

# An LMI Approach with Pole Placement Objective for the Design of Robust SSSC Controller for Damping Inter-Area Mode Oscillation Considering Global Signal

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## Abstract

This paper presents robust Static Synchronous Series Capacitor (SSSC) controller design using LMI approach with pole placement objective. The controller is designed for damping of inter-area mode oscillations in two area four machine test system considering a global signal. The control signals are obtained from wide area measurements (WAMs) system. Residue analysis is performed to decide best input signal to the controller. The mixed sensitivity approach in Linear Matrix Inequality (LMI) formulation is used to design the  $H_\infty$  damping controller. The uncertainty in load model composition results in the inaccurate estimation of the designed controller capability. So, different load model compositions including static and dynamic loads are considered here. Various contingency conditions are applied for testing the designed controller. The designed controller provides sufficient damping for such contingent conditions.

**Keywords:** Inter-area mode oscillations,  $H_\infty$  control, Linear Matrix Inequalities (LMI), Load composition uncertainties, Phasor Measurement Unit (PMU), Static Synchronous Series Capacitor (SSSC), Wide Area Measurements (WAMs), Wide Area Damping Controller (WADC).

## 1. Introduction

The deregulated electricity market and the increased demand for electrical energy necessitate interconnections of various power networks which are geographically separated [1]. It also ensures stable, reliable and economical operation of power system [1]. Continuously varying load demand and subsequent adjustments in generation results in oscillations of generator rotors [2-3]. The power transmission capability of the given network is restricted due to these oscillations. The conventional solution for damping these oscillations is the Power system stabilizers (PSSs). The PSSs are reliable for damping local mode oscillations. However, due to its location in excitation system of the generator, they are not suitable for damping inter-area mode oscillations [4]. Due to its location in the transmission network, FACTS devices are more suitable as supplementary damping controller for damping inter-area mode oscillations [5-8]. The PMUs are located strategically in the multi-machine system and captures a real-time picture of given power networks. The PMUs measure positive sequence currents and voltages in different areas of the grid and can deliver as frequently as once per cycle of power frequency [9]. Robust and stable operation of power system was ensured through  $H_\infty$  based controller design in [10]. A  $H_\infty$  controller for TCSC was designed to damp inter-area modes oscillations in large power system [10]. The controller designed through  $H_\infty$  control technique based on Riccati equation approach suffers from pole-zero cancellation [11].

The numerical solutions of Riccati equation based on LMI formulation do not suffer from pole-zero cancellation [12]. Further minimum damping ratio can be achieved through pole placement in LMI formulation. Damping controller design based on LMI

approach was demonstrated in [13] for PSSs but the performance of PSSs is found to be ineffective for damping inter-area mode oscillations [5]. LMI design for TCSC was discussed in [14] but the coordinated approach was missing. Mixed-sensitivity approach for designing robust controllers for various FACTS devices was discussed in [15-16] but local control signals were used for controller design.

This paper proposes a wide area measurement based SSSC controller for providing effective damping of critical inter-area mode oscillation under various operating conditions through global control signals. The  $H_\infty$  control design based on the mixed-sensitivity formulation in LMI for FACTS devices ensures higher damping ratios for critical inter-area modes.

This paper is organized as follows: In section 1, the need for the WAMs based FACTS controller is discussed. Section 2 discusses basic structure of WAMs system two area test system with SSSC. Section 3 describes the general procedure for damping controller design based LMI approach. Section 4 discusses the design of SSSC WADC. The testing of the designed controller for different contingency conditions is carried out in this section. Section 5 discusses the conclusion.

## 2. Basic Structure of WAMS System and Modified Test System

The basic structure of wide area measurements system is as shown in Figure 1. In two area test system, PMUs are optimally located in each area.

PMUs capture important system dynamic variables of that area. These variables may be bus voltages, line power flow, frequency and remote generator speed etc. PMUs transmit these data to

WAMs center which preprocesses the selected useful data and transmit it to SCADA. The control signals to be applied to FACTS controller (SSSC in this case) are predetermined on the basis of residue analysis. These control signals are applied to respective

FACTS controller through WAC Centre. With appropriate input signals, FACTS controller ensures secure and stable operation of power system.

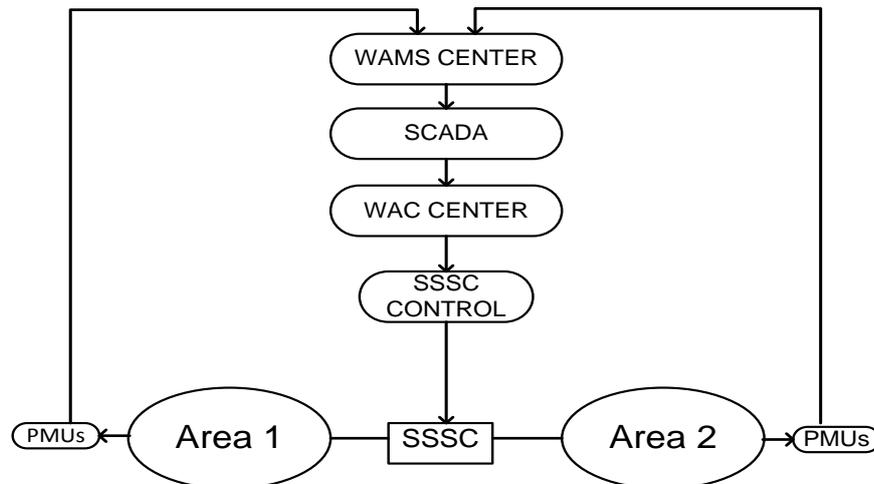


Fig. 1: Basic Wide Area Measurement (WAM) structure

The modified two-area four-machine system is as shown in Figure 2. Each generator is rated at 900 MVA. Area 1 (comprises of Generators 1, 2) and area 2 (comprises of Generators 3, 4) are connected by a tie line. The detail information regarding the test system components can be found in [17]. The power transfer from

area 1 to area 2 is 376 MW. To facilitate the power transfer in the tie-line, Static Synchronous Series Capacitor (SSSC) is employed in the tie-line. The power flow  $P_{tie}$  from area 1 to area 2 through the tie line is 376 MW (3.76 p.u.) and is chosen as equilibrium point. By varying the load at various buses, the power flow in the tie-line can be varied.

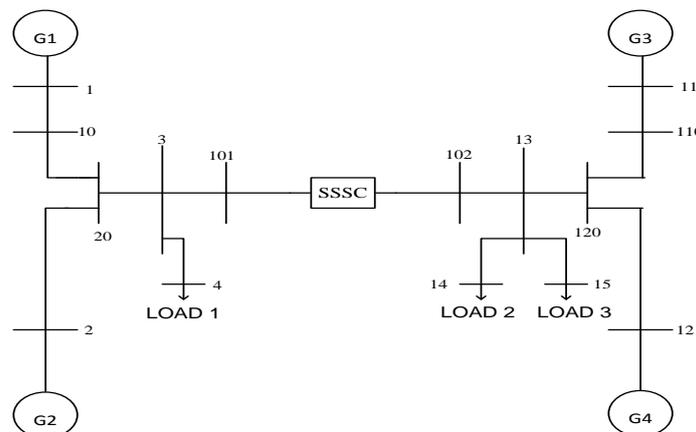


Fig. 2: Two area four machine test system

### 3. LMI Based Damping Controller Design

Before initiating the design of the controller, a nonlinear model of the power system and its components is obtained. This model is linearized around the equilibrium point (operating point in this

case). Modal analysis is carried out for this linearized model to find out various state space matrices A, B, C and D of the test system.

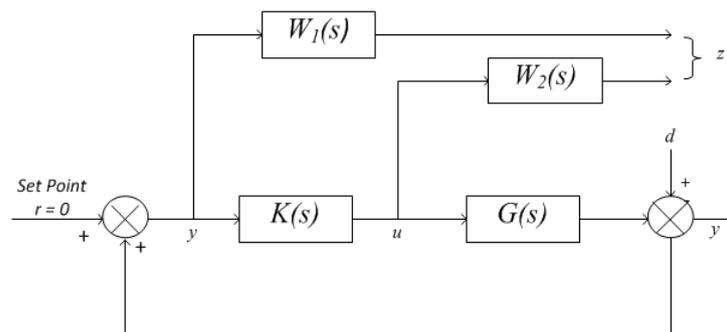


Fig. 3: Mixed sensitivity output disturbance rejection configuration

Continuous change in the loading conditions, response of AVT to short circuit faults etc. are the main reasons for producing oscillations in the power system. For the FACTS controllers, these oscillations are appeared as disturbances and they must act promptly to minimize the effect of these oscillations on power system. Figure 3 represents the mixed sensitivity output disturbance rejection configuration, where  $G(s)$  is the open loop plant and  $K(s)$  is the controller to be designed.  $W_1(s)$  and  $W_2(s)$  are the weights for shaping the open loop plant characteristics. The design objective is to minimize the weighted transfer function  $S(s) = (I-G(s)K(s))^{-1}$  for necessary disturbance rejection and  $K(s)S(s)$  which ensures robustness and minimized control efforts. This objective can be represented in the form of equation as [18]

$$\left\| \begin{bmatrix} W_1(s)S(s) \\ W_2(s)K(s)S(s) \end{bmatrix} \right\|_{\infty} < 1 \quad (1)$$

The augmented plant in state space is given by

$$\begin{bmatrix} \dot{x} \\ y \\ z \end{bmatrix} = \begin{bmatrix} A & B_1 & B_2 \\ C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & 0 \end{bmatrix} \begin{bmatrix} x \\ d \\ u \end{bmatrix} \quad (2)$$

Where,  $x$  is the state variable vector,  $d$  is the disturbance,  $u$  is the plant input,  $y$  is the plant output and  $z$  is the regulated output. The state space representation of the controller is given by

$$\dot{x}_k = A_k x_k + B_k y \quad (3)$$

$$u = C_k x_k + D_k y \quad (4)$$

where  $x_k$  represents the controller states. The transfer matrix between  $d$  and  $z$  is given by

$$T_{zd}(s) = \begin{bmatrix} W_1(s)S(s) \\ W_2(s)K(s)S(s) \end{bmatrix} = C_{cl}(sI - A_{cl})^{-1}B_{cl} + D_{cl} \quad (5)$$

Where,

$$A_{cl} = \begin{bmatrix} A + B_2 D_k C_2 & B_2 C_k \\ B_k C_2 & A_k \end{bmatrix} \quad (6)$$

$$B_{cl} = \begin{bmatrix} B_1 + B_2 D_k D_{21} \\ B_k D_{21} \end{bmatrix} \quad (7)$$

$$C_{cl} = [C_1 + D_{12} D_k C_2 \quad D_{12} C_k] \quad (8)$$

$$D_{cl} = [D_{11} + D_{12} D_k D_{21}] \quad (9)$$

The closed loop system in equation (5) is asymptotically stable if there exist  $X = X^T > 0$  such that

$$\begin{bmatrix} A_{cl}^T X + X A_{cl} & B_{cl} & X C_{cl}^T \\ B_{cl}^T & -I & D_{cl}^T \\ C_{cl} X & D_{cl} & -I \end{bmatrix} < 0 \quad (10)$$

The pole placement objective is formulated in terms of LMI regions of the complex plane. A conic sector of inner angle  $\theta$  and

apex at the origin is the appropriate region for ensuring minimum damping to closed loop poles (refer Fig. 4). If  $X > 0$  exist such as given in equation (11), minimum damping can be ensured for all poles within the conic region, for given value of inner angle  $\theta$ .

$$\begin{pmatrix} \sin \theta (A_{cl} X + X A_{cl}^T) & \cos \theta (A_{cl} X - X A_{cl}^T) \\ \cos \theta (X A_{cl}^T - A_{cl} X) & \sin \theta (A_{cl} X + X A_{cl}^T) \end{pmatrix} < 0 \quad (11)$$

So the problem simplifies to solving the matrix inequalities contained in (10) and (11). To convert the nonlinearity in equation (10) and (11) into linear one, controllable variable change is necessary. These new controller variables are given by

$$\hat{A} = N A_k M^T + N B_k C_2 R + S B_2 C_k M^T + S(A + B_2 D_k C_2) R \quad (12)$$

$$\hat{B} = N B_k + S B_2 D_k \quad (13)$$

$$\hat{C} = C_k M^T + D_k C_2 R \quad (14)$$

$$\hat{D} = D_k \quad (15)$$

Where,  $R, S, M$  and  $N$  are sub-matrices of  $X$ . Hence the revised solution to design problem is given by

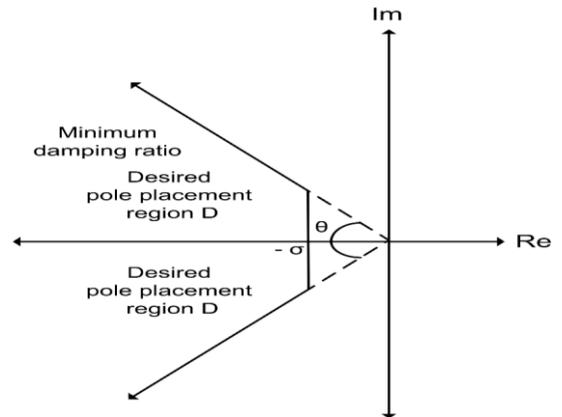


Fig. 4: LMI region for pole placement

$$\begin{bmatrix} R & I \\ I & S \end{bmatrix} > 0 \quad (16)$$

$$\begin{bmatrix} \varphi_{11} & \varphi_{21}^T \\ \varphi_{21} & \varphi_{22} \end{bmatrix} < 0 \quad (17)$$

where  $\varphi_{11}, \varphi_{21}$  and  $\varphi_{22}$  are given as follows:

$$\varphi_{11} = \begin{pmatrix} AR + RA^T + B_2 \hat{C} + \hat{C}^T B_2^T & B_1 + B_2 \hat{D} D_{21} \\ (B_1 + B_2 \hat{D} D_{21})^T & I \end{pmatrix} = [D_{11} + D_{12} D_k D_{21}] \quad (18)$$

$$\varphi_{21} = \begin{pmatrix} \hat{A} + (A + B_2 \hat{D} C_2)^T & S B_1 + \hat{B} D_{21} \\ C_1 R + D_{12} \hat{C} & D_{11} + D_{12} \hat{D} D_{21} \end{pmatrix} \quad (19)$$

$$\varphi_{22} = \begin{pmatrix} A^T S + S A + \hat{B} C_2 + C_2^T \hat{B}^T & (C_1 + D_{12} \hat{D} C_2)^T \\ C_1 + D_{12} \hat{D} C_2 & -I \end{pmatrix} \quad (20)$$

The new control variables are obtained by solving LMIs in equation (16) and (17) as optimization problem. Once new control variables are available,  $A_k, B_k, C_k$  and  $D_k$  can be obtained by solving the equation (12)-(15).

### 4. Design of SSSC WADC

The design of SSSC WADC includes following steps:

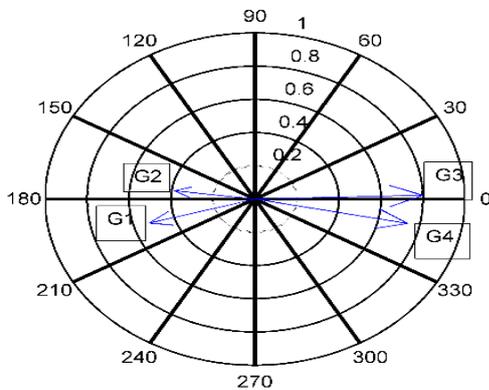
#### 4.1 Modal Analysis of the Test System

Table 1 shows the critical inter-area modes of modified test system with their frequency, mode shape and damping ratio. Further, their mode shape is indicated in Figure 5 through compass chart. The information from the compass charts are shown in Table 1.

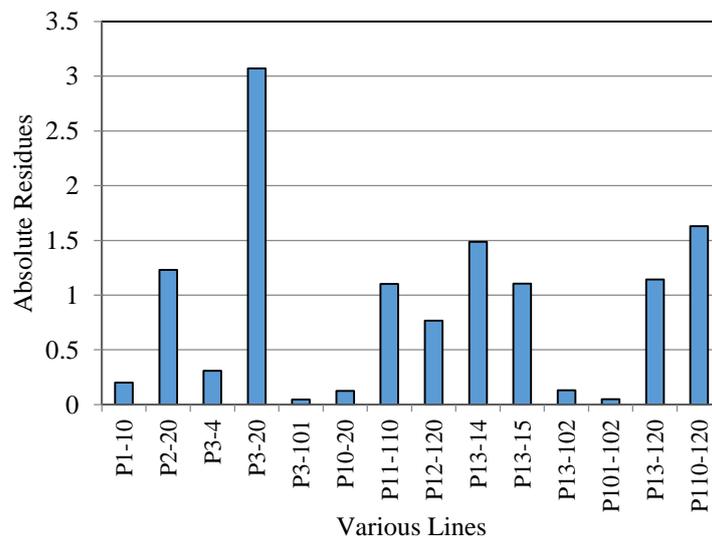
**Table 1:** Details of the inter-area mode oscillation of the test system

Mode Index	Eigen value	$f$ (Hz)	Damping ratio $\zeta$	Mode Shape
1	-0.161 ±3.604i	0.573	0.0447	G1-G2 Vs.G3-G4 (Area 1 Vs Area2)

Table 1 clearly indicates that presence of one inter-area mode oscillation with 4.47% damping ratio. Further, figure 5 shows mode shape for this inter-area mode which shows generator 1, 2 are oscillating against generator 3,4. The objective of the SSSCWADC design is to improve damping ratio of this inter-area mode.



**Fig. 5:** Compass chart for the inter-area mode



**Fig. 6:** Residue analysis for a critical mode with line power as an input signal

#### 4.3 Selection of Weighing Matrices

The selection of weights in Riccati based approach is of utmost importance. The weight  $W_1(s)$  is for output disturbance rejection. Hence,  $W_1(s)$  is a high gain low pass filter. The weight  $W_2(s)$  is for control effort optimization for taking care of plant model uncertainty. Hence, weight  $W_2(s)$  should be a high pass filter. As all the inter-area oscillations occurring in the system are below 1.0

#### 4.2 Selection of Suitable Global Control Signal

Selection of suitable input signal is of the prime importance for the effective design of the damping controller. Line current, line power, the voltage at the bus, generator rotor speeds etc. are the probable candidate as input signals. The signal with the highest controllability/observability measures is the best candidate for the critical mode. As the main purpose of the SSSC in two area test system is to improve the transmission capacity of the tie-line, the best location for SSSC, in this case, is the tie-line. Hence, only observability is to be checked. The bus voltage angle difference or generator rotor speed signals may contain some spurious signals. Hence, they are not selected as input signals. The line active power is found to be free from above-mentioned eventuality. Hence, line active power is selected as an input signal. A model residue analysis is carried out to identify most stabilizing global signals. The inter-area mode is found to be highly observable in the active power flow in the various transmission lines. Figure 6 shows the residue values of different lines for line power as input signal for inter-area mode. Line 4 (connecting bus 3- bus 20) has the highest residue for the inter-area mode. So, active power in the line connecting bus 3- bus 20 is selected as an input signal.

Hz frequency range, the weights selected are such that they intersect at around 1.0 Hz. The selected weights  $W_1$  and  $W_2$  for SSSC controller are given as follows:

$$W_1(s) = \frac{10}{s + 10} \tag{21}$$

$$W_2(s) = \frac{s}{s+10} \tag{22}$$

### 4.4 Model Order Reduction

The order of the controller in linear control techniques is higher than the system order[19]. Thus for simplification of the design procedure, model order reduction is required. The reduced order plant must be a close approximation of original full order plant. Schur balanced model reduction procedure is followed for reducing the order of the plant [19]. For the given higher order plant model  $G(S)$ , the  $k^{th}$  order reduced order plant order  $G_r(S)$  must be such that

$$\|G - G_r\|_{\infty} \leq 2 \sum_{i=k+1}^n \sigma_i \tag{23}$$

where  $\sigma_i$  is the Hankel singular value of  $G(j\omega)$  and is given by

$$\sigma_i = \sqrt{\gamma_i(PQ)} \tag{24}$$

where  $\gamma_i$  is the largest eigen value of  $(PQ)$  and  $P, Q$  is a solution of  $PA^T + AP + BB^T = 0$  (Controllability grammian)  $\tag{25}$

$QA + A^TQ + C^TC = 0$  (Observability grammian)  $\tag{26}$

In the present case, balanced reduction function of Matlab is used to reduce the order of the system from 59<sup>th</sup> to 10<sup>th</sup> order. The frequency responses of the full-order model and reduced-order model are shown in Figure 7. From the frequency response, it is quite clear that in the interested frequency range (i.e. less than 10 rad/sec), the full and reduced-order model is closely matching. Hence, the reduced order system is reliable for robust controller design.

### 4.5 Load Modeling

The load modeling has great significance in power system damping controller design [20]. Conventionally, power system loads are represented by constant impedance. However, it may results in the false estimation of the damping capability of the controller [21]. Different load models may have a different effect on power system dynamic behavior. Conventionally, ZIP load model is used to represent power system static loads and it is given by following equations [22]:

$$P = P_0 \left[ p_1 \left( \frac{V}{V_0} \right)^2 + p_2 \left( \frac{V}{V_0} \right) + p_3 \right] \tag{27}$$

$$Q = Q_0 \left[ q_1 \left( \frac{V}{V_0} \right)^2 + q_2 \left( \frac{V}{V_0} \right) + q_3 \right] \tag{28}$$

In both the equations, constant impedance load is represented by first term. Constant current load is represented by second term and constant power load is represented by the third term. The constants  $p_1+p_2+p_3=q_1+q_2+q_3=1$ . However, in practice, the load is a mixture of above three type as well as dynamic load. Induction motors at various load buses are considered as dynamic loads in this paper. An attempt should be made to create actual power system load composition conditions. Considering these facts, various load models resembling actual power system are considered here which are as shown in Table 2.

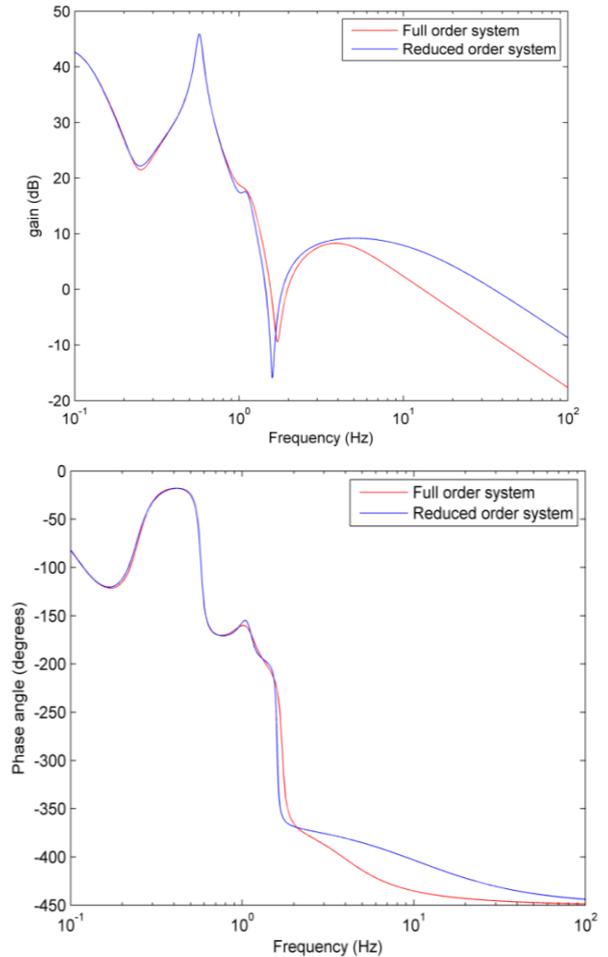


Fig. 7: Frequency response of full order and reduced order system

Table 2: Proposed load Models

Load Model	Fraction of Constant Impedance load	Fraction of Constant Power load	Fraction of Constant current load	Dynamic load
I	1.0	0	0	0
II	0.5	0.5	0	0
III	0.5	0	0.5	0
IV	0.5	0	0	0.5

### 4.6 Controller Synthesis

For the reduced order model obtained in section 4.4, a multi-objective SSSC wide-area damping controller is designed in this section. For achieving robust controller, the mixed  $H_2/H_{\infty}$  output feedback control with pole placement in LMI framework is developed. For achieving the minimum damping of 10%, the placement of the poles must be in the D region (figure 4).The inner angle  $\theta$  is set at 82° for this purpose. The design problem is solved by the function *hinfmix*, which is available in the LMI

control toolbox of MATLAB [100].

By combining the weights with the reduced order plant, the controller design is carried out in LMI framework with pole placement. However, the order of the controller obtained is relatively high and it is difficult to implement practically. So, the order of the controller is further reduced. The finally obtained controller is of 9<sup>th</sup> order. The controller matrices are shown in Appendix.

### 4.7 Performance Evaluation of the Controller

The robustness of the controller must be evaluated for various loading conditions and various compositions of load models. The closed loop performance of the designed SSSC WADC is as shown in Table 3. This result is for base case i.e. the equilibrium

point for which the controller is designed. Table 3 clearly shows improvement in damping ratio of inter-area mode from 4.47% to 14.28%.

**Table 3:** Closed-loop performance of SSSC WADC

System	$f$ (Hz)	Damping ratio $\zeta$
Open loop	0.573	0.0447
SSSC WADC	0.57	0.1428

The load models suggested in Table 2 are applied for testing the designed controller. Table 4 indicates the robustness of the designed controller for proposed load models. Sufficient improvement in the damping ratio is observed in all cases. The

damping ratio improvement in case of proposed load model II and IV is less as compared to other two cases. In all cases, the minimum damping ratio is more than 10% which is considered as an acceptable value in the literature.

**Table 4:** Performance of SSSC WADC with proposed load models

Proposed load models	Open loop system		Closed-loop system with SSSC WADC	
	Damping ratio ( $\zeta$ )	Frequency (Hz)	Damping ratio ( $\zeta$ )	Frequency (Hz)
I	0.0447	0.573	0.1428	0.570
II	0.0422	0.558	0.1131	0.549
III	0.0441	0.563	0.1381	0.562
IV	0.0440	0.566	0.1198	0.551

**Table 5:** Performance of SSSC WADC for various tie-line power flows

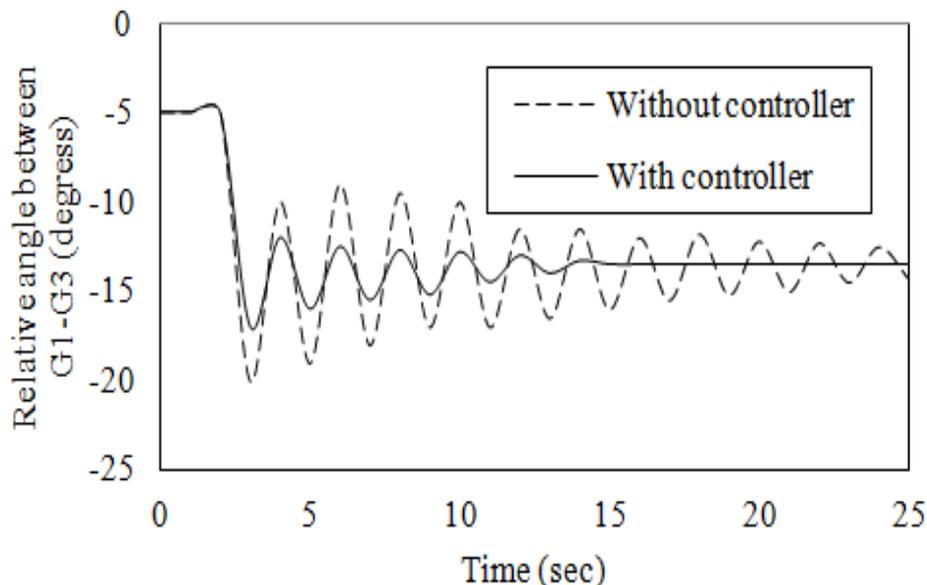
Tie line power flows (MW)	Open loop system		Closed loop system with SSSC WADC	
	Damping ratio ( $\zeta$ )	Frequency (Hz)	Damping ratio ( $\zeta$ )	Frequency (Hz)
100	0.0512	0.558	0.1897	0.598
200	0.0476	0.546	0.1605	0.584
300	0.0451	0.540	0.1519	0.572
400	0.0437	0.533	0.1409	0.562
500	0.0428	0.517	0.1219	0.537

The robustness of the designed SSSC WAD controller is also tested for various tie-line power flows. Tie-line power flow is increased from 100MW to 500 MW in steps. The results are shown in Table 5. The results show the robustness of the designed controller for various operating conditions.

### 4.8 Nonlinear Time Domain Simulations

In a practical power system, the inter-area modes are excited by different contingency conditions emerging in power system. Such typical condition includes line faults, load shedding etc. The designed controller must be able to damp the oscillations produced

in the power system due to these contingency conditions. Nonlinear time domain simulations are performed to prove the robustness of the designed controller under the influence of these conditions. Two such contingency conditions are simulated here:  
 1) Line fault: In this case, a transient line to ground fault near bus 20 in the line connecting bus 20 and bus 3 is simulated to check the performance of the designed controller. The dynamic response of the controller where the variations of the relative angle between generator 1 and generator 3 located in the different areas as shown in Figure 8. It can be seen that transient faults excite the oscillation in the test system. The SSSC WADC damped the oscillations within 15 seconds.



**Fig. 8:** Dynamic response of SSSC WADC during line fault at bus 20 (Relative angle between G1-G3)

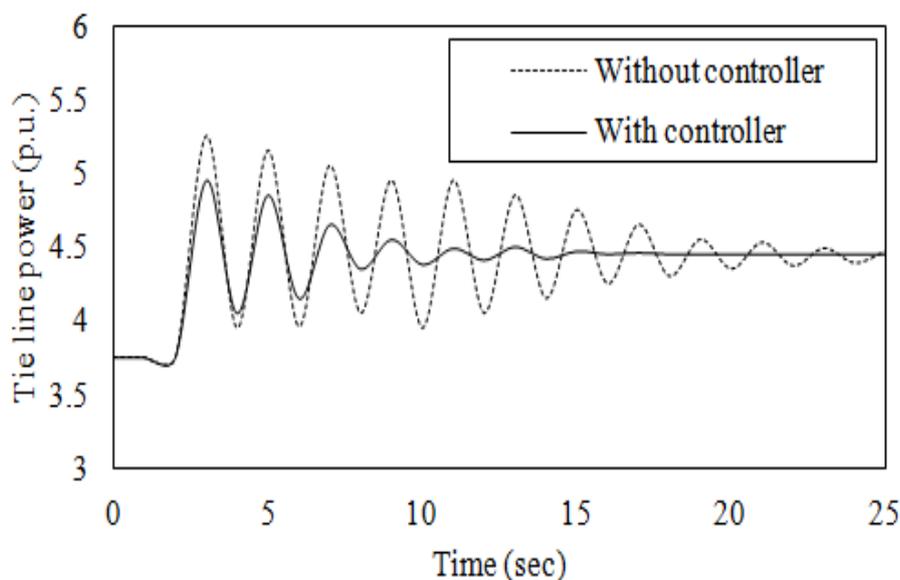


Fig. 9: Dynamic response of SSSC WADC for increased loading at bus 15

2) Increased loading: For observing the performance of the SSSC WADC, the load on bus 15 is increased from 900 MW to 1000 MW (i.e. 9.00 p.u. to 10.0 p.u.). The resulting change in the power flow in tie-line is as shown in Figure 9. The oscillations produced due to load shedding are clearly damped within 15 seconds. Hence, we can conclude that the performance of the designed SSSC WADC is satisfactory for various operating conditions in the test system and for various contingency conditions.

## 5. Conclusion

This paper proposes a SSSC damping controller based mixed sensitivity LMI approach with pole placement objective. Residue analysis with line active power as the input signal is carried out to determine signal with the highest residue for the inter-area mode. The mixed sensitivity based LMI approach with pole placement objective is applied for designing the SSSC WADC. Various load compositions model resembling actual power system loading conditions are considered for accurate estimation of the damping capability of the designed SSSC WADC. The performance of the designed controller is tested for robustness in the frequency domain for various loading conditions and various load compositions. The designed controller provides damping ratio more than 10% in all cases. The designed SSSC WADC is also tested in time domain simulations for various contingency conditions like line faults and increased line loading. The designed controller is able to damp the oscillations caused due to these conditions within 15 seconds. The designed SSSC WADC is found to be robust enough to meet various power system contingencies.

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## Appendix

The reduced order SSSC wide area controller can be represented as following transfer function:

$$K_{SSSC}(s) = \frac{N_{SSSC}(s)}{D_{SSSC}(s)}$$

where,

$$N_{SSSC}(s) = -5.36 * 10^5 s^{11} - 2.33 * 10^7 s^{10} - 6.54 * 10^8 s^9 - 1.06 * 10^9 s^8 - 2.987 * 10^{10} s^7 \\ - 6.745 * 10^{10} s^6 - 4.564 * 10^{11} s^5 - 9.856 * 10^{11} s^4 - 4.576 * 10^{11} s^3 - 1.989 * 10^{11} s^2 \\ - 9.485 * 10^{11} s + 5.939 * 10^{10}$$

$$D_{SSSC}(s) = s^{12} + 2.384 * 10^5 s^{11} + 7.944 * 10^9 s^{10} + 8.567 * 10^{11} s^9 + 5.687 * 10^{11} s^8 + 1.723 * 10^{12} s^7 \\ + 5.678 * 10^{13} s^6 + 8.692 * 10^{14} s^5 + 7.347 * 10^{14} s^4 + 9.325 * 10^{14} s^3 + 2.873 * 10^{14} s^2 \\ + 2.746 * 10^{14} s + 6.374 * 10^{14}$$