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***Research paper***

# A Study on Fuzzy TΓ-Ideals in Ternary Γ-Semiring

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**Abstract**

The concepts of fuzzy theory in TΓ-semiring in terms of fuzzy TΓ-ideals in TΓ-semirings are introduced and also we made a study on some properties of fuzzy TΓ-Ideals in TΓ-semiring.

***Keywords***: *TΓ-Semiring, TΓ-ideal, fuzzy TΓ-ideal, TΓ-subsemiring, regular TΓ-semiring.*

## 1. Introduction

The concept of TΓ-semiring was introduced by S. Lavanya, D. M. Rao and syam Julius Rajendra[2] In the year 2015, Later they [3], [4] studied about the characteristics of TΓ-ideals in TΓ-semirings. In the year 2005 T. K. Dutta and T. Chanda[1] investigated about fuzzy Γ-ideals in Γ-rings. In 2004, K. Syam Prasad and Bh. Satyanarayana [5] studied about prime fuzzy ideals in Γ-near rings. In this paper fuzzy concept in TΓ-semiring was introduced.

## 2. Preliminaries

**Note 2.8:** For preliminaries refer the references [2], [3].

## 3. Fuzzy TΓ-ideals:

**Def 3.1**: A fuzzy set $\xi$ of a TΓ-semiring is said to be a ***fuzzy TΓ-sub semiring*** of Q if

(i) $\xi(u+v) \geq \min\{\xi(u), \xi(v)\}$

(ii) $\xi(u\gamma v\delta w) \geq \min\{\xi(u), \xi(v), \xi(w)\}$ $\forall$ $u, v, w \in$ M and $\gamma, \delta \in \Gamma$.

**Ex: 3.2**: Let M be the set of rational numbers and Γ is the set of natural numbers. Define a mapping from M× Γ × M × Γ × M to M by usual addition and ternary multiplication defined by $a\alpha b\beta c$ = usual product of $a, \alpha, b, \beta, c$; for $a, b, c \in$ Q, $\alpha, \beta \in \Gamma$. Then M is a T Γ-semi ring. Define $\mu$:M → [0, 1] as $\mu(x) = \begin{cases} 1 & \text{if } x \in Q \\ 0, & \text{otherwise} \end{cases}$

Then $\mu$ is a fuzzy T Γ-semi ring of M.

**Example 3.4:** Consider the set Z = { 0, 1, -1, 2, -2, ……} and Γ be the set of even natural numbers. Then with respect to usual addition Z and multiplication is infinite T Γ-semiring. Clearly 2Z is a proper Γ-subsemiring of Z. Define $\mu: z \to [0,1]$ by $\xi(x) = \begin{cases} 0.9 & \text{if } x \in 2Z \\ 0.8, & \text{if } x \in 2Z+1 \end{cases}$ It is easy to verify that ξ is a fuzzy T Γ-sub semi ring of the TΓ-semi ring Z.

**Def 3.5:** A fuzzy T Γ -sub semi ring $\mu$ of a TΓ-semi ring M is called ***improper*** if $\mu$ is constant on the T Γ-semi ring T, otherwise $\mu$ is termed as ***proper.***

**The 3.6 : Let M be a TΓ-semiring and F(M) be the set of all non-empty fuzzy subsets of TΓ-semiring M. If *e, f, g, h* ∈ F(M), then**

**(i)** $e \cap h \cap (f \cup g) = (e \cap h \cap f) \cup (f \cap h \cap g)$

**(ii)** $(e \cup f) \cap g \cap h = (e \cap f \cap g) \cup (f \cap g \cap h)$

**(iii)** $g \cap (e \cup f) \cap h = (g \cap e \cap h) \cup (g \cap f \cap h)$

**The 3.7: Suppose Q be a TΓ-semi ring and $\varrho$ a fuzzy sub set of M. Then (i) $\varrho(d\chi m\psi k) = \min\{\varrho(d), \varrho(m), \varrho(k)\}$ ∀ *d, m, k* ∈ Q and $\chi, \psi \in \Gamma$ and $\varsigma'(d\chi m\psi k) = \vee\{\varsigma'(d), \varsigma'(m), \varsigma'(k)\}$ are equivalent ∀ *d, m, k* ∈ Q and $\chi, \psi \in \Gamma$.**

***Proof***: Assume that $\varrho(d\chi m\psi k) = \min\{\varrho(d), \varrho(m), \varrho(k)\}$ ∀ $d, m, k \in$ Q and $\chi, \psi \in \Gamma$. We may assume that $\varrho(d\chi m\psi k) = \varrho(d)$. Then $\varrho(d) \leq \varrho(m)$ and $\varrho(d) \leq \varrho(k)$, so $\varsigma'(p\gamma f\delta k) = 1 - \varsigma(d\chi m\psi k) = 1 - \varsigma(d) = \varsigma'(d)$ and $\varsigma'(d) = 1 - \varsigma(d) \geq 1 - \varsigma(m) = \varsigma'(m)$ and $\varsigma'(m) = 1 - \varsigma(d) \geq 1 - \varsigma(k) = \varsigma'(k)$. Therefore, $\varsigma'(d\chi m\psi k) = \vee\{\varsigma'(d), \varsigma'(m), \varsigma'(k)\}$ ∀ $d, m, k \in$ Q and $\chi, \psi \in \Gamma$.

Conversely, suppose that $\varsigma'(d\chi m\psi k) = \vee\{\varsigma'(d), \varsigma'(m), \varsigma'(k)\}$ ∀ $d, m, k \in$ Q and $\chi, \psi \in \Gamma$. We may assume that $\varsigma'(d\chi m\psi k) = \varsigma'(m)$. Then $\varsigma'(d) \geq \varsigma'(m)$ and $\varsigma'(d) \geq \varsigma'(k)$ so $1 - \varsigma(d\chi m\psi k) = 1 - \varsigma(d)$, $1 - \varsigma(d) \geq 1 - \varsigma(m)$ and $1 - \varsigma(m) \geq 1 - \varsigma(k)$. Thus $\varsigma(d\chi m\psi k) = \varsigma(d)$, $\varsigma(d) \leq \varsigma(m)$ and $\varsigma(d) \leq \varsigma(k)$. Hence $\varsigma(d\chi m\psi k) = \min\{\varrho(d), \varrho(m), \varrho(k)\}$ ∀ $d, m, k \in$ Q and $\chi, \psi \in \Gamma$.

**Th 3.8: Let R be a TΓ-semi ring and K $\subseteq$ R, K $\neq \varnothing$. Then K is a TΓ-sub semi ring of R iff the fuzzy subset $\pi_K$ is a fuzzy TΓ-sub-semiring of R.**

***Proof*** **:** Obviously, $\pi_K$ is a fuzzy subset of R. Let $u, v, w \in$ R and $\gamma, \delta \in \Gamma$. If $u \notin$ K or $v \notin$ K or $w \notin$ K, then $\pi_K(u) = 0$ or $\pi_K(v) = 0$ or $\pi_K(w) = 0$, then $\pi_K(u+v) \geq 0 = \min\{\pi_K(u), \pi_K(v)\}$ and $\pi_K(u\gamma v\delta w) \geq 0 = \min\{\pi_K(u), \pi_K(v), \pi_K(w)\}$. Let $u, v, w \in$ K. Then $\pi_K(u) = \pi_K(v) = \pi_K(w) = 1$. Since $u+v \in$ K, $u\gamma v\delta w \in$ K $\Gamma$ KΓK $\subseteq$ K, we have $\pi(u+v) = 1 \geq 1 = \min\{\pi_K(u), \pi_K(v)\}$ and $\pi_K(u\gamma v\delta w) = 1$. Thus $\pi_K(u\gamma v\delta w) = 1 \geq 1 = \min\{\pi_K(u), \pi_K(v), \pi_K(w)\}$. Therefore the fuzzy subset $\pi_K$ is a fuzzy TΓ-sub-semi ring of R.

Conversely, suppose that $u, v, w \in$ K and $\gamma, \delta \in \Gamma$. Then $\pi_K(u) = \pi_K(v) = \pi_K(w) = 1$. Since $\pi_K$ is a fuzzy TΓ-sub semiring of R, we have $\pi_K(u+v) \geq \min\{\pi_K(u), \pi_K(v)\} = \min\{1, 1\} = 1$. Thus $u + v \in$ K and $\pi_K(u\gamma v\delta w) \geq \min\{\pi_K(u), \pi_K(v), \pi_K(w)\} = \min\{1, 1, 1\} = 1$. Thus $\pi_K(u\gamma v\delta w) = 1$ and hence $u\gamma v\delta w \in$ K. Therefore K is a TΓ-sub semi ring of R.

**Def 3.9:** A non-empty fuzzy sub-set $\varrho$ of a TΓ-semiring $Q$ is called a ***fuzzy L(La, R) TΓ-ideal*** or simply ***fuzzy left TΓ-ideal*** of $Q$ if

(i) $\varrho(u+v) \geq \wedge\{\varrho(u), \varrho(v)\}$

*(ii)* $\varrho(u\gamma v\delta w) \geq \varrho(w)$[ $\varrho(u\gamma v\delta w) \geq \varrho(v)$, $\varrho(u\gamma v\delta w) \geq \varrho(u)$] $\forall$ $u, v, w \in Q$, $\forall \gamma, \delta \in \Gamma$.

**Ex 3.10:** Let Q = {0, $s$, $t$, $u$} and Γ be the non-empty set of binary operations such that $\alpha, \beta \in \Gamma$ is defined below:

| + | 0 | *s* | *t* | *u* |
|---|---|---|---|---|
| 0 | 0 | *s* | *t* | *u* |
| *s* | *s* | *t* | *u* | *s* |
| *t* | *t* | *u* | *u* | *s* |
| *u* | *u* | *t* | *s* | *t* |

| $\alpha$ | *0* | *s* | *s* | *u* |
|---|---|---|---|---|
| *0* | 0 | 0 | 0 | 0 |
| *s* | *0* | *s* | *t* | *u* |
| *t* | 0 | 0 | 0 | 0 |
| *u* | *0* | *s* | *t* | *u* |

| $\beta$ | *0* | *s* | *t* | *u* |
|---|---|---|---|---|
| *0* | 0 | 0 | 0 | 0 |
| *s* | *s* | *s* | *s* | *s* |
| *t* | 0 | 0 | 0 | *t* |
| *u* | *s* | *s* | *s* | *u* |

Clearly Q is a TΓ-semi ring. Define a fuzzy subset $\pi$: Q → [0, 1] by $\pi(x) = \begin{cases} 1 \text{ if } x=0, a \\ 0, \text{ otherwise} \end{cases}$. Clearly, $\pi$ is a fuzzy L-TΓ-ideal of Q.

**Def 3.11:** A non-empty fuzzy sub-set $\varpi$ of a TΓ-semi ring $Q$ is called a ***fuzzy TΓ-ideal*** of $Q$ if

(i) $\varpi(f+g) \geq \min\{\pi(f), \pi(g)\}$

(ii) $\varpi(f\gamma g\delta h) \geq \varpi(f) \vee \varpi(g) \vee \varpi(h)$ for any $f, g, h \in Q$ and $\gamma, \delta \in \Gamma$.

**Ex 3.12:** Let the set of -ve integers with 0 be P and the set of negative even integers with 0 be Γ. Then $P$ is a TΓ-semi ring if $u + v$, $u\gamma v\delta w$ as well as $\alpha u\beta v\gamma$ denote the usual multiplication of integers $u, \gamma, v, \delta, w$ as well as $\alpha, u, \beta, v, \gamma$ respectively where $u, v, w \in Q$ as well as $\alpha. \beta, \gamma, \delta \in \Gamma$. Let $\eta$ be a fuzzy sub-set of $Q$, defined as follows:

$$\eta(m) = \begin{cases} 1, & \text{if m} = 0 \\ 0.1, & \text{if m} = -1, -2 \\ 0.2, & \text{if m} < -2 \end{cases}$$

Then $\eta$ is a fuzzy TΓ-ideal of Q.

**Ex 3.13:** Let M = {0, $u$, $v$, $w$} and Γ = {$\alpha, \beta$} be the non-empty set of binary operations defined below:

| + | 0 | *u* | *v* | *w* |
|---|---|---|---|---|
| 0 | 0 | *u* | *v* | *w* |
| *u* | *u* | *v* | *w* | *u* |
| *v* | *v* | *w* | *w* | *u* |
| *w* | *w* | *v* | *u* | *v* |

| $\alpha$ | 0 | *u* | *v* | *w* |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| *u* | 0 | *v* | 0 | *u* |
| *v* | 0 | *v* | 0 | *w* |
| *w* | 0 | 0 | 0 | *v* |

| $\beta$ | 0 | *u* | *v* | *w* |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| *u* | *u* | *u* | *u* | *u* |
| *v* | 0 | 0 | 0 | 0 |
| *w* | *u* | *u* | *u* | *w* |

Obviously, M is a TΓ-semiring. Moreover the fuzzy set $\pi$ : M → [0, 1] defined by $\pi(0) = 0.7$, $\pi(u) = 0.6$, $\pi(v) = \pi(w) = 0.3$ is a fuzzy TΓ-ideal of Q.

**Th 3.14: Suppose $\{\pi_i : i \in I\}$ is a family of fuzzy L-(La, R) T$\Gamma$-ideals of T$\Gamma$-semi ring T, then $(\bigcap_{i \in I} \pi_i)(u)$ is a fuzzy L-(La, R, T$\Gamma$-ideal) T$\Gamma$-ideal of Q.**

***Proof*** **:** Let $\{\pi_i : i \in I\}$ be a family of fuzzy L-T$\Gamma$-ideals of T$\Gamma$-semi ring T. Let $\pi = \bigcap_{i \in I} \pi_i$. Let $u, v, w \in$ Q, $\gamma, \delta \in \Gamma$. Since $(\bigcap_{i \in I} \pi_i)(u) = \min\{\pi_i(u) : i \in I\}$ and each $\pi_i(u)$ is a fuzzy L-T$\Gamma$-ideal of Q. Therefore,

$$(\bigcap_{i \in I} \pi_i)(k+v) \geq \min\{\pi_i(k+v)\}$$
$$= \min\{\min\{\pi_i(k), \pi_i(v)\}$$
$$= \min\{\min\{\ \pi_i(k)\ \}, \min\{\{\ \pi_i(v)\ \}\}.$$ and

$\pi_i(k\gamma v\delta o) \geq \pi_i(o)$, $i \in I$ and $\forall k, v, o \in Q$, $\forall \gamma, \delta \in \Gamma$.

$$\text{Now } \pi(k\gamma v\delta o) = (\bigcap_{i \in I} \pi_i)(k\gamma v\delta o)$$
$$= \min\{\ (\pi_i)(k\gamma v\delta o) : i \in I\}$$
$$\geq \min\{\pi_i(o) : i \in I\}$$
$$= (\bigcap_{i \in I} \pi_i)(o) = \pi(o).$$

Therefore $\pi(k\gamma v\delta o) \geq \pi(o)$ and hence $\pi = \bigcap_{i \in I} \pi_i$ is a L- fuzzy T$\Gamma$-ideal of Q. Similarly, we can prove the remaining two parts.

**Theorem 3.15: If $\{\pi_i : i \in I\}$ is a family of fuzzy left (lateral, right) T$\Gamma$-ideals of $\Gamma$-semi ring T, then $(\bigcup_{i \in I} \pi_i)(g)$ is a fuzzy left (lateral, right, T$\Gamma$-ideal) T$\Gamma$-ideal of H.**

***Proof*** **:** Let $\{\pi_i : i \in I\}$ be a family of fuzzy left T$\Gamma$-ideals of T$\Gamma$-semi ring H. Let $\pi = \bigcup_{i \in I} \pi_i$. Let $k, v, o \in$ H, γ, δ $\in \Gamma$. Since $(\bigcup_{i \in I} \pi_i)(k) = \max\{\pi_i(k) : i \in I\}$ and $\pi_i(k)$ is a fuzzy left T$\Gamma$-ideal of S for some $i \in I$. Therefore,

$$\bigcup_{i \in I} (\pi_i)(k+v) \geq \max\{\pi_i(k+v)\}$$
$$= \max\{\min\{\pi_i(k), \pi_i(v)\}$$
$$= \max\{\min\{\ \pi_i(k)\ \}, \min\{\{\ \pi_i(v)\ \}\}$$ and

$\pi_i(k\gamma v\delta o) \geq \pi_i(o)$, $i \in I$ and $\forall k, v, o \in H$, $\forall \gamma, \delta \in \Gamma$.

$$\text{Now } \pi(k\gamma v\delta o) = (\bigcup_{i \in I} \pi_i)(k\gamma v\delta o)$$
$$= \max\{\ (\pi_i)(k\gamma v\delta o) : i \in I\} \geq \max\{\pi_i(o) : i \in I\}$$
$$= (\bigcup \pi_i)(o) \text{ for some } i \in I = \pi(w).$$

Therefore $\pi(k\gamma v\delta o) \geq \pi(o)$ and hence $\pi = \bigcup_{i \in I} \pi_i$ is a L- fuzzy T$\Gamma$-ideal of Q. Similarly, prove the other parts.

**Th 3.16: Let P(≠∅) be a subset of a TΓ-semiring Q and $\pi_I$ be the characteristic function of I, then I is a L(La, R) TΓ-ideal of Q iff $\pi_I$ is a fuzzy L(La, R) TΓ-ideal of Q.**

***Proof***: Let P be a L-TΓ-ideal of a TΓ-semi ring Q. Let $x, y, z \in$ Q and $\gamma, \delta \in \Gamma$, then $x + y \in$ I and $x\gamma y\delta z \in$ I if $z \in$ I. It follows that $\mathcal{X}_I(x + y) = 1$ and $\mathcal{X}_I(x\gamma y\delta z) = 1 = \mathcal{X}_I(z)$. If $y \notin$ I, then $\mathcal{X}_I(z) = 0$. In this case $\mathcal{X}_I(x\gamma y\delta) \geq 0 = \mathcal{X}_I(z)$. Therefore $\mathcal{X}_I$ is a fuzzy L-TΓ-ideal of Q.
Conversely, suppose that $\mathcal{X}_I$ be a fuzzy left Γ-ideal of S. Let $x, y \in$ I, then if $x, y \in$ I, then $\mathcal{X}_I(x) = \mathcal{X}_I(y) = 1$ and $\mathcal{X}_I(x + y) \geq$ min{ $\mathcal{X}_I(x), \mathcal{X}_I(y)$} = min {1, 1} = 1. Thus $x + y \in$ I. $\mathcal{X}_I(x) = \mathcal{X}_I(y) = 1$. Now let $x \in$ I and $s, t \in$ Q, , $\delta \in \Gamma$. Then $\mathcal{X}_I(x) = 1$. Also $\mathcal{X}_I(s\gamma t\delta x) \geq \mathcal{X}_I(x) = 1$. Thus $s\gamma t\delta x \in$ I. So I is a L-TΓ-ideal of Q.
In the similar manner one can prove remaining two parts.

**Th 3.17: Suppose X be a L(La, R) TΓ-ideal of a TΓ-semi ring V and $\gamma \leq \theta \neq 0$ be any 2 elements in [0, 1], then the fuzzy subset $\pi$ of X, defined by** $\pi(x) = \begin{cases} \theta \text{ if } x \in I \\ \gamma \text{ otherwise} \end{cases}$ **is a fuzzy L(La, R) TΓ-ideal of X.**

*Proof*: Let X be a L-TΓ-ideal of a TΓ-semi ring V and $\alpha, \beta \in$ [0,1]. Let $s, t, x, y \in$ T and $\gamma, \delta \in \Gamma$. If $x, x + y \in$ I, $\pi(x + y) \geq$ min {$\pi(x)$, $\mu(x)$} = $\alpha \Rightarrow \alpha \leq \mu(x + y)$ and if $x \notin$ I, $x + y \notin$ I, then $\alpha = \mu(x + y) \geq$ min {$\pi(x)$, $\pi(x)$} = α. Therefore, $\pi(x + y) \geq$ min { $\pi(x)$, $\pi(x)$}. Now $x \in$ X, then $s\gamma t\delta x \in$ X and $\pi(x) = \theta = \pi(s\gamma t\delta x)$. Therefore $\pi(x) = \pi(s\gamma t\delta x)$. If $x \notin$ X then $\mu(x) = \gamma \leq \theta = \pi(s\gamma t\delta x)$ and then $\pi(s\gamma t\delta x) \geq \pi(x)$. Thus $\pi(s\gamma t\delta x) \geq \pi(x)$ ∀ $s, t, x \in$ V and $\gamma, \delta \in \Gamma$. Hence $\pi$ is a fuzzy L-TΓ-ideal of V. Similarly, prove the other parts.

**Th 3.18: Let V be a TΓ-semi ring and $\pi$ be a non-empty fuzzy subset of V, then $\pi$ is a fuzzy L(La. R) TΓ-ideal of V if and only if $\pi_t$'s are L(La. R) TΓ-ideal of V for all $t \in$ Im($\pi$) where $\pi_t = \{x \in V : \mu(x) \geq t\}$.**

***Proof***: Let $\pi$ be a fuzzy L-TΓ-ideal of V. Let $t \in$ Im(π), then ∃ $\alpha \in$ V ∋ $\pi(\alpha) = t$ and so $\alpha \in \pi_t$. Thus $\pi_t \neq \emptyset$. Let $a,q \in \pi_t$ then $a + q \geq t$ implies that $\pi(a + q) \geq$ min {$\pi(a)$, $\pi(q)$} $\geq t$ and hence $\pi(a + p) \geq$ t. Therefore $a + q \in \pi_t$ . Now let $a \in \pi_t$ then $\pi(a) \geq t$. Again let $s, t \in$ V, $e \in \pi_t$ and $\gamma, \delta \in \Gamma$. Now $\pi(s\gamma t\delta e) \geq \pi(e) \geq t$. Therefore, $s\gamma t\delta e \in \pi_t$. Thus $\pi_t$ is a L-TΓ-ideal of V.
Conversely, suppose that $\pi_t$'s are L-TΓ-ideals of V for all $t \in$ Im($\pi$). Again let $a, q \in$ V, $s, t \in$ V and $\gamma, \delta \in \Gamma$, then $\pi(a) = \pi(q) = t$. Since $\pi_t$ is a L-TΓ-ideal of V and hence $a + q$, $s\gamma t\delta a \in \pi_t$. Therefore $\pi$ $\pi(a + q) \geq$ min {$\pi(a)$, $\pi(q)$} = t and $\pi(s\gamma t\delta a) \geq t = \pi(a)$. Hence $\pi$ is a fuzzy L-TΓ-ideal of V. Similarly, prove the other parts.

**Def 3.19:** Let V be a TΓ-semi ring and $\pi$ be a fuzzy L(La, R) TΓ-ideal of a TΓ-semi ring V. Then the TTΓ-ideals $\pi_t$'s are know as ***level L*(La, R) *TΓ-ideal*** of $\pi$ where $t \in$ Im($\pi$).

**Th 3.20: Let $\eta$ be a fuzzy L(La, R) TΓ-ideal of a TΓ-semiring V and $t_1 > t_2$. Then $\eta_{t_1} \subseteq \eta_{t_2}$ where $t_1, t_2 \in$ Im($\eta$), equality occurs iff there is no $x \in$ V ∋ $t_1 \leq \eta(x) < t_2$.**

***Proof.*** The 1st section of the theorem follows easily. Now let $\eta$ be a fuzzy L-TΓ-ideal of $V$ ∋ $\eta_{t_1} = \eta_{t_2}$ . If possible ∃ $u \in V$ ∋ $t_1 \leq \eta(x) < t_2$. Then $x \in \eta_{t_1}$ but $x \notin \eta_{t_1}$ , it is contradiction. So ∃ no $x \in V$ ∋ $t_1 \leq \eta(x) \leq t_2$.
Conversely, Let $\eta$ is a fuzzy L-TΓ-ideal of $V$ ∋ there does not exist $x \in$ V with $t_1 \leq \eta(x) < t_2$. ∵ $t_1 < t_2$, then $\eta_{t_1} \subseteq \eta_{t_2}$ (by Definition 3.15). If possible $\eta_{t_1} \neq \eta_{t_2}$ . Then there is some $y \in V$ ∋ $y \in \eta_{t_1}$ but $y \notin \eta_{t_2}$ , i.e, $\eta(y) \geq t_1$ but $\eta(y) < t_2$, i.e, $t_1 \leq \eta(y) < t_2$, it is a contradiction to our assumption. Hence, $\eta_{t_1} = \eta_{t_2}$ . Similarly, prove the other sections of the statement.

**Th 3.21 : Suppose V be a TΓ-semi ring. Every L(R)-Γ-ideal of V is two-sided iff every fuzzy L(R)-Γ-ideal of V is fuzzy two-sided.**

**Th 3.22: Let $\varsigma$ be a fuzzy subset of a TΓ-semi ring M. Then $\varsigma$ is a fuzzy TΓ-ideal of M iff ∀ $t \in$ [0, 1], if $\varsigma_t^S \neq \emptyset$, then $\varsigma_t^S$ is a TΓ-ideal of M.**

***Proof* :** Assume $\varsigma$ is a fuzzy TΓ-ideal of M. Then $\varsigma(a+p) \geq \wedge\{\varsigma(a), \varsigma(p)\}$ and $\varsigma(a\gamma p\delta w) \geq \varsigma(a) \vee \varsigma(p) \vee \varsigma(w)$ for any $a, p, w \in$ M and $\gamma, \delta \in \Gamma$. Thus $\varsigma(a) > t$. Since $\varsigma$ is a fuzzy TΓ-ideal of M, $\varsigma(a + p) \geq$ min{ $\varsigma(a), \varsigma(p)$} $> t$ and
$\varsigma(a\gamma p\delta w) \geq \varsigma(a) \vee \varsigma(p) \vee \varsigma(w) \geq \varsigma(a) > t$.
Therefore, $a + p$, $a\gamma p\delta w \in \varsigma_t^S$ .
Similarly, $a + p$, $a\gamma p\delta w \in \varsigma_t^S$ and $a + p$, $a\gamma p\delta w \in \varsigma_t^S \Rightarrow \varsigma_t^S$ is a TTΓ-ideal of M.
Conversely, suppose that for all $t \in$ [0, 1], if $\varsigma_t^S \neq \emptyset$, then $\varsigma_t^S$ is a TΓ-ideal of M. Let $a, p, w \in$ M and $\gamma, \delta \in \Gamma$.
Case 1: $\varsigma(a) \geq \varsigma(p) \geq \xi(w)$. Thus $a \in \varsigma_t^S \ \forall\ t < \varsigma(a)$.
By our supposition, we have $\varsigma_t^S$ is a TΓ-ideal of M $\forall\ t < \varsigma(a)$.
So $a\gamma p\delta w \in \varsigma_t^S \ \ \forall\ t < \varsigma(a)$.
Then $\varsigma(a\gamma p\delta w) > t \ \ \forall\ t < \varsigma(a)$.
Then $\varsigma(a\gamma p\delta w) \geq \varsigma(a) = \varsigma(a) \vee \varsigma(p) \vee \varsigma(w)$.
Case 2: $\varsigma(a) < \varsigma(p) < \varsigma(w)$. Then, $p \in \varsigma_t^S \ \ \forall\ t < \varsigma(w)$.
By our supposition, we have $\varsigma_t^S$ is a TΓ-ideal $\forall\ t < \varsigma(w)$.
So $a\gamma p\delta w \in \varsigma_t^S \ \ \forall\ t < \varsigma(w)$.
Then, $\varsigma(a\gamma p\delta w) > t \ \ \forall\ t < \varsigma(w)$.
Then $\varsigma(a\gamma p\delta w) \geq \varsigma(w) = \varsigma(a) \vee \varsigma(p) \vee \varsigma(w)$.
Therefore $\varsigma$ is a fuzzy TΓ-ideal of M.

## 4. Conclusion

many of the researchers were studied about fuzzy ideals in semigroups, γ-semigroups, ternary semigroups, semirings and near rings. here, we extended those concepts to fuzzy tγ-ideals in ternary γ-semirings.

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