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***Research paper***

# Completely Prime and Prime Fuzzy TΓ-Ideals in TΓ-Semi rings

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**Abstract**

C-prime fuzzy TΓ-ideals and prime fuzzy TΓ-ideals are studied, then proved some theorem and characterized the C-prime and prime fuzzy TΓ-ideals in TΓ-semirings.

***Keywords***: *TΓ-semiring, C-prime fuzzy TΓ-ideals, prime fuzzy TΓ-ideals, commutative TΓ-semiring.*

## 1. Introduction

Most of the papers on fuzzy theory appeared showing the importance of the concept and applications to logic, topology, theory of algebraic structures, etc. Here, we introduce about C-prime , prime, C-semiprime and semiprime fuzzy TΓ-ideals in TΓ-semirings.

## 2. Preliminaries:

For preliminaries refer the references

## 3. C-Prime and Prime Fuzzy TΓ-Ideals:

**Def 3.1 :** A fuzzy TΓ-ideal $\mu$ of a TΓ-semiring Q is known as ***C-prime fuzzy TΓ-ideal*** provided $\mu$ : Q → [0. 1] is a non-constant function and for any three fuzzy points $a_t$, $b_r$ , $c_s$ of T, $a_t\Gamma b_r\Gamma c_s \leq \mu$ implies either $a_t \in \mu$ or $b_r \in \mu$ or $c_s \in \mu$.

**Ex 3.2:** Let $Q$ be the set all 1× 2 matrices over 2 $GF$ (the finite field with two elements) and Γ be the set of all 2×1matrices over 2 $GF$. Then $Q$ is a TΓ -semiring where $a\alpha b\beta c$ and αaβbγ ($a, b, c \in$ Q, α ,β, γ ∈Γ) denote usual matrix product. Let $\pi : Q \rightarrow [0,1]$ by μ ($x$) = 0.3, if $x$ = (0,0) and 0.4, otherwise. Then $\pi$ is a C-prime fuzzy TΓ-ideal of Q.

**Def 3.3:** Suppose Q be a TΓ-semiring. A fuzzy sub set $\mu$ of T is said to be a ***fuzzy c-system*** of Q if for each $\mu_s$, $\mu_t$, $\mu_r$ of $\mu$ there exist an element $\alpha, \beta \in \Gamma$ such that $\mu_s\alpha\mu_t\beta\mu_r \in \mu$.

**Th 3.4 : Every fuzzy TΓ-sub semi ring of a TΓ-semi ring is a fuzzy *c-system*.**

***Proof*:** Let $\xi$ is a fuzzy TΓ-sub semiring of a TΓ-semiring M and $\xi_s$, $\xi_t$, $\xi_r$ $\in \xi$. Since $\xi$ is a fuzzy TΓ-sub semi ring of M. So, $\xi(u\gamma v\delta w) \geq \xi(u) \vee \xi(v) \vee \xi(w)$ ∀ $u, v, w \in M$ and $\gamma, \delta \in \Gamma$.
Since $\xi_s$, $\xi_t$, $\xi_r \in \xi$. Therefore $\xi(u) = s$, $\xi(v) = t$ and $\xi(z) = r$.
If $\xi_s(x) = s$, $\xi_t(v) = t$ and $\xi_r$wz) = $r$ for $u$, $v$, $w$ ∈ M. Then

$$(\xi_s\Gamma\xi_t\Gamma\xi_r)(p) = \bigvee_{p=u\gamma v\delta w}\{\xi_s(u) \wedge \xi_t(v) \wedge \xi_z(w)\}$$

$= \min(s, t, r) = \xi(u) \wedge \xi(v) \wedge \xi(w) \leq \xi(u\gamma v\delta w)$
$= \xi(p)$ and hence $\xi_s\Gamma\xi_t\Gamma\xi_r \leq \xi \Rightarrow \xi_s\alpha\xi_t\beta\xi_r \in \xi$, for $\alpha$ , $\beta \in$ Γ.
Therefore $\xi$ is a fuzzy *c-system* of M.
Now $(\xi_s\Gamma\xi_t\Gamma\xi_r)(p) = 0$ if $p \neq u\gamma v\delta w$, then it follows that
$(\xi_s\Gamma\xi_t\Gamma\xi_r)(p) = 0 \leq (p) \Rightarrow \xi_s\Gamma\xi_t\Gamma\xi_r \leq \xi \Rightarrow \xi_s\alpha\xi_t\beta\xi_r \in \xi$, for $\alpha$ , $\beta \in$ Γ.
Therefore $\mu$ is a fuzzy *c-system* of M.

**Th 3.5: A fuzzy TΓ-ideal $\pi$ of a TΓ-semiring Q is C-prime fuzzy TΓ-ideal iff its complement $\pi' = 1 - \pi$ is a *fuzzy c-system*.**

***Proof*:** Suppose $\pi$ is a C-prime fuzzy TΓ-ideal of Q. Suppose $a_s$, $b_t$, $c_r \in \pi'$ . Then $a_s \notin \pi$, $b_t \notin \pi$ and $c_r \notin \pi$.
Suppose if possible $a_s\Gamma b_t\Gamma c_r \nleq \pi'$, then $a_s\Gamma b_t\Gamma c_r \leq \pi$. ∵ $\pi$ is C-prime fuzzy TΓ-ideal of Q, either $a_s \in \pi$ or $b_t \in \pi$ or $c_r \in \pi$. It is a contradiction. Therefore if $a_s$, $b_t$, $c_r \in \pi'$, then $a_s\Gamma b_t\Gamma c_r \leq \pi'$ and hence $\pi'$ is a *fuzzy c-system*.

Conversely, suppose that $\pi'$ is a *fuzzy c-system* of Q.
Let $a_s$, $b_t$, $c_r$ ∈ Q and $a_s\Gamma b_t\Gamma c_r \leq \pi$. Suppose if possible $a_s \notin \pi$, $b_t \notin \pi$ and $c_r \notin \pi$. Then $a_s$, $b_t$, $c_r \in \pi'$ . Since $\pi'$ is a *fuzzy c-system* and hence $a_s\Gamma b_t\Gamma c_r \leq \pi'$ . Thus $a_s\Gamma b_t\Gamma c_r \nleq \pi$.
It is a contradiction. Hence either $a_s \in \pi$ or $b_t \in \pi$ or $c_r \in \pi$.
Therefore $\pi$ is a C-prime fuzzy TΓ-ideal of Q.

**Def 3.6:** A fuzzy subset π of a TΓ-semiring M is said to be ***prime*** if for any fuzzy subsets $\lambda$, $\nu$, $\xi$ of M, $\lambda o \nu o \xi \leq \pi \Rightarrow \lambda \leq \pi$ or $\nu \leq \pi$ or $\xi \leq \pi$.

**Th 3.7: Let Q be a TΓ-semiring and $\varsigma$ a fuzzy subset of Q. Then $\varsigma$ is prime iff**
$\varsigma(p\gamma w\delta a) \leq \max\{\varsigma(p), \varsigma(w), \varsigma(a)\}$ **for all *p, w, a* ∈ T and *γ, δ* ∈ Γ.**

***Proof***: Suppose that $\varsigma$ is prime. Let *p, w, a* ∈ Q.
Since $p\alpha w\beta a$ ∈ Q for some $\alpha, \beta \in \Gamma$.
We have $\varsigma(p\alpha w\beta a) \in [0, 1]$. We put $\lambda = \varsigma(p\alpha w\beta a) \rightarrow (1)$.
Since *p, w, a* ∈ Q and $\lambda \in [0, 1]$, the fuzzy points $(p\alpha w\beta a)_\lambda$, $p_\lambda$, $w_\lambda$, $a_\lambda$ are defined. Let $x \in$ Q. If $x \neq p\alpha w\beta a$ for $\alpha, \beta \in \Gamma$, then $(p\alpha w\beta a)_\lambda(x) = 0$. ∵ $\varsigma$ is a fuzzy subset of Q,
we have $\varsigma(x) \in [0, 1]$, $\varsigma(x) \geq 0$. Then $\varsigma(p\alpha w\beta a)_\lambda \leq \varsigma(x)$.
If $x = p\alpha w\beta a$, then $(p\alpha w\beta a)_\lambda(x) = \lambda$. Then by (1),
$(p\alpha w\beta a)_\lambda(x) = \varsigma(p\alpha w\beta a) = \varsigma(x)$.
Therefore $(p\alpha w\beta a)_\lambda(x) \leq \varsigma(x)$. We have $(p\alpha w\beta a)_\lambda \leq \varsigma \rightarrow (2)$.
Since $\varsigma$ is prime, by (2), we have $p_\lambda \leq \varsigma$ or $w_\lambda \leq \varsigma$ or $a_\lambda \leq \varsigma$.
Then $\lambda = p_\lambda(p) \leq \varsigma(p)$ or $\lambda = w_\lambda(w) \leq \varsigma(w)$ or or $\lambda = a_\lambda(a) \leq \varsigma(a)$.
Therefore $\varsigma(p\alpha w\beta a) \leq \varsigma(p)$ or $\varsigma(p\alpha w\beta a) \leq \varsigma(w)$ or $\varsigma(p\alpha w\beta a) \leq \varsigma(a)$ for some $\alpha, \beta \in \Gamma$.
Thus $\varsigma(p\gamma w\delta a) \leq \max\{\varsigma(p), \varsigma(w), \varsigma(a)\}$

Let $x, y, z \in$ Q, $\alpha, \beta \in \Gamma$ and $\lambda \in [0, 1]$, $x_\lambda \alpha y_\lambda \beta z_\lambda \leq \varsigma$. Let $q \in$ Q.
If $q \neq x$, $q \neq y$ and $q \neq z$, then $x_\lambda(q) = 0$, $y_\lambda(q) = 0$ and $z_\lambda(q) = 0$.
Since $\varsigma$ is a fuzzy subset of Q, $\varsigma(z) \in [0, 1]$, so $0 \leq \varsigma(z)$.
i.e. $x_\lambda(q) \leq \varsigma(z)$, $y_\lambda(q) \leq \varsigma(z)$ and $z_\lambda(q) \leq \varsigma(z)$.
If $q = x$ or $q = y$ or $q = z$, then $x_\lambda(q) = \lambda$ or $y_\lambda(q) = \lambda$ or $z_\lambda(q) = \lambda$.
Since $x_\lambda \alpha y_\lambda \beta z_\lambda \leq \varsigma$, we have
$\lambda = (x_\lambda \alpha y_\lambda \beta z_\lambda)\gamma(x\alpha y\beta z) = (x\alpha y\beta z)_\lambda\gamma(x\alpha y\beta z) = \varsigma(x\alpha y\beta z)$.
Then, by hypothesis, we have
$\lambda \leq \varsigma(x\alpha y\beta z) \leq \varsigma(x) = \varsigma(y) = \varsigma(z) = \varsigma(p)$
and hence $x_\lambda(q) = y_\lambda(q) = z_\lambda(q) = \varsigma(q)$.

**Th 3.8: Let M be a TΓ-semi ring and ∅≠ I⊆ M. Then I is a prime subset of T iff the fuzzy subset $\xi_I$ is a prime fuzzy sub set of M.**

***Proof*** : Obviously, $\xi_I$ is a fuzzy subset of M. Let *e, p, v* ∈ M and $\gamma, \delta \in \Gamma$. If $e\gamma p\delta v \notin$ I, then
$\xi_I(e\gamma p\delta v) = 0 \leq \max\{\xi_I(e), \xi_I(p), \xi_I(v)\}$.
Let $e\gamma p\delta v \in$ I. Then $\xi_I(e\gamma p\delta v) = 1$. Since I is a prime sub set of M, we have *e* ∈ I or *p* ∈ I or *v* ∈ I.
Thus $\xi_I(e) = 1$ or $\xi_I(p) = 1$or $\xi_I(v) = 1$ and so
$\xi_I(e\gamma p\delta v) = 1 \leq 1 = \max\{\xi_I(e), \xi_I(p), \xi_I(v)\}$.

Therefore, the fuzzy subset $\xi_I$ is a prime fuzzy sub set of M.

Conversely, suppose that *e, p, v* ∈ M and $\gamma, \delta \in \Gamma$ be such that $e\gamma p\delta v \in$ I. Then $\xi_I(e\gamma p\delta v) = 1$. Since $\xi_I$ is a prime fuzzy sub set of M, we have $1 = \xi_I(e\gamma p\delta v) = \max\{\xi_I(e), \xi_I(p), \xi_I(v)\}$.
Thus $\xi_I(e) = 1$ or $\xi_I(p) = 1$or $\xi_I(v) = 1$ and so *e*∈ I or *p*∈ I or *v*∈ I.
Therefore, I is a prime sub set of M.

**Def 3.9:** A fuzzy TΓ-ideal $\pi$ of a TΓ-Semiring M is known as ***prime fuzzy TΓ-ideal*** provided for any fuzzy TΓ-ideals $\nu$, ξ, $\eta$ of M, $\nu\Gamma\xi\Gamma\eta \leq \mu \Rightarrow \nu \leq \mu$ or $\xi \leq \mu$ or $\eta \leq \mu$.

**Th 3.10: A fuzzy TΓ-ideal $\xi$ of a TΓ-semi ring M is said to be *prime fuzzy TΓ-ideal* iff $\xi(s\gamma e\delta d) = \max\{\xi(s), \xi(e), \xi(d)\}$ for any *s, e, d* ∈ M & *γ, δ*∈ Γ.**

***Proof*** : Suppose that $\xi$ is a prime fuzzy TΓ-ideal. Then $\xi$ is fuzzy TΓ-ideal of M ⟹ for any *s, e, d* ∈ M & *γ, δ* ∈ Γ, we have $\xi(s\gamma e\delta d) \geq \max\{\xi(s), \xi(e), \xi(d)\}$ &
$\xi$ is a fuzzy prime ⟹ $\xi(s\gamma e\delta d) \leq \max\{\xi(s), \xi(e), \xi(d)\}$ and hence $\xi(s\gamma e\delta d) = \max\{\xi(s), \xi(e), \xi(d)\}$.

Conversely, suppose that for any *s, e, d* ∈ M and *γ, δ*∈ Γ, $\xi(s\gamma e\delta d) = \max\{\xi(s), \xi(e), \xi(d)\}$.
Then we have $\xi(s\gamma e\delta d) \geq \max\{\xi(s), \xi(e), \xi(d)\}$ &
$\xi(s\gamma e\delta d) \leq \max\{\xi(s), \xi(e), \xi(d)\}$. Therefore $\xi$ is a fuzzy TΓ-ideal of M. By th 3.7, $\xi$ is prime fuzzy subset, so $\xi$ is prime fuzzy TΓ-ideal of M.

**Corollary 3.11: A fuzzy TΓ-ideal ξ of a TΓ-semiring Q is said to be *prime fuzzy* TΓ-ideal if**
$\inf_{\gamma,\delta \in \Gamma} \xi(s\gamma f\delta l) = \max\{\xi(s), \xi(f), \xi(l)\}\ \forall\ s, f, l \in Q$.

***Proof***: Since $\xi$ is a prime fuzzy TΓ-ideal of Q. Then
$\inf_{\gamma,\delta \in \Gamma} \xi(s\gamma f\delta l) = \xi(s\gamma f\delta l) = \max\{\xi(s), \xi(f), \xi(l)\}\ \forall\ s, f, l \in Q$

**Ex 3.12 :** Let Q be the set of all 1x2 matrices over $GF_2$ ( the finite field with two elements) and Γ be the set of all 2x1 matrices over $GF_2$. Then T is a TΓ-semi ring where $s\alpha t\beta u$ and $\lambda s\mu t\nu$ for all *s, t, u* ∈ Q and $\lambda, \mu, \nu \in \Gamma$ denotes the usual matrix product. Let $\xi: Q \rightarrow [0,1]$ be defined by $\xi(x) = \begin{cases} 0.3 \text{ if } x = (0,0) \\ 0.2 \text{ otherwise} \end{cases}$. Then ξ is a fuzzy prime TΓ-ideal of Q.

**Th 3.13 : Let Q be a TΓ-semiring and ∅≠ I ⊆ Q. Then**
**(i) I is a prime TΓ-ideal of Q.**
**(ii) The characteristic function $\mu_I$ of I is a prime fuzzy TΓ-ideal of T are equivalent.**

***Proof*** : (i) ⇒ (ii) **:** Let I be a prime TΓ-ideal of Q and $\mu_I$ be the characteristic function of I. Since I ≠ ∅, $\mu_I$ is non-empty. Let *r, f, v* ∈ Q. Suppose $r\Gamma f\Gamma v \subseteq$ I. Then $\mu_I(r\gamma f\delta v) = 1$ for $\gamma, \delta \in \Gamma$. Hence $\inf_{\gamma,\delta\in\Gamma} \mu_I(r\gamma f\delta v) = 1$.
Now I being prime, then we have, $r \in I$ or $f \in I$ or $v \in I$.
Hence $\mu_I(r) = 1$ or $\mu_I(f) = 1$ or $\mu_I(v) = 1$ which gives $\max\{\mu_I(r), \mu_I(f), \mu_I(v)\} = 1$. Thus we see that
$\inf_{\gamma,\delta\in\Gamma} \mu_I(r\gamma f\delta v) = \max\{\mu_I(r), \mu_I(f), \mu_I(v)\}$.
Now suppose that $r\Gamma f\Gamma v \not\subseteq$ I. Then for $\gamma, \delta \in \Gamma$, $r\gamma f\delta v \notin I$
which means that $\mu_I(r\gamma f\delta v) = 0$.
Consequently, $\inf_{\gamma,\delta\in\Gamma} \mu_I(r\gamma f\delta v) = 0$.
Now since I is a prime of Q, $r \notin$ I, f$\notin$ I & $v \notin$ I.
Hence $\mu_I(r) = 0$ or $\mu_I(f) = 0$ or $\mu_I(v) = 0$
Consequently, $\max\{\mu_I(r), \mu_I(f), \mu_I(v)\} = 0$.
Thus we see that in this case also
$\inf_{\gamma,\delta\in\Gamma} \mu_I(r\gamma f\delta v) = \max\{\mu_I(r), \mu_I(f), \mu_I(v)\}$.
(ii) ⟹(i): Let $\mu_I$ be a fuzzy prime TΓ-ideal of Q. Then $\mu_I$ is a TΓ-ideal of T. So, I is a TΓ-ideal of Q. Let *r, f, v*∈ Q ∋ $r\Gamma f\Gamma v \subseteq$ I. Then $\mu_I(r\gamma f\delta v) = 1$ for $\gamma, \delta \in \Gamma$. Hence $\inf_{\gamma,\delta\in\Gamma} \mu_I(r\gamma f\delta v) = 1$.
Let $r \notin$ I, $f \notin$ I and v $\notin$ I. Then $\mu_I(r) = 0$ or $\mu_I(f) = 0$ or $\mu_I(v) = 0$ Which means $\max\{\mu_I(r), \mu_I(f), \mu_I(v)\} = 0$. ⟹ $\inf_{\gamma,\delta\in\Gamma} \mu_I(r\gamma f\delta v) = 0$. Thus we get a contradiction. Hence $r \in$ I, $f \in$ I & $v \in$ I. Thus we see that I is a prime TΓ-ideal of Q.

**Th 3.14: If Q be a TΓ-semiring and $\pi$ be a non-empty fuzzy subset of Q. Then.**
**(i) $\pi$ is fuzzy prime TΓ-ideal of Q.**
**(ii) For any *t*∈ [0,1] the *t*-level subset of $\pi$ (if it is non-empty) is a prime TΓ-ideal of Q are equivalent.**

**Th 3.15: Let $\rho$ be a fuzzy subset of a Γ-semi ring R. Then $\rho$ is a fuzzy prime of R iff ∀ *t*∈ [0, 1], $\rho_t^R \neq \emptyset$, then $\rho_t^R$ is a prime of R.**

***Proof*** **:** Assume $\rho$ is a fuzzy prime TΓ-ideal of R. Then is a fuzzy TΓ-ideal of R. Assume that $\rho_t^R \neq \emptyset$. By known theorem, $\rho_t^S$ is a fuzzy TΓ-ideal of R. Let $p, w, h \in$ R and $\gamma, \delta \in \Gamma$ such that $p\gamma w\delta h \in \rho_t^R$. Then $\rho(p\gamma w\delta h) > t$. Since $\rho$ is a fuzzy prime of R, $\rho(p\gamma w\delta h) = \rho(p)$ or $\rho(p\gamma w\delta h) = \rho(w)$ or $\rho(p\gamma w\delta h) = \rho(h)$.
$\Rightarrow \rho(p) > t$ or $\rho(w) > t$ or $\rho(h) > t$.
Hence, $p \in \rho_t^R$ or w $\in \rho_t^R$ or h $\in \rho_t^R$.
Therefore $\rho_t^R$ is a prime of R.

Conversely, assume for all $t \in [0,1]$, if $\rho_t^R \neq \emptyset$, then $\rho_t^R$ is a prime TΓ-ideal of R.
Let $p, w, h \in$ R and $\gamma, \delta \in \Gamma$. Then we have, $\rho$ is a fuzzy prime of R. This implies
$\rho(p\gamma w\delta h) \geq \rho(p)$, $\rho(p\gamma w\delta h) \geq \rho(w)$ and $\rho(p\gamma w\delta h) \geq \rho(h)$.
We have, $p\gamma w\delta h \in \rho_t^R$ for all $t < \rho(p\gamma w\delta h)$.
Since $\rho_t^R$ is a fuzzy prime TΓ-ideal of R for all $t < \rho(p\gamma w\delta h)$, $p \in \rho_t^R$ or w $\in \rho_t^R$ or h $\in \rho_t^R$ for all $t < \rho(p\gamma w\delta h)$. This implies that $\rho(p) > t$ or $\rho(w) > t$ or $\rho(h) > t$ for all $t < \rho(p\gamma w\delta h)$.
Then $\rho(p) \geq \rho(p\gamma w\delta h)$ or $\rho(w) \geq \rho(p\gamma w\delta h)$ or $\rho(h) \geq \rho(p\gamma w\delta h)$.
Hence $\rho(p\gamma w\delta h) = \rho(p)$ or $\rho(p\gamma w\delta h) = \rho(w)$ or $\rho(p\gamma w\delta h) = \rho(w)$.
Hence is a fuzzy prime TΓ-ideal of R.

**Th 3.16: Every completely prime fuzzy TΓ-ideal of a TΓ-semiring Q is a prime fuzzy TΓ-ideal of Q.**
***Proof*** **:** Suppose that $\mu$ is a fuzzy completely prime TΓ-ideal of a TΓ-semiring Q.
Let $\nu$, ξ, η be fuzzy TΓ-ideals of T such that $\nu o \xi o \eta \leq \mu$.
Suppose $\nu \nleq \mu$ and $\xi \nleq \mu$.
Then there exists $x \in$ T and $y \in$ T such that $\mu(x) < \nu(x)$, $\mu(y) < \xi(y)$.
Let $\nu(x) = r$ and $\xi(y) = s$.
Take any element $z \in$ Q, $\gamma, \beta \in \Gamma$ and let $(z) = t$.
Then $x_r\Gamma y_s\Gamma z_t(x\gamma y\delta z) = \min(r, s, t)$. But $\mu(x\gamma y\delta z) \geq \nu\Gamma\xi\Gamma\eta(x\gamma y\delta z) \geq \min(\nu(x), \xi(y), \eta(z)) = \min(r, s, t) = x_r\Gamma y_s\Gamma z_t(x\gamma y\delta z)$.
Since $x_r\Gamma y_s\Gamma z_t(p) = 0$ if $p \neq x\gamma y\delta z$, it follows that $x_r\Gamma y_s\Gamma z_t \leq \mu$.
So by hypothesis, either $x_r \leq \mu$ or $y_s \leq \mu$ or $z_t \leq \mu$. Since $r \nleq \mu(x)$ and $s \nleq \mu(y)$, it follows that $\eta(z) = t \leq \mu(y)$. Hence $\eta \leq \mu$.

**Th 3.17: Let T be a commutative TΓ-semiring. Then a prime fuzzy TΓ-ideal $\mu$ of Q is a fuzzy completely prime of Q.**

***Proof***: Suppose that $\mu$ is a fuzzy prime TΓ-ideal in a commutative TΓ-semiring Q. Suppose $x_r, y_s, z_t$ are three fuzzy points of T and $\gamma, \delta \in \Gamma$, such that $x_r\Gamma y_t\Gamma z_t \leq \mu$.
Then $x_r\Gamma y_s\Gamma z_t(x\gamma y\delta z) \leq \mu(x\gamma y\delta z)$.
Hence $\min(r, s, t) \leq \mu(x\gamma y\delta z) \rightarrow (1)$.
Let fuzzy subsets $\nu, \xi$, be defined by

$$\nu(p) = \begin{cases} r & \text{if } p \in <x> \\ 0 & \text{otherwise} \end{cases}$$

$$\xi(p) = \begin{cases} s & \text{if } p \in <y> \\ 0 & \text{otherwise.} \end{cases}$$

$$\eta(p) = \begin{cases} t & \text{if } p \in <z> \\ 0 & \text{otherwise.} \end{cases}$$

Clearly if $p$ is not expressible in the form $p = u\alpha v\beta w$ for some $u \in <x>$, $v \in <y>$, $w \in <z>$ and $\alpha, \beta \in \Gamma$. Hence $\nu\Gamma\xi\Gamma\eta(p) = 0$.
Otherwise $\nu\Gamma\xi\Gamma\eta(p) = \sup_{p=u\alpha v\beta w, u\in<x>, v\in<y>, w\in<z>, \alpha,\beta\in\Gamma}\{\min(\nu(u), \xi(v), \eta(w))\} = \min(r, s, t)$.
Since T is commutative, $u \in <x>$ implies $u = x\gamma b\delta c$ for some $b$, $c \in$ T, $\gamma, \delta \in \Gamma$. Similarly $v \in <y>$ implies $v = y\theta d\varepsilon e$ for some $d, e \in$ T, $\theta, \varepsilon \in \Gamma$ and $w \in <z>$ implies $w = z\zeta f\lambda g$ for some $f, g \in$ T and $\zeta, \lambda \in \Gamma$. So by commutatively again,
$u\alpha v\beta w = (x\gamma b\delta c)\alpha(y\theta d\varepsilon e)\beta(z\zeta f\lambda g) = x\gamma b\delta c\alpha y\theta d\varepsilon e\beta z\zeta f\lambda g$
$= x\gamma y\delta z\alpha b\theta c\delta d\beta e\zeta f\lambda g = x\gamma y\delta z\alpha d$ for some $d \in$ T.
Since $\mu$ is a fuzzy TΓ-ideal and hence $\mu(u\alpha v\beta w) \geq \mu(x\gamma y\delta z) \geq \min(r, s, t)$ by (1). Thus $\nu\Gamma\xi\Gamma\eta \leq \mu$. From the definition of $\nu, \xi, \eta$ it is easily shown that , $\xi$ and $\eta$ are fuzzy TΓ-ideals of Q. Since $\mu$ is a fuzzy prime TΓ-ideal, it follows that either $\nu \leq \mu$ or $\xi \leq \mu$ or $\eta \leq \mu$. So either $\nu(x) \leq \mu(x)$ or $\xi(x) \leq \mu(x)$ or $\eta(x) \leq \mu(x)$. Thus either $x_r \in \mu$ or $y_s \in \mu$ or $z_t \in \mu$.

**Def 3.18:** A non-empty fuzzy subset $\tau$ of a TΓ-semiring W is said to be a ***fuzzy m-system*** provided for any fuzzy points $x_p, y_q, z_r \in \tau \ni$ $W'\Gamma W'\,\Gamma x_p\,\Gamma W'\Gamma W'\,\Gamma W'\Gamma W'\,\Gamma z_r\Gamma W'\Gamma W'\,\Gamma y_q\,\Gamma W'\Gamma W' \wedge \tau \neq \emptyset$.

**Th 3.19: A fuzzy TΓ-ideal $\tau$ of a TΓ-semiring W is a fuzzy prime of W iff $\tau'$ is a fuzzy *m*-system of W.**
***Proof***: Suppose that τ is a fuzzy prime of a TΓ-semiring W and $\tau' \neq \emptyset$.
Let $x_p, y_q, z_r \in \tau'$. Then $x_p \notin \tau$, $y_q \notin \tau$ and $z_r \notin \tau$.
Suppose if possible $W'\Gamma W'\,\Gamma x_p\,\Gamma W'\Gamma W'\,\Gamma W'\Gamma W'\,\Gamma z_r\Gamma W'\Gamma W'\,\Gamma y_q\,\Gamma W'\Gamma W' \wedge \tau' = \emptyset$.
$W'\Gamma W'\,\Gamma x_p\,\Gamma W'\Gamma W'\,\Gamma W'\Gamma W'\,\Gamma z_r\Gamma W'\Gamma W'\,\Gamma y_q\,\Gamma W'\Gamma W' \wedge \tau' = \emptyset$.
$\Rightarrow W'\Gamma W'\,\Gamma x_p\,\Gamma W'\Gamma W'\,\Gamma W'\Gamma W'\,\Gamma z_r\Gamma W'\Gamma W'\,\Gamma y_q\,\Gamma W'\Gamma W' \subseteq \tau$.
Since $\tau$ is fuzzy prime, either $x_p \in \tau$ or $y_q \in \tau$ or $z_r \in \tau$.
It is a contradiction. Therefore
$W'\Gamma W'\,\Gamma x_p\,\Gamma W'\Gamma W'\,\Gamma W'\Gamma W'\,\Gamma z_r\Gamma W'\Gamma W'\,\Gamma y_q\,\Gamma W'\Gamma W' \wedge \tau' \neq \emptyset$.
Hence $\tau'$ is a fizzy *m*-system.
Conversely suppose that $\tau'$ is either a *m*-system of T or $\tau' = \emptyset$.
If $\tau' = \emptyset$, then T = $\tau$ and hence $\tau$ is a fuzzy prime of W.
Assume that $\tau'$ is a fuzzy *m*-system of Q. Let $x_p, y_q, z_r \in$ W and $< x_p > \Gamma < y_q > \Gamma < z_r > \subseteq \tau$.
Suppose if possible $x_p \notin \tau$, $y_q \notin \tau$ & $z_r \notin \tau$.
Then $x_p, y_q, z_r \in \tau'$. Sine $\tau'$ is a fuzzy *m*-system,
$\Rightarrow W'\Gamma W'\,\Gamma x_p\,\Gamma W'\Gamma W'\,\Gamma W'\Gamma W'\,\Gamma z_r\Gamma W'\Gamma W'\,\Gamma y_q\,\Gamma W'\Gamma W' \wedge \tau' \neq \emptyset. \Longrightarrow W'\Gamma W'\,\Gamma x_p\,\Gamma W'\Gamma W'\,\Gamma W'\Gamma W'\,\Gamma z_r\Gamma W'\Gamma W'\,\Gamma y_q\,\Gamma W'\Gamma W' \nsubseteq \tau \Longrightarrow < x_p > \Gamma < y_q > \Gamma < z_r > \nsubseteq \tau$. It is a contradiction.
Therefore $x_p \in \tau$ or $y_q \in \tau$ or $z_r \in \tau$.
Hence $\tau$ is a fuzzy prime of W.

## 4. Completely Semi prime Fuzzy TΓ-Ideals and Semi prime Fuzzy TΓ-Ideals:

**Def 4.1 :**A fuzzy TΓ-ideal ρ of a TΓ-semiring M is said to be ***fuzzy irreducible TΓ-ideal*** provided for any fuzzy TΓ-ideals $\nu, \xi, \eta$ of T, $\nu \wedge \xi \wedge \eta = \rho \Rightarrow \nu = \rho$ or $\xi = \rho$ or $\eta = \rho$.

**Th 4.2: Let *I* be a nonempty subset of a TΓ-semiring *M*. Then**
**(1) *I* is completely semi prime.**
**(2) The characteristic function $\xi_I$ of *I* is fuzzy completely semi prime are equivalent.**
***Proof*** :Suppose that I is completely semi prime. Let *u* be any element of *M*. If $u\gamma u\delta u \in$ I, then, since *I* is completely semi prime, we have $u \in I$. Thus $\xi_I(u) = 1 = \xi_I(u\gamma u\delta u)$. If $u\gamma u\delta u \notin I$, then we have $\xi_I(u) \geq 0 = \xi_I(u\gamma u\delta u)$.Therefore we have $\xi_I(u) \geq \xi_I(u\gamma u\delta u)$ for all $u \in M, \gamma, \delta \in \Gamma$ and $\xi_I$ is a completely semi prime fuzzy subset of *M*.

Conversely suppose that the characteristic function $\xi_I$ of I is a fuzzy completely semi prime. Let $u\gamma u\delta u \in I$, $u \in M$, $\gamma, \delta \in \Gamma$. Then, since $\xi_I$ is fuzzy completely semi prime, we have $\xi_I(u) \geq \xi_I(u\gamma u\delta u) \geq 1$. Since $\xi_I$ is a fuzzy subset of *M* and $\xi_I(u) \leq 1$ for any $u \in M$, so we have $\xi_I(u) = 1$, which implies that $u \in I$. It thus follows that *I* is completely semi prime.

**Th 4.3: Let $\mu$ be any fuzzy TΓ-ideal of a TΓ-semiring *M*. Then**
**(1) $\xi$ is fuzzy completely semi prime.**
**(2) $\xi(u) = \xi(u\gamma u\delta u)$ for all $u \in$ M, $\gamma, \delta \in \Gamma$.**
**(3) $\xi(u) = \xi[(u\gamma)^{n-1} u]$ for all $u \in$ M, $\gamma \in \Gamma$ and *n* is odd number are equivalent.**

**Th 4.4: Let $\xi$ be a fuzzy TΓ-ideal of a TΓ-semiring M. Then $\xi$ is completely semi prime iff for any fuzzy points $u_\lambda \in$M, ∀λ ∈(0, 1], $u_\lambda o u_\lambda o u_\lambda \leq \xi$ implies $u_\lambda \in \xi$.**

***Proof*** **:** Let $\xi$ be a fuzzy TΓ-ideal of a Γ-semiring $M$ and $u \in$ M. Then $\xi(u) \geq \xi(u\gamma u\delta u)$.
Since $u_\lambda o u_\lambda o u_\lambda = (u\gamma u\delta u)_\lambda$. If $u_\lambda o u_\lambda o u_\lambda \leq \xi \Rightarrow (u\gamma u\delta u)_\lambda \in \xi$, $\lambda \in (0, 1]$. Then $\xi(u\gamma u\delta u) \geq \lambda$, and so $\xi(u) \geq \lambda$, which implies $u_\lambda \in \xi$. Therefore, $u_\lambda o u_\lambda o u_\lambda \leq \xi$ implies $u_\lambda \in \xi$.

Conversely, let $u$ be any element of $M$. Put $\lambda = \xi(u\gamma u\delta u)$. If $\lambda \in (0, 1]$, since $u_\lambda o u_\lambda o u_\lambda \in \xi$, then, by hypothesis, we have $u_\lambda \in \xi$.Which implies $\xi(a) \geq \lambda = \xi(u\gamma u\delta u)$.

**Th 4.5: If $\xi$ is a completely semi prime fuzzy TΓ-ideal of a TΓ-semi ring $M$, then $\xi(a\gamma b\delta c) = \xi(b\delta c\gamma a) = \xi(c\gamma a\delta b)$ for all $a, b, c \in M$ and $\gamma, \delta \in \Gamma$.**
***Proof :*** Suppose that $\xi$ is a completely semi prime fuzzy TΓ-ideal of a TΓ-semiring $M$.
For all $a, b, c \in$ T, $\gamma, \delta \in \Gamma$, by Th 4.3, we have
$\xi(a\gamma b\delta c) = \xi[(a\gamma b\delta c)\gamma(a\gamma b\delta c)\gamma(a\gamma b\delta c)]$
$= \xi(a\gamma b\delta c\gamma a\gamma b\delta c\gamma a\gamma b\delta c) \geq \xi(b\delta c\gamma a)$.
Similarly, $\xi(b\delta c\gamma a) \geq \xi(a\gamma b\delta c)$.
It thus follows that $\xi(a\gamma b\delta c) = \xi(b\delta c\gamma a)$.
Similarly, prove the remaining part.

**Th 4.6: Every completely prime of a TΓ-semiring Q is a completely semi prime fuzzy TΓ-ideal of Q.**

**Def 4.7:** Suppose M be a TΓ-semiring. A fuzzy subset $\pi$ of M is known as a ***fuzzy d-system*** of T if for each $w_t \in \pi$ there exists an element $\alpha, \beta \in \Gamma$ such that $w_t \alpha w_t \beta w_t \in \pi$.

**Th 4.8: A fuzzy TΓ-ideal $\varsigma$ of a TΓ-semiring H is completely semi prime fuzzy TΓ-ideal iff its complement $\varsigma' = 1 - \varsigma$ is a fuzzy *d*-system.**

***Proof***: Suppose that $\varsigma$ is a completely semi prime fuzzy TΓ-ideal of H. Let the fuzzy point $u_p \in \varsigma'$. Then $u_p \notin \varsigma$. Suppose if possible there exists no $\alpha, \beta \in \Gamma$ such that $u_p \alpha u_p \beta u_p \in \varsigma'$. Then $u_p \alpha u_p \beta u_p \in \varsigma$. Since $\varsigma$ is completely semi prime fuzzy TΓ-ideal of H and hence $u_p \in \varsigma$. It is a contradiction. So, $u_p o u_p o u_p \in \varsigma'$. Therefore $\varsigma'$ is a fuzzy *d*-system of H.

Conversely, let $\varsigma'$ is a fuzzy *d*-system of H. Let $u_p \in$ M and $u_p \alpha u_p \beta u_p \in \varsigma$. Suppose if possible the fuzzy point $u_p \notin \varsigma$. Then $u_p \in \varsigma'$. Since $\varsigma'$ is a fuzzy *d*-system then there exist $\alpha, \beta \in \Gamma$ such that $u_p \alpha u_p \beta u_p \in \varsigma'$. Thus $u_p \alpha u_p \beta u_p \notin \varsigma$. It is a contradiction. Hence, $u_p \in \varsigma$. Therefore $\varsigma$ is a completely semi prime fuzzy TΓ-ideal of H.

**Def 4.9:** A fuzzy TΓ-ideal π of a TΓ-semiring Q is known as ***semi prime fuzzy*** provided for any fuzzy TΓ-ideals $\nu$ of T, $\nu o \nu o \nu \leq \pi \Rightarrow \nu \leq \pi$ **.**

**Th 4.10: If P be a TΓ-semiring and $\tau$ a fuzzy subset of P. Then $\tau$ is semi prime iff $\tau(u) \geq \tau(u\gamma u\delta u)$.**

**Lemma 4.11:** A fuzzy TΓ-ideal ξ of a TΓ-semiring P is ***semi prime fuzzy*** if $\xi(u) \geq \inf_{\gamma,\delta\in\Gamma} \xi(u\gamma u\delta u)$ .

**Th 4.12: For any non-empty fuzzy subset $\pi$ of a TΓ-semiring M, then**

**(i) $\pi$ is a fuzzy semi prime ,**

**(ii)** $\pi(a) = \inf_{\gamma,\delta\in\Gamma} \pi(a\gamma a\delta a) \forall a \in M$ **are equivalent**.

**Th 4.13 : Let M be a commutative TΓ-semiring and $\mu$ be fuzzy TΓ-ideal of T. Then the following are equivalent:**

**(i)** $x_\alpha \Gamma x_\alpha \Gamma x_\alpha \subseteq \mu \Rightarrow x_\alpha \subseteq \mu$ **where $x_\alpha$ is fuzzy point of M.**

**(ii) $\mu$ is a semi prime fuzzy TΓ-ideal of T.**

**(iii)** $\sigma\Gamma\sigma\Gamma\sigma \subseteq \mu \Rightarrow \sigma \subseteq \mu$ **.**

***Proof*** **:** (i) ⇒ (ii): Let $\sigma\Gamma\sigma\Gamma\sigma \subseteq \mu$ and $\sigma \not\subseteq \mu$. Then ∃ $x \in$ T such that $\sigma(x) > \mu(x)$. Let $\sigma(x) = \alpha$. By (i) $x_\alpha \Gamma x_\alpha \Gamma x_\alpha \subseteq \mu \Rightarrow x_\alpha \subseteq \mu$. This shows that $x_\alpha(x) \subseteq \mu(x)$

$\Rightarrow \alpha = \sigma(x) > \mu(x)$. This is a contradiction.

(ii) ⇒ (iii): Trivial.

(iii) ⇒ (i): Let $x_\alpha \Gamma x_\alpha \Gamma x_\alpha \subseteq \mu$ , where $x_\alpha$ is fuzzy point of T. Assuming that $x_\alpha = \sigma$ is a fuzzy TΓ-ideal of T such that $\sigma(y) = 0$ for all $y \in$ T\{$x$} and $\sigma(x) = \beta$. $\sigma\Gamma\sigma\Gamma\sigma \subseteq \mu$. Then it can be said that $x_\alpha \Gamma x_\alpha \Gamma x_\alpha \subseteq \mu$ , since $\sigma \subseteq \mu$ , $\sigma$ can be obtain as $\sigma = x_\alpha \subseteq \mu$ .

**Th 4.14: Let B(≠∅)⊆H where H is aTΓ -semiring . Then**
**1) B is semiprime**
**2) The characteristic function $\mu_B$ of I is fuzzy semi prime are equivalent.**

***Proof*** **:** (i) ⇒ (ii) **:** Let B be a semiprime of T and $\mu_B$ be the characteristic function of I. Since B ≠ ∅, $\mu_B$ is non-empty. Let $l \in$ H. Suppose $l\,\Gamma\, l\Gamma l \subseteq$ B. Then $\mu_B(l\gamma l\delta l) = 1$ for $\gamma, \delta \in \Gamma$. Hence $\inf_{\gamma,\delta\in\Gamma} \mu_B(l\gamma l\delta l) = 1$. Now B being semiprime, we get $l \in B$. Hence $\mu_B(l) = 1$.
Thus we see that $\inf_{\gamma,\delta\in\Gamma} \mu_B(l\gamma l\delta l) \geq \max \mu_B(l)$ .
Now suppose that $l\,\Gamma\, l\Gamma l \not\subseteq$ B. Then for $\gamma, \delta \in \Gamma$, $l\gamma l\delta l \notin B$ which means that $\mu_B(l\gamma l\delta l) = 0$. Consequently, $\inf_{\gamma,\delta\in\Gamma} \mu_B(l\gamma l\delta l) = 0$. Now since B is a semiprime of H, $l \notin$ B. Hence $\mu_B(l) = 0$ . Consequently, $\max \mu_B(l) = 0$. Thus we see that in this case also $\inf_{\gamma,\delta\in\Gamma} \mu_B(l\gamma l\delta l) = \max \mu_B(l)$ .
(ii) ⟹ (i) : Let $\mu_B$ be a fuzzy semiprime of H. Then $\mu_B$ is a TΓ-ideal of H. So, B is a TΓ-ideal of H. Let $l \in$ H ∋ $l\Gamma l\Gamma l \subseteq$ B. Then $\mu_B(l\gamma l\delta l) = 1$ for $\gamma, \delta \in \Gamma$ .Hence $\inf_{\gamma,\delta\in\Gamma} \mu_B(l\gamma l\delta l) = 1$ . Let $l \notin$ B. Then $\mu_B(l) = 0$ . Which means $\max \mu(l) = 0$. This implies that $\inf_{\gamma,\delta\in\Gamma} \mu_B(l\gamma l\delta l) = 0$ . hus we get a contradiction. Hence $l \in$ B. Thus we see that I is a semiprime TΓ-ideal of H.

**Th 4.15: Let T be a commutative TΓ-semiring. A fuzzy TΓ-ideal $\mu$ of T is fuzzy C-semi prime if and only if fuzzy semi prime of T.**
***Proof***: Suppose that $\mu$ is a completely semi prime fuzzy TΓ-ideal of T. Then, $\mu$ is a semi prime fuzzy TΓ-ideal of T.

Conversely, suppose that $\mu$ is a semi prime fuzzy TΓ-ideal of T. Let $a_p$ is a fuzzy point of T and $a_p \Gamma a_p \Gamma a_p \leq \mu$. Then $a_p \Gamma a_p(x\gamma y \delta z) \leq \mu(x\gamma y \delta z)$.
Hence min(p) = $p \leq \mu(x\gamma y \delta z)$ → (1). Let fuzzy subset $\nu$ be defined by

$$\nu(z) = \begin{cases} r & \text{if } z \in \langle a \rangle, \text{ where } \langle a \rangle \text{ is the T}\Gamma\text{-ideal generated by } a \\ 0 & \text{otherwise} \end{cases}$$

Clearly if $z$ is not expressible in the form $z = u\alpha v\beta w$ for some $u, v, w \in \langle a \rangle$ and $\alpha, \beta \in \Gamma$. Hence $\nu o \nu o \nu(z) = 0$. Otherwise, $\nu o \nu o \nu(z) = \mathop{Sup}_{z=u\alpha v\beta w \text{ and } u,v,w\in\langle a\rangle,\alpha,\beta\in\Gamma}\{\min(\nu(u),\nu(v),\nu(w))\} = \min(p) = p$. Since T is commutative, $u, v, w \in \langle a \rangle$ implies $u = a\gamma b\delta c$, $v = a\varepsilon d\zeta e$ and $w = a\eta f\theta g$ for some $b, c, d, e, f, g \in$ T, $\gamma, \delta, \varepsilon, \zeta, \eta, \theta \in \Gamma$. So by commutatively again, $u\alpha v\beta w$ = $(a\gamma b\delta c)\,\alpha\,(a\varepsilon d\zeta e)\beta(a\eta f\theta g)$ = $a\gamma b\delta c\alpha a\varepsilon d\zeta e\beta a\eta f\theta g = a\gamma a\delta a\alpha b\varepsilon c\zeta d\eta e\theta f\beta g = a\gamma a\delta a\alpha d$ for some $d \in$ T. Since $\mu$ is a fuzzy TΓ-ideal and hence $\mu(u\alpha v\beta w) \geq \mu(a\gamma a\delta a) \geq$ min(p) = p by (1). Thus, $\nu\Gamma\nu\,\Gamma\nu \leq \mu$. From the definition of $\nu$ it is easily shown that $\nu$ is fuzzy TΓ-ideal of T.

Since $\mu$ is a fuzzy semi prime TΓ-ideal, it follows that $\nu \leq \mu$. So $a_p \in \mu$.

**Th 4.16: A fuzzy TΓ-ideal of a TΓ-semiring H is fuzzy prime iff it is fuzzy semi prime and fuzzy irreducible.**

**Def 4.17:** A TΓ-semiring H is known as ***fully fuzzy prime*** if each of its fuzzy TΓ-ideal is prime.

**Def 4.18:** A TΓ-semiring H is known as ***fully fuzzy semiprime*** if each of its fuzzy TΓ-ideal is semiprime.

**Def 4.19:** A TΓ-semiring H is known as ***right weakly regular*** provided for each $l \in H$, $l \in l\Gamma H\Gamma l\Gamma H\Gamma l\Gamma H$.

**Th 4.20: Let H be a TΓ-semiring. A fuzzy semiprime irreducible right TΓ-ideal of T is a fuzzy prime right TΓ-ideal.**

## 5. Conclusion

Mainly we investigate completely prime and prime fuzzy TΓ-ideals in TΓ-semirings.

## References


[1] T. K. Dutta and Chanda. T., *Structres of Fuzzy Ideals of Γ-Ring*; Bull. Malays. Math. Sci. Soc. (2) 28(1) (2005), 9-18.

[2] Y. B. Jun, J. Neggers, H. S. Kim,On L-fuzzy ideals in semir ingd I, Czechoslovak Math. Journal, 48 (123) (1998) 669-675.

[3] J. Kavikumar, Azme Bin Khamis, Fuzzy ideals and Fuzzy Quasi-ideals of Ternary Semirings, IAENG International Journalof Applied Mathematics, 37: 2 (2007) 102-106.

[4] C. B. Kim, Mi-Ae Park,k-Fuzzy ideals in Semirings, Fuzzy Sets and Systems 81 (1996) 281-286.

[5] W. Liu, Fuzzy invariant subgroups and Fuzzy ideals, Fuzzy Sets and Systems, 8 (1982) 133-139.

[6] M. Sajani Lavanya, Dr. D. Madhusudhana Rao, and V. Syam Julius Rajendra; *On Lateral Ternary Γ-Ideals of Ternary Γ-Semirings*-American International Journal of Research in Science, Technology, Engineering &Mathematics (AIJRSTEM), 12(1), September-November, 2015, pp: 11-14.

[7] M. Sajani Lavanya, Dr. D. Madhusudhana Rao, and Prof. K. Panduranga Rao; *On simple Ternary* Γ*-Smiring*, International Journal of Engineering Research and Application, Vol. 7, Issue 2, (Part-5) February 2017, pp: 01-06.

[8] M. Sajani Lavanya, Dr. D. Madhusudhana Rao, and VB Subrahmanyeswara rao Seetam Raju; *On Right Ternary* Γ*-Ideals of Ternary* Γ*-Semiring*, International Journal of Research in Applied Natural and Social Sciences, Vol. 4, Issue 5, May 2016, 107-114.

[9] K. Syam Prasad and Bh. Satyanarayana., *Fuzzy Prime Ideal of a Gamma Near-Ring*; Soochow Journal of Mathematics, Volume 31, No. 1, January 2005, pp: 121-129.

[10] L. A. Zadeh, Fuzzy Sets, Information and Control, 8 (3)(1965) 338-353.