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Research paper

Adaptive Synchronization of Chaotic Genesio–Tesi Systems Via A Nonlinear Control

Esmat Sadat Alaviyan Shahri

Faculty of Computer & Electrical Eng., University of Gonabad, Gonabad, Iran
**Corresponding author E-mail: ss_alaviyan@yahoo.com*

Abstract

The paper presents the stabilization and adaptive synchronization problem of a class of chaotic systems (Genesio–Tesi system) with three unknown parameters. A novel nonlinear control effort is proposed and an adaptive strategy is presented in order to the states of two Genesio–Tesi systems were asymptotically synchronized. The known Lyapunov method guarantees the presented stability analysis and design. An illustrative simulation result is given to demonstrate the effectiveness of the proposed chaos synchronization scheme.

Keywords:

1. Introduction

The insurgent and classically unpredictable behavior presented by nonlinear systems is termed as chaos. It happens practically in every field, such as applied mechanical, electronics, chemical and systems, etc. [1]. Control of chaos by altered methodologies have been addressed including turbulent fluid, magneto-elastic oscillators, oscillating chemical reactions, etc. [2]. Among of them, chaos synchronization and control has grown into a topic of great interest [3] [4]. Numerous techniques have been offered in the last decades to succeed the synchronization and stabilization of chaotic systems, comprising parametric perturbation, adaptive sliding mode control (ASMC), observer based synchronization, etc. The popular of these applications has been addressed by using fractional order chaotic systems [5]. Authors proposed a fractional order control scheme for the stabilization of the nonlinear chaotic Genesio-Tesi systems in [6]. Furthermore, the synchronization problem of the system via feedback control was investigated in [7] [8]. However, in these works, the exactly knowing of the system parameters were assumed but in real world and application, some or all of the parameters are unknown. A dynamic surface control scheme by applying feedback controller was offered for the uncertain Genesio-Tesi chaotic system in [9]. In the present work, the problem of chaos synchronization of Genesio-Tesi system is considered with all unknown parameters. For chaotic synchronization of the considered system, a new Theorem is derived base on the Lyapunov stability and update rules are achieved. The simulation results prove that the proposed control structure has adequate efficiency to asymptotically stability.

The paper is organized in four sections as follows. Section II introduces Genesio–Tesi, slave and master system, then by applying the proposed controller and updating rules adaptive synchronization is proven. Simulation results are given in section III. The work is concluded in section IV.

2. Adaptive Synchronization of Genesio–Tesi system

The Genesio–Tesi system is one of models of chaos nonlinear systems since it captures many features of chaotic systems. It contains a simple square part and three regular differential equations depend on three positive parameters. The dynamic equation of the system is shown as:

$$\begin{cases} \dot{x} = y \\ \dot{y} = z \\ \dot{z} = -cx - by - az + x^2 \end{cases} \quad (1)$$

where x, y and z are state, a, b and c are the positive real constants satisfying $ab < c$. For example, $a = 1.2$, $b=2.92$ and $c = 6$ are chosen that the system has chaotic behavior, where the chaotic motion of the system is illustrated in Fig. 1 with the initial condition $\begin{bmatrix} x & y & z \end{bmatrix}^T = \begin{bmatrix} 0.2 & -0.3 & 1 \end{bmatrix}$. Furthermore, the states behaver of open loop system is shown in Fig. 2.

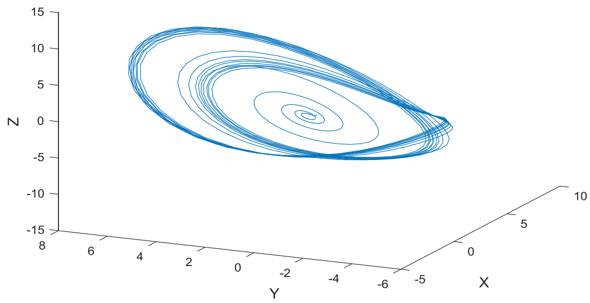

Fig. 1. The chaotic trajectories of Genesio–Tesi system.

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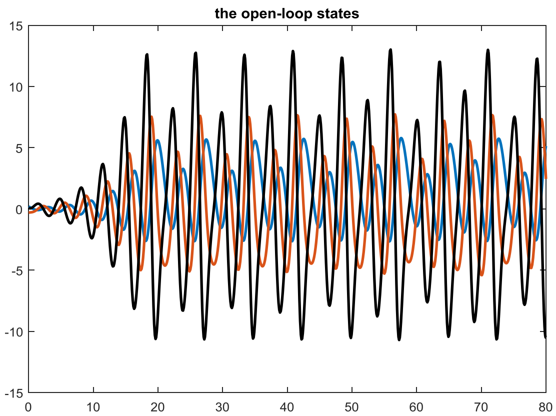

Fig. 2. The open loop states of Gensio-Tesi system

To detect the synchronization behavior in response (slave) system when all parameters of the system are unknown and different with those of the drive (master) system, assume that two Genesio–Tesi systems exist, where the drive (with the subscript m) and response (with the subscript s) systems are defined respectively

$$\begin{cases} \dot{x}_m = y_m \\ \dot{y}_m = z_m \\ \dot{z}_m = -cx_m - by_m - az_m + x_m^2 \end{cases} \tag{2}$$

And

$$\begin{cases} \dot{x}_s = y_s \\ \dot{y}_s = z_s \\ \dot{z}_s = -\hat{c}x_s - \hat{b}y_s - \hat{a}z_s + x_s^2 \end{cases} \tag{3}$$

where $\hat{a}, \hat{b}$ and $\hat{c}$ are the estimate of the parameters a, b, and c.

The objective of this paper is to design a controller $u(t)$ such that the controlled response system with three unknown parameter. consider the following Gensi-Tesi system

$$\begin{cases} \dot{x}_s = y_s \\ \dot{y}_s = z_s \\ \dot{z}_s = -\hat{c}x_s - \hat{b}y_s - \hat{a}z_s + x_s^2 + u(t) \end{cases} \tag{4}$$

Defining error signal as

$$\begin{cases} e_1 = x_s - x_m \\ e_2 = y_s - y_m \\ e_3 = z_s - z_m \end{cases} \tag{5}$$

The error signals between drive system (2) and the controlled response system (4) is

$$\begin{cases} \dot{e}_1 = e_2 \\ \dot{e}_2 = e_3 \\ \dot{e}_3 = -\hat{c}x_s - \hat{b}y_s - \hat{a}z_s + x_s^2 + u(t) + cx_m + by_m + az_m - x_m^2 \end{cases} \tag{6}$$

Theorem: Consider the system dynamics (4), if the system is controlled by

$$u(t) = -(3 + x_s + x_m - \hat{c})e_1 - (5 - \hat{b})e_2 - (3 - \hat{a})e_3 \tag{7}$$

with adaptation law

$$\begin{aligned} \dot{\hat{a}} &= x_3(2x_1 + 2x_2 + x_3) \\ \dot{\hat{b}} &= x_2(2x_1 + 2x_2 + x_3) \\ \dot{\hat{c}} &= x_1(2x_1 + 2x_2 + x_3) \end{aligned} \tag{8}$$

then the controlled Genesio Tesi chaotic system (4) with three unknown parameters is synchronous with drive Genesio Tesi chaotic system (2).

Proof: Consider error system with Eq.(6) and the following Lyapunov function as

$$\begin{aligned} V(e_1, e_2, e_3, \hat{a}, \hat{b}, \hat{c}) &= \frac{1}{2}\left(e_1^2 + (e_1 + e_2)^2 + (2e_2 + 2e_2 + e_3)^2\right) \\ &+ \frac{1}{2}\left((\hat{a} - a)^2 + (\hat{b} - b)^2 + (\hat{c} - c)^2\right) \end{aligned} \tag{9}$$

The time derivative of Lyapanov function, we have

$$\begin{aligned} \dot{V} &= \left(e_1\dot{e}_1 + (e_1 + e_2)(\dot{e}_1 + \dot{e}_2) + (2e_2 + 2e_2 + e_3)(2\dot{e}_2 + 2\dot{e}_2 + \dot{e}_3)\right) \\ &+ \left((\hat{a} - a)\dot{\hat{a}} + (\hat{b} - b)\dot{\hat{b}} + (\hat{c} - c)\dot{\hat{c}}\right) \end{aligned} \tag{10}$$

we try to simplify the above Eq. (10)as

$$e_1\dot{e}_1 = e_1e_2 = -e_1^2 + e_1(e_1 + e_2)$$

$$(e_1 + e_2)(\dot{e}_1 + \dot{e}_2) = (e_1 + e_2)(e_2 + e_3) = -(e_1 + e_2)^2 + (e_1 + e_2)(e_1 + 2e_2 + e_3)$$

$$(2e_1 + 2e_2 + e_3)(2\dot{e}_1 + 2\dot{e}_2 + \dot{e}_3) = -(2e_1 + 2e_2 + e_3)^2 + (2e_1 + 2e_2 + e_3)\left((2e_1 + 2e_2 + e_3) + 2e_2 + 2e_3 + \dot{e}_3\right)$$

$$= -(2e_1 + 2e_2 + e_3)^2 + (2e_1 + 2e_2 + e_3)(2e_1 + 4e_2 + e_3 + \dot{e}_3)$$

Substituting Eq. (6) in (12), the result can be rewritten as

$$\begin{aligned} &(2e_1 + 2e_2 + e_3)(2\dot{e}_1 + 2\dot{e}_2 + \dot{e}_3) = -(2e_1 + 2e_2 + e_3)^2 \\ &+ (2e_1 + 2e_2 + e_3)\left(2e_1 + 4e_2 + 3e_3 - \hat{c}x_s - \hat{b}y_s - \hat{a}z_s + x_s^2 + u(t) + cx_m + by_m + az_m - x_m^2\right) \end{aligned}$$

To streamline Eq. (13), $\hat{c}x_m, \hat{b}y_m$ and $\hat{a}z_m$ Add and subtract as

$$\begin{aligned} &(2e_1 + 2e_2 + e_3)(2\dot{e}_1 + 2\dot{e}_2 + \dot{e}_3) = -(2e_1 + 2e_2 + e_3)^2 \\ &+ (2e_1 + 2e_2 + e_3)\left((2 - \hat{c})e_1 + (4 - \hat{b})e_2 + (3 - \hat{a})e_3 + x_s^2 + (c - \hat{c})x_m + (b - \hat{b})y_m + (a - \hat{a})z_m - x_m^2 + u(t)\right) \end{aligned}$$

Substituting Eqs.(11) and (14) in Eq. (10), we have

$$\begin{aligned} \dot{V} &= -e_1^2 + e_1(e_1 + e_2) - (e_1 + e_2)^2 + (e_1 + e_2)(e_1 + 2e_2 + e_3) - (2e_1 + 2e_2 + e_3)^2 \\ &+ (2e_1 + 2e_2 + e_3)\left((2 - \hat{c})e_1 + (4 - \hat{b})e_2 + (3 - \hat{a})e_3 + x_s^2 + (c - \hat{c})x_m + (b - \hat{b})y_m + (a - \hat{a})z_m - x_m^2 + u(t)\right) \\ &+ \left((\hat{a} - a)\dot{\hat{a}} + (\hat{b} - b)\dot{\hat{b}} + (\hat{c} - c)\dot{\hat{c}}\right) \\ &= -e_1^2 - (e_1 + e_2)^2 - (2e_1 + 2e_2 + e_3)^2 + \\ &(2e_1 + 2e_2 + e_3)\left((3 - \hat{c})e_1 + (5 - \hat{b})e_2 + (3 - \hat{a})e_3 + x_s^2 + (c - \hat{c})x_m + (b - \hat{b})y_m + (a - \hat{a})z_m - x_m^2 + u(t)\right) \\ &+ \left((\hat{a} - a)\dot{\hat{a}} + (\hat{b} - b)\dot{\hat{b}} + (\hat{c} - c)\dot{\hat{c}}\right) \end{aligned}$$

By utilizing the control input (7) and update rule (8), we have

$$\dot{V} = -e_1^2 - \left(e_1 + e_2\right)^2 - \left(2e_2 + 2e_2 + e_3\right)^2 \le 0 \tag{13}$$

Since $\dot{V} \le 0$, we not authorize immediately obtain that the origin of error system (6) is asymptotically stable. From (13) we can easily indicate that the square of $e_i, (i = 1, 2, 3)$ is integrable with respect to time, in other hands, $e_1(t), e_2(t), e_1(t) \to 0$ as $t \to \infty$. Thus it concludes that the response system (6) synchronize the drive system (2) by the controller (7). This completes the proof.

3. Simulation Results

In this section, to verify and validate the effectiveness of the proposed controller, we argue the simulation result for Genesio–Tesi system. The simulation results are carried out using the MATLAB. The master system is chosen as stable system with parameters $a = 3, b = 3$ and $c = 6$. Fig. 3 shows the behaver of master states.

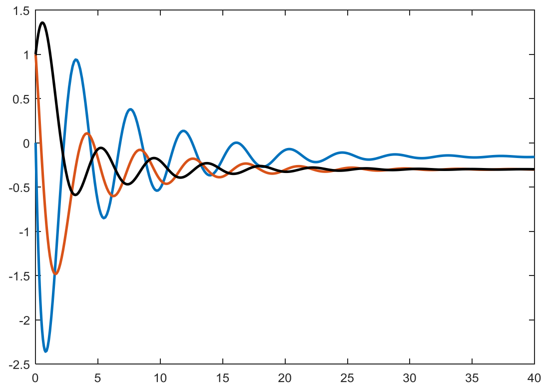

Fig. 3. The behavior of master states

By applying adaptation law, Eq. (8), and the proposed controller, Eq. (7), to the slave system with unknown parameters, Eq (4), the closed-loop system is stable. Figs. 4 and 5 present the error and state trajectories of master and slave systems, respectively. Fig. 6 also show the estimation of parameters $\widehat{a}, \widehat{b}$ and $\widehat{c}$.

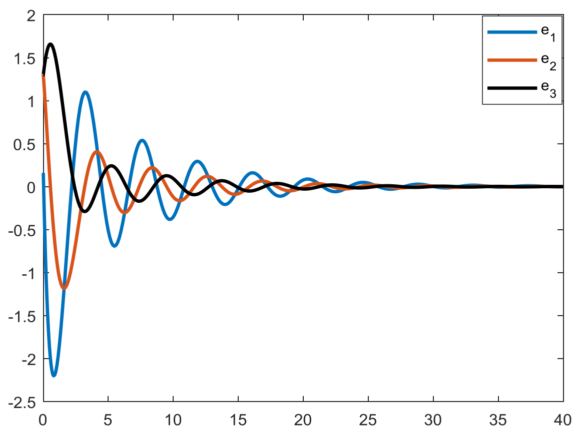

Fig. 4. Synchronization errors, e1, e2, e3 with time t.

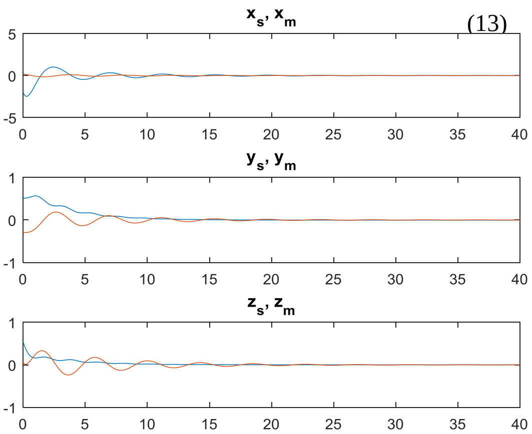

Fig. 5. State trajectories of master and slave systems

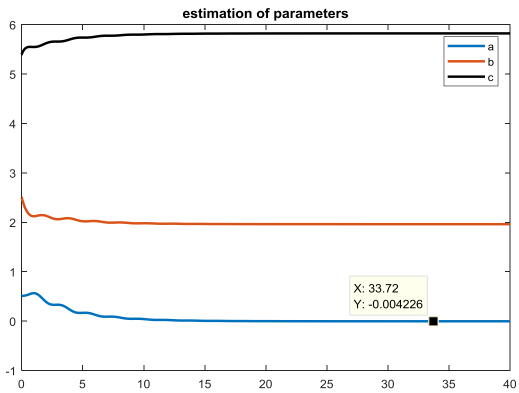

Fig. 6. Estimation of parameters $\widehat{a}, \widehat{b}$ and $\widehat{c}$.

Form the charts, it is obviously, the proposed controller and update rules have the adequate performance and the synchronization of slave and master system is achieved.

4. Conclusions

In this paper, the problem of synchronization for controlled Genesio-Tesi chaotic systems with unknown parameters has been considered. Using the novel nonlinear control scheme and the Lyapunov stability theory, asymptotic chaos synchronization was studied and update rules were achieved. Finally, the numerical simulations appearances the effectiveness of our technique proposed. Furthermore, synchronization of fractional order Gensio Tesi chaotic system can be investigated with unknown parameters by applying nonlinear feedback controller as continue the research.

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