes 15 minutes. The discharge current during radiation power measurement in the sections of a tube with a cylindrical shape assumes the following values: 0.32 A, 0.49 A, 0.65 A, 0.81 A.

The discharge current $I_Ц$, flowing in the plasma of circular cross-section tube sections is connected with the current I_C on the tube sections with a sickle-shaped cross-section by the following formula:

$$I_с = I_ц S_с / \pi R^2, \qquad (24)$$

where S_C is the area of a tube with a crescent-shaped cross-section.

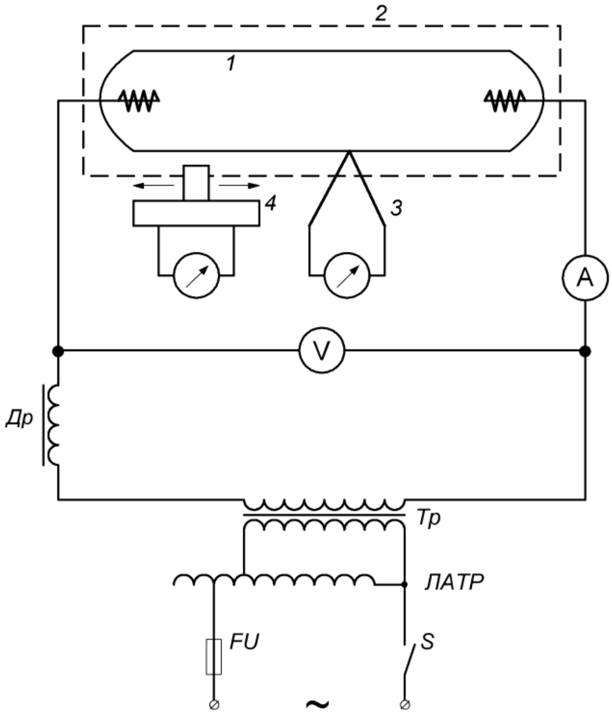

Figure 4: Device diagram to study the effect of a tube geometry on sodiumdischarge power: 1 - discharge tube; 2 - thermostat; 3 - thermocouple; 4 - photocell.

In order to verify the abovementioned procedure to calculate the power of sodium radiation in a DT of the crescent section let's replace it by a tube of a rectangular cross section equal to it in area with the height *l* = *R* and the length *m*.

Let us calculate the power of the resonance radiation for the DT of the rectangular cross section using the following condition. The neon pressure is P_{Ne} = 10,6 kPa, the wall temperature of a DT makes 260 °C, m = $1.35 \cdot 10^{-2}$ m, l = $0.8 \cdot 10^{-2}$ m, Spr = $0.88 \cdot 10^{-4}$ m^2, Sc = $2.08 \cdot 10^{-4}$ m^2. The discharge current takes the value of 0.4 A, 0.6 A, 0.8 A, 1 A.

Using the formula (23), we determine the radius of an equivalent cylindrical tube $R_э$ =$0{,}53 \cdot 10^{-2}$ m. The discharge current $I_ц = I_п \frac{\pi R^2}{S}$ = 0,32 A; 0,49 A; 0,65 A.

Table 1: shows the experimental values of the light current F of an equivalent DT from the current in relative units.

$I_Ц$, A	0,32	0,49	0,65	0,85
F, rel. un.	25	31	34	41

Table 2: shows the results of $W_п$ and *F* calculations according to the formula (21) and experimental data.

I, A	*F calc.*, rel. un.	*F exp.*, rel. un.	Discrepancy %
0,4	30,5	31,2	2,3
0,6	37,8	39,0	3,1
0,8	42,7	44,2	3,4
1	50	52	3,8

It follows from the table that at the discharge current of 0.4 A, the discrepancy in the determination of F by the calculated and experimental method makes 2.3%, while at the current of 1.0 A it does not exceed 3.8%.

4. Conclusions

1. The specific features of a discharge tube with a direct sodium lamp allow its crescent section to be replaced by a rectangular cross-section equal in area and the height equal to the radius of the circumference around the circumference section during calculation.
2. A tube with a rectangular cross-section corresponds to an equivalent cylindrical tube with the radius defined by the formula (23).
3. The power of the resonant radiation for the crescent-section tube, expressed in terms of the radiation power generated by an equivalent discharge in a cylindrical shape tube, has the following form:

$$W_с = 0{,}934 \frac{ml}{\pi R_э^2}.$$

4. The power of resonant radiation generated by an equivalent discharge is the greater, the larger the cross section of a tube, which agrees with the data given by Kayless.
5. They considered the scheme of the device and the method of sodium lamp luminous flux determination emitted by a tube.
6. The discrepancy between the calculated and the experimental data during resonance radiation power determination is less than 4%.

5. Summary

The article proposed the method to calculate the power of resonant radiation from a direct low-pressure sodium lamp. They considered the effect of a discharge tube geometry on the relative yield of the sodium discharge. It is shown that the power of resonance radiation increases with a tube cross-section increase. It is established that a discharge tube with a rectangular cross section corresponds to an equivalent cylindrical tube. The specific features of a discharge tube with a direct sodium lamp allowed its sickle-like cross-section to be replaced by a rectangular cross-section equal in area and height, equal to the radius of the circumference circumscribed around the cross-section. Thus, the calculation of the sodium discharge resonance radiation power in a tube of a crescent-shaped cross-section is reduced to the radiation power calculation in an equivalent tube of a cylindrical shape.

References

[1] Modifications and technical characteristics of PHILIPS MASTER SOX-E lamps. URL: http://www.lighting.philips.com/main/prof/conventional-lamps-and-tubes/high-intensity-discharge-lamps/sox-low-pressure-sodium/master-sox-e (reference date: 05.02.2017).

[2] Sveshnikov V.K. Modernization of sodium lamp design with low pressure / V.K. Sveshnikov, A.F. Bazarkin, A.M. Kokinov // The

International Scientific Institute "EDUCATIO". - No. 6. - 2014. - pp. 13-16.

[3] Sveshnikov V.K. The physics of the sodium discharge with low pressure / V.K. Sveshnikov. - Moscow: MGPI, 1987. - 99 p.

[4] Sveshnikov V.K. The study of a discharge tube geometry influence on discharge radiation power / V.K. Sveshnikov // Teaching Experiment in Education. - No. 4. - 2011. - pp. 65-75.

[5] Cayless, M. A. Production of resonans in discharge tubes of non circular cross-section / M. A. Cayless // Brit. Journ. of applied physics. - 1960. - №11. - pp. 492-497.

[6] Svechnikov, V. K. The Aspects Of Increase Of Luminous Efficacy Of The Low Voltage Sodium Vapor Lamp DNaO-85 / V. K. Svechnikov, A. V. Kurenshchikov, A. F. Bazarkin // International Journal of Applied Engineering Research – 2015. – V.10. – № 24. – pp. 44879 – 44883.

[7] Sveshnikov, V. K. Non-Destructive Method of Electron Concentration De-Terminationin Discharge Tubes / V. K. Sveshnikov, A. V. Kurenshikov // Journal of Engineering and Applied Sciences. – 2017. – Vol. 12. - issue №13. – pp. 3374-3337.

[8] Sveshnikov, V. K. Quality Control of Sodium Lamps Discharge Tubes / V. K. Sveshnikov, A. V. Kurenshchikov, K. A. Ilyushkin // International Journal of Pharmacy & Technology. – 2016. – V. 8. – № 3. – pp. 15247-15252.