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Research paper

On J_D-Fuzzy Ideal of BH-Algebra

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Abstract

In this analysis, you think about the construct JD-fuzzy ideal of BH-Algebra and we can study a number of properties, theorem and that we can provide some examples then we gave the construct JD –fuzzy extension ideal.

Keywords: *Analysis, algebra.*

1. Introduction

After the introduction of fuzzy sets by Zaden there are a variety of generalization of this elementary construct. In 2001 the notion Fuzzy BH algebras introduced by alphabetic character. Zhang, E,H Roh and Y.B.Jun [1]. Later in 2006 the construct fuzzy ideal in BH- pure mathematics was introduce by C.H. Park In 2003 the construct of homomorphy was conferred by a tanassov K.T.[2].

2. Preliminaries

In this chapter we'll gift some of the ideas that we'll like

Definition (2.1) [3]

A BH-Algebra is a nonempty set X with a constant (0) and binary operation (*) satisfying the following condition.

1. $x * x = 0 \quad \forall\, x \in X.$
2. $x * y = 0 \wedge y * x = 0$ imply $, x = y.$
3. $x * 0 = x \quad \forall\, x \in X.$

Definition (2.2) [4]

A fuzzy subset $\Im_A$ of a BH-Algebra X is said to be a fuzzy ideal if and only if

1. $\forall x \in X \quad \Im_A(0) \geq \Im_A(x)$
2. $\forall x, y \in X, \Im_A(x) \geq \min\{\Im_A(x * y), \Im_A(y)\}$

Definition (2.4)[2]

Let X and Y be two BG-algebras, then a mapping $f : X \to Y$ is said to be homomorphism if

$$f(x * y) = f(x) * f(y) \quad \forall x, y \in X.$$

Definition (2.6) [4]

If β is a fuzzy subset of Y, then the fuzzy subset μ = β o *f* in X (i.e the fuzzy subset defined by μ (x) = β (*f* (x)) for all x ∈ X) is called the pre-image of β under *f* .

Definition (2.7)[3]

Let X be a nonempty set and $\Im_A$ be a fuzzy subset of X let $\alpha \in [0, T]$.

A mapping $\Im_{A\alpha}^{T} : X \to [0,1]$ is called a fuzzy translation of $\Im_A$ if it fulfills $\Im_A\ \Im_{A\alpha}^{T}(x) = \Im_A(x) + \alpha, \forall x \in X$.

Definition (2.8)[4]

Let $\Im_{A_1}$ and $\Im_{A_2}$ be fuzzy subset of X.

If $\Im_{A_1} \leq \Im_{A_2}, \forall x \in X$,then we say that $\Im_{A_2}$ is a fuzzy extension of $\Im_{A_1}$.

3. The Mean Result

In this section, we tend to recall some definitions and results which can be utilized in what follows. Thought the paper we tend to devoted JD –fuzzy ideal of BH algebras X, and provides some properties, theorem concerning this ideal.

Definition (3.1)

A fuzzy set $\Im_A(x)$ is called J_D -fuzzy ideal of BH-Algebra X if fulfills ; $\Im_A(x.(y.x)) \geq \min\{\Im_A(x.(y.(y.x)), \Im_A(x.y))\}$.

Examples (3.2)

Let BH-algebra X={0,1,2} with the accompanying table :

.	0	1	2
0	0	1	2
1	1	0	1
2	2	2	0

$$\Im_A(x) = \begin{cases} .6 & x=0 \\ .5 & x=1 \\ .3 & x=2 \end{cases}$$

$\Im_A(x)$ is J_D -fuzzy ideal of X.

Theorem (3.3)

Let $\Im_A(x)$ be J_D -fuzzy ideal of BH-algebra X and let $\Im^*{}_A(x) = \Im_A(x) + 1 - \Im_A(0), \forall x \in X$ then $\Im^*{}_A(x)$ is J_D -fuzzy ideal.

Proof

Let $\Im^*{}_A(x) = \Im_A(x) + 1 - \Im_A(0)$ where

$\Im^*_A(x.(y.x)) = \Im_A(x.(y.x)) + 1 - \Im_A(0)$

$\geq \min\{\Im_A(x.(y.(y.x))), \Im_A(x.y)\} + 1 - \Im_A(0)$

$\geq \min\{\Im_A(x.(y.(y.x))) + 1 - \Im_A(0), \Im_A(x.y)$

$+1 - \Im_A(0)\}$

$\geq \min\{\Im^*_A(x.(y.(y.x))), \Im^*_A(x.y)\}$

$\Im^*_A(x.(y.x)) \geq \min\{\Im^*_A(x.(y.(y.x))), \Im^*_A(x.y)\}$

$\Rightarrow \Im^*_A(x)$ is J_D -fuzzy ideal of X .

Definition (3.4)

Let $\Im_A(x)$ and $\Psi_A(x)$ be any J_D -fuzzy ideal of BH-algebra X. then

1. $\Im_A(x).\Psi(x) = \Upsilon_{x.y}\{\Im_A(y).\Psi(z)\}$.

2. $\Im_A(x) + \Psi(x) = \Upsilon_{x+y}\{\Im_A(y).\Psi(z)\}$.

Where $\forall x, y, z \in X$.

Theorem (3.5)

Let $\Im_A(x)$ be a J_D -fuzzy ideal of algebra X. and $F:[0,\Im_A(0)] \to [0,1]$ is increasing function then $\Im_A f(x) = f(\Im_A(x))$ is a J_D -fuzzy ideal of X.

Proof

Let $x, y \in X$

$\Im_A f((x.(y.x))) = f\,\Im_A(x.(y.x))$

$\geq f(\min\{\Im_A(x.(y.(y.x)))), \Im_A(x.y)\}$

$\geq \min\{f\,\Im_A(x.(y.(y.x))), f\,\Im_A(x.y)\}$

$\geq \min\{\Im_A f\,(x.(y.(y.x))), \Im_A f\,(x.y)\}$

$\Rightarrow \Im_A f(x)$ is J_D -fuzzy ideal of X .

Proposition (3.6)

If $\{\Im_{A_j} \mid j \in J\}$ is a family of J_D -fuzzy ideal of BH-algebra X , then $\prod_{j\in J} \Im_{A_j}$ is J_D -fuzzy ideal of X .

Proof

Let $\{\Im_{A_j} \mid j \in J\}$ is a family of J_D-fuzzy ideal of X , $\forall x, y \in X$ we have $\prod_{j\in J} \Im_{A_j}(x) \geq \inf\{\Im_{A_j}(x)\}$

$\prod_{j\in J} \Im_{A_j}(x.(y.x)) \geq \inf_{j\in J}\{\min\{\Im_{A_j}(x.(y.(y.x)))$

$, \Im_{A_j}(x.y)\}$

$\geq \min\{\inf_{j\in J} \Im_{A_j}(x.(y.(y.x))), \inf_{j\in J} \Im_{A_j}(x.y)\}$

$= \min\{\prod_{j\in J} \Im_{A_j}(x.(y.(y.x))), \prod_{j\in J} \Im_{A_j}(x.y)\}$

$\prod_{j\in J} \Im_{A_j}(x.(y.x))$

$\geq \min\{\prod_{j\in J} \Im_{A_j}(x.(y.(y.x))), \prod_{j\in J} \Im_{A_j}(x.y)\}$.

$\prod_{j\in J} \Im_{A_j}$ is J_D -fuzzy ideal of X .

Theorem (3.7)

Let $\Im_A(x)$ and $\Psi_A(x)$ be any J_D -fuzzy ideal of BH-algebra X. then

$\Im_A(x) \cap \Psi(x)$ is J_D -fuzzy ideal of BH-algebra X.

Proof

We have

$\Im_A(x.(y.x)) \cap \psi(x.(y.x)) = \min\{\Im_A(x.(y.x)), \psi(x.(y.x))\}$

$\geq \min\{\min\{\Im_A(x.(y.(y.x))), \Im_A(x.y)\}, \min\{\psi_A(x.(y.(y.x))), \psi_A(x.y)\}$

$\geq \min\{\min\{\Im_A(x.(y.(y.x))), \Im_A(x.y)\}, \psi_A(x.(y.(y.x))), \psi_A(x.y)\}\}$

$\geq \min\{\Im_A(x.(y.(y.x))), \psi_A(x.(y.(y.x))), \Im_A(x.y)\}, \psi_A(x.y)\}\}$

$\geq \min\{\min\{\Im_A(x.(y.(y.x))), \psi_A(x.(y.(y.x)))\}, \min\{\Im_A(x.y)\}, \psi_A(x.y)\}\}$

$\Rightarrow \Im_A(x) \cap \Psi(x)$ J_D -fuzzy ideal of BH-algebra X

Proposition (3.8)

Let $\Im_A(x)$ be J_D -fuzzy ideal of BH-algebra X and let $\overline{\Im}_A(x) = 1 - \Im_A(x)$ then $\overline{\Im}_A(x)$ is J_D -fuzzy ideal .

Proof

Let $\Im_A(x)$ is J_D -fuzzy ideal of X and

$\overline{\Im}_A(x) = 1 - \Im_A(x)$

$\overline{\Im}_A(x.(y.x)) = 1 - \Im_A(x.(y.x)) \geq 1 - \min\{\Im_A(x.(y.(y.x))), \Im_A(x.y)\}$

$\geq 1 - \min\{\Im_A(x.(y.(y.x))), \Im_A(x.y)\} \geq \min\{1 - \Im(x.(y.(y.x))), 1 - \Im_A(x.y)\}$

$\geq \min\{\overline{\Im}_A(x.(y.(y.x))), \overline{\Im_A}(x.y)\} \overline{\Im}_A(x.(y.x)) \geq \min\{\overline{\Im}_A(x.(y.(y.x))), \overline{\Im_A}(x.y)\}$

$\overline{\Im}_A(x)$ is J_D -fuzzy ideal of X

Proposition (3.9)

Let $\Im_A(x)$ and $\Psi_A(x)$ be any J_D -fuzzy ideal of BH-algebra X. then

$\Im_A(x) \times \Psi(x)$ is J_D -fuzzy ideal of BH-algebra X.

Proof

Let $\Im_A(x)$ and $\Psi_A(x)$ be any J_D -fuzzy ideal of X.

$\Im_A(x.(y.x)) \times \psi(x.(y.x)) =$

$\min\{\Im_A(x.(y.x)), \psi(x.(y.x))\}$

$\geq \min\{\min\{\Im_A(x.(y.(y.x))), \Im_A(x.y)\}$

$, \min\{\psi_A(x.(y.(y.x))), \psi_A(x.y)\}$

$\geq \min\{\min\{\Im_A(x.(y.(y.x))),\psi_A(x.(y.(y.x)))\}$

$,\min\{\Im_A(x.y),\psi_A(x.y)\}\}$

$\geq \min\{\Im_A(x.(y.(y.x)))\times\psi_A(x.(y.(y.x)))$

$,\Im_A(x.y)\times\psi_A(x.y)\}\}$

Then $\Im_A(x)\times\Psi(x)$ is J_D-fuzzy ideal of BH-algebra X.

Theorem (3.10)

Let $\Im_A$ be a fuzzy subset of BH-algebra X and $\Im_{A\alpha}^T$ is fuzzy translation of $\Im_A$ for $\alpha\in[0,T]$

$\Im_A$ is a J_D-fuzzy ideal of X if and only if $\Im_{A\alpha}^T$ is J_D-fuzzy ideal of X.

Proof

$\rightarrow$ Assume $\Im_A$ be a J_D-fuzzy ideal of X and let $\alpha\in[0,T]$.For all $x,y\in X$ we have

$\Im_{A\alpha}^T(x.(y.x))=\Im_A(x.(y.x))+\alpha$

$\geq\min\{\Im_A(x.(y.(y.x))),\Im_A(x.y)\}+\alpha$

$\geq\min\{\Im_A(x.(y.(y.x)))+\alpha,\Im_A(x.y)+\alpha\}$

$\geq\min\{\Im_{A\alpha}^T(x.(y.(y.x))),\Im_{A\alpha}^T(x.y)\}$.

Hence $\Im_{A\alpha}^T$ is J_D-fuzzy ideal of X.

$\leftarrow$Assume the fuzzy translation $\Im_{A\alpha}^T$ is J_D-fuzzy ideal of X for some $\alpha\in[0,T]$ let $x,y\in X$ we have

$\Im_A(x.(y.x))\geq\min\{\Im_A(x.(y.(y.x))),\Im_A(x.y)\}$

$\Im_A(x.(y.x))+\alpha\geq\min\{\Im_A(x.(y.(y.x))),\Im_A(x.y)\}+\alpha$

$\Im_{A\alpha}^T(x.(y.x))\geq\min\{\Im_A(x.(y.(y.x)))+\alpha,\Im_A(x.y)+\alpha\}$

$\geq\min\{\Im_{A\alpha}^T(x.(y.(y.x))),\Im_{A\alpha}^T(x.y)\}$.

Hence $\Im_{A\alpha}^T$ *is* J_D-fuzzy ideal of X.

Definition (3.11)

Let $\Im_{A_1}$ and $\Im_{A_2}$ be fuzzy subsets of X .Then $\Im_{A_2}$ is called JD –fuzzy extension ideal of $\Im_{A_1}$ in the event that the accompanying hold

(1) $\Im_{A_2}$ is a fuzzy extension ideal of $\Im_{A_1}$.

(2) If $\Im_{A_1}$ is a J_D –fuzzy ideal of X, then $\Im_{A_2}$ is a J_D –fuzzy ideal of X.

Proposition (3.12)

Let $\Im_A$ is a J_D –fuzzy ideal of X, and $\alpha,\lambda\in[0,T]$.If $\alpha\geq\lambda$, then the fuzzy translation $\Im_{A\alpha}^T$ of $\Im_A$ is J_D –fuzzy extension ideal of the fuzzy translation $\Im_{A\lambda}^T$ of $\Im_A$.

Proof

Let $\Im_A$ be a J_D-fuzzy ideal of X then by the (2-10) the fuzzy translation $\Im_{A\lambda}^T$ of $\Im_A$ and the fuzzy translation $\Im_{A\alpha}^T$ of $\Im_A$ are J_D-fuzzy ideal of X , for all $\alpha,\lambda\in[0,T]$ since $\alpha\geq\lambda$,

$\Im_A(x.(y.x))+\alpha\geq\Im_A(x.(y.x))+\lambda$, for all $x\in X$ .Therefore, $\Im_{A\alpha}^T(x.(y.x))\geq\Im_{A\lambda}^T(x.(y.x))$ .Hence $\Im_{A\alpha}^T$ is a J_D –fuzzy extension ideal of $\Im_{A\lambda}^T$.

Theorem (3.13)

The intersection of any J_D-fuzzy translation ideal of BH-algebra X is also J_D-fuzzy translation) ideal of X.

Proof

$(\bigcap_{i\in\eta}(\Im_{A\alpha}^T)_i)(x.(y.x)))=\inf\{(\Im_{A\alpha}^T)(x.(y.x))\}=\inf\{\Im_A(x.(y.x))+\alpha\}$

$\geq\inf\{\min\{\Im_A(x.(y.(y.x))),\Im_A(x.y)\}+\alpha\}\geq\inf\{\min\{\Im_A(x.(y.(y.x)))+\alpha,\Im_A(x.y)+\alpha\}$

$\geq\min\{\inf\{\Im_A(x.(y.(y.x)))+\alpha\},\inf\{\Im_A(x.y)+\alpha\}\}$.

$\geq\min\{\bigcap_{i\in\eta}\Im_{A_i}(x.(y.(y.x)))+\alpha,\bigcap_{i\in\eta}\Im_{A_i}(x.y)+\alpha\}$

$\geq\min\{(\bigcap_{i\in\eta}\Im_{A\alpha}^T)_i(x.(y.(y.x))),(\bigcap_{i\in\eta}\Im_{A\alpha}^T)_i(x.y)\}$.

$\Rightarrow(\bigcap_{i\in\eta}(\Im_{A\alpha}^T)_i)$ is J_D –fuzzy translation ideal of BH-algebra X.

Proposition (3.14)

The intersection of any set of J_D-fuzzy extension ideal of J_D-fuzzy ideal $\Im_A$ of BH-algebra X, is J_D-fuzzy extension ideal of $\Im_A$.

Proof

Let $\{\Im_{A_i}:i\in\eta\}$ be a family of J_D-fuzzy extension ideal of J_D-fuzzy ideal $\Im_A$ of X

$\Im_{A_i}(x.(y.x))\geq\Im_A(x.(y.x))\quad\forall i\in\eta, x,y\in X$

Since $\Im_A$ is J_D-fuzzy ideal of *X*. $\Im_{A_i}$ Are J_D-fuzzy ideal of X $\forall i\in\eta$,Then $\bigcap_{i\in\eta}\Im_{A_i}$ is also J_D-fuzzy ideal of X

$\Rightarrow(\bigcap_{i\in\eta}(\Im_{A_i})(x.(y.x))=\inf_{i\in\eta}\{\Im_{A_i}(x.(y.x)$

$\geq\inf\{\Im_{A_i}(x.(y.x)\}=\Im_A(x.(y.x)$

Hence $\bigcap_{i\in\eta}\Im_{A_i}$ is J_D-fuzzy extension ideal of $\Im_A$.

Theorem (3.15)

Let $f:X\rightarrow Y$ be a homomorphism of BH-algebra X to BH-algebra Y and $\Im_{A\alpha}^T$

Is defined as $f^{-1}(\Im_{A\alpha}^T)=\Im_{A\alpha}^T(f(x)),\forall x\in X$, if $\Im_A$ is J_D-fuzzy ideal of Y, then $f^{-1}(\Im_{A\alpha}^T)$

Is J_D-fuzzy ideal of X .

Proof

Let $\Im_A$ be J_D-fuzzy ideal of Y ,let $x,y\in X$

$f^{-1}(\Im_{A\alpha}^T(x.(y.x))=\Im_{A\alpha}^T(f(x.(y.x)))=\Im_A(f(x.(y.x)))+\alpha$

$\geq\min\{\Im_A(f(x.(y.(y.x))),\Im_A(f(x.y))\}+\alpha$

$\geq\min\{\Im_A(f(x.(y.(y.x)))+\alpha,\Im_A(f(x.y))+\alpha\}$

$\geq\min\{\Im_{A\alpha}^T(f(x.(y.(y.x))),\Im_{A\alpha}^T(f(x.y))\}$

$$\geq \min\{ f^{-1}(\Im^{T}_{A\alpha}(x.(y.(y.x)))), f^{-1}(\Im^{T}_{A\alpha}(x.y))\}$$

$$\Rightarrow f^{-1}(\Im^{T}_{A\alpha}(x.(y.x)) \geq \min\{ f^{-1}(\Im^{T}_{A\alpha}(x.(y.(y.x)))), f^{-1}(\Im^{T}_{A\alpha}(x.y))\}$$

Hence $f^{-1}(\Im^{T}_{A\alpha})$ is J_D -fuzzy ideal of X.

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