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Research paper

Weibull Parameter Estimation Using Particle Swarm Optimization Algorithm

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Abstract

Many research works on Weibull parameter estimation has focused on graphical or analytical techniques, with little effort devoted towards the use of population based optimization algorithm. Accurate estimation of failure distributive parameter such as Weibull is a key requirement for efficient reliability analysis. In this study Particle Swarm Optimization Algorithm (PSOA), with particle position and velocity iteratively updated was used to estimate Weibull parameters. Probability density function and reliability plots were generated using the results obtained. Generally, PSOA shows better parameter estimation in comparison with analytical method based on Maximum Likelihood Estimator (MLE).

Keywords: *Algorithm; Parameter estimation; Particle Swarm Optimization; Reliability; Weibull distribution*

1. Introduction

Reliability is defined as the probability that a product performs its intended function without failure under specified conditions for a specified period of time. This important characteristic of a system must be considered during every engineering system design program, especially for complex or safety critical systems [1]. The ability to accurately predict, the reliability of a component, subsystem or system is mainly dependent on the identification, selection and implementation of such component, subsystem and system appropriate failure distribution. Although, exponential and Weibull distribution are the most commonly used failure distribution in reliability analysis. In many applications, Weibull is the best choice for modeling not only the life but also the product's properties, such as the performance characteristics [2]. This choice is hinged towards Weibull's distribution capability of modeling other distributions such as exponential and Rayleigh. In this paper, Weibull distribution is considered. The probability density ($f(t)$) and reliability function ($R(t)$) based on Weibull distribution is given in (1) and (2).

$$f(t) = \frac{\beta}{\eta}\left(\frac{t_i}{\eta}\right)^{\beta-1} exp^{\left[-\left(\frac{t}{\eta}\right)\right]^{\beta}} \quad (1)$$

$$R(t) = exp^{\left[-\left(\frac{t}{\eta}\right)\right]^{\beta}} \quad (2)$$

where $\boldsymbol{\beta}$ ***and*** $\boldsymbol{\eta}$ are the weibull shape and characteristic parameters respectively. Weibull parameter estimation has been studied extensively and quite a number of techniques have been generated. Al-fawzan [3] classified the parameter estimation techniques into two: graphical and analytical methods. The common graphical methods used are the Weibull probability and hazard plotting techniques. The frequent use of the graphical method is mainly associated with their simplicity and speed. However, this technique involves a great probability of error. Analytical methods such Maximum likelihood estimator (MLE), method of moments (MOM) and Least Squares Method (LSM) are preferred because of lower probability of estimation error. Numerical methods such as Newton-Raphson methods and Monte-Carlo simulation are commonly implemented to solve the complex models generated by these analytical methods such as MLE, MOM and LSM [2, 3]. In order to accurately predict the failure cumulative distributive function or reliability index, it is crucial to carry out parameter estimation based on experimental demonstration or field data. Recent studies have focused more on the use of artificial intelligence based techniques such as Genetic algorithm and Particle swarm optimization Algorithm (PSOA) for parameter estimation. Ref [5] implemented PSOA in their maintainability parameter estimation study. This paper extend our past work on maintainability parameter estimation based on exponential and log normal distribution [5], to reliability analysis using Weibull distribution. PSOA is a population based stochastic optimization technique inspired by social behavior of bird flocking or fish schooling [6]. PSOA possesses many similarities with evolutionary computation techniques such as Genetic Algorithms (GA). For instance, it is initialized with a population of random solutions and searches for optimal solution is derived by updating generations. However, unlike GA, PSOA has no evolution operators such as crossover and mutation. So the implementation of PSOA is more convenient in comparison to GA and some other evolutionary computation techniques [7]. In this study PSOA, was used to estimate Weibull parameters. Yang and Nie implemented an advanced numerical algorithm, used for the maximum likelihood estimation of three parameter Weibull distribution. The algorithm was found to be stable and does not require initial value input [8]. Zhao and Jinxian presented a new algorithm for calculating the maximum value of the likelihood functions for maximum likelihood estimation. The algorithm was found to generate a more precise result in comparison to common maximum likelihood method [9]. Least square estimation is also a commonly used method in the field of parameter estimation; Ref [10] implemented a novel least-squares procedure for estimating the shape parameter of Weibull distribution and the simulation results show that their approach is better than common least-squares estimation. Ref [11] proposed an alternative procedure of unknown parameter estimation; their method mainly involves rough parameter estimation and the treatment of errors. Generally, from the past works reviewed above, linear transformation some distribution, usually increases computational complexity. Techniques such as PSOA which does not require such linear transformation will be computationally less tasking. The assessment of Weibull parameter estimation efficiency using PSOA in comparison with analytical method such as MLE is the main focus of the paper.

2. Maximum Likelihood Estimation Implementation

Adopting the two parameter Weibull function given in (2), the likelihood function of as stated by ref [12] was reached as shown in (3) and (4).

$$L(t,\eta,\beta) = \prod_{i=1}^{d} f(t_i) \tag{3}$$

$$L(t,\eta,\beta) = \prod_{i=1}^{d} \frac{\beta}{\eta}\left(\frac{t_i}{\eta}\right)^{\beta-1} exp^{\left[-\left(\frac{t_i}{\eta}\right)\right]^{\beta}} \tag{4}$$

The log likelihood is shown in (5).

$$LnL = dln(\beta) - d\beta\ln(\eta) + (\beta-1)\sum_{i=1}^{d}\ln(t_i) - \sum_{i=1}^{d}\left(\frac{t_i}{\eta}\right)^{\beta} \tag{5}$$

Differentiating (5) with respect to α and β and equating to zero, will result to (6) and (7).

$$\eta = \left[\frac{1}{d}\sum_{i=1}^{d} {t_i}^{\beta}\right]^{1/\beta} \tag{6}$$

$$\frac{d}{\beta} + \sum_{i=1}^{d}\ln\left(\frac{t_i}{\eta}\right) - \sum_{i=1}^{d}\left(\frac{t_i}{\eta}\right)^{\beta}\ln\left(\frac{t_i}{\eta}\right) = 0 \tag{7}$$

Substituting (6) into (7) gives (8).

$$\frac{d}{\beta} + \sum_{i=1}^{d}\ln\left(\frac{t_i}{\left[\frac{1}{d}\sum_{i=1}^{d}{t_i}^{\beta}\right]^{1/\beta}}\right) - \sum_{i=1}^{d}\left(\frac{t_i}{\left[\frac{1}{d}\sum_{i=1}^{d}{t_i}^{\beta}\right]^{1/\beta}}\right)^{\beta}\ln\left(\frac{t_i}{\left[\frac{1}{d}\sum_{i=1}^{d}{t_i}^{\beta}\right]^{1/\beta}}\right) = 0 \tag{8}$$

Let $f(\beta)$ represent (8) above, then taking the first differential of $f(\beta)$, we have (9).

$$f^1(\beta) = -\frac{d}{\beta^2} - \sum_{i=1}^{d}\left(\frac{t_i}{\left[\frac{1}{d}\sum_{i=1}^{d}{t_i}^{\beta}\right]^{1/\beta}}\right)^{\beta}\ln^2\left(\frac{t_i}{\left[\frac{1}{d}\sum_{i=1}^{d}{t_i}^{\beta}\right]^{1/\beta}}\right) = 0 \tag{9}$$

Implementing Newton-Raphson iterative procedure, β is estimated by assuming an initial value and solving (10) repeatedly till converges, after which α can be determined using (6).

$$\beta_{i+1} = \beta_i - \left\{\frac{\frac{d}{\beta}+\sum_{i=1}^{d}\ln\left(\frac{t_i}{\left[\frac{1}{d}\sum_{i=1}^{d}{t_i}^{\beta}\right]^{1/\beta}}\right)-\sum_{i=1}^{d}\left(\frac{t_i}{\left[\frac{1}{d}\sum_{i=1}^{d}{t_i}^{\beta}\right]^{1/\beta}}\right)^{\beta}\ln\left(\frac{t_i}{\left[\frac{1}{d}\sum_{i=1}^{d}{t_i}^{\beta}\right]^{1/\beta}}\right)}{-\frac{d}{\beta^2}-\sum_{i=1}^{d}\left(\frac{t_i}{\left[\frac{1}{d}\sum_{i=1}^{d}{t_i}^{\beta}\right]^{1/\beta}}\right)^{\beta}\ln^2\left(\frac{t_i}{\left[\frac{1}{d}\sum_{i=1}^{d}{t_i}^{\beta}\right]^{1/\beta}}\right)}\right\} \tag{10}$$

3. PSOA Implementation

PSOA just like GA is a population based algorithm. It population and individuals are called Swarm and particles respectively [6]. Each particle keeps track of it coordinates in the problem space which are usually associated with the best solution obtained so far. This best solution position (value) is termed "$\boldsymbol{pb}$". PSOA also uses another best value termed *"gb"* which represents the global best value in comparison with any particles. Generally, PSOA involves changing the velocity of particle towards its *"pb"* and *"gb"* values. The velocity changing rate is weighted by a random number, with different random number generated for the velocity computation as shown in (15). Figure 1 shows PSOA implementation flowchart.

3.1. PSOA Distribution Parameter Coding

As stated earlier, in PSOA each particle has both position and velocity. The particle position represents the parameter (s) to be estimated. The particle velocity and position vector are shown in (12) and (13).

$$V_j = (v_i)\forall\ i = 1,2,\dots,p \tag{11}$$

$$P_j = (\vartheta_i)\ \forall\ i = 1,2,\dots,p \tag{12}$$

where ϑ_i and v_i are the values of the ith parameter and velocity respectively.

3.2. PSOA Determination of the Population Initial Member

Prior to initial population member determination, it is essential to determine the coding space for each particles. Confidence interval of each parameter based on given confidence contains the optimium value of the parameter, so it can be taken as coding space of each parameter. In this work $1-\alpha$ confidence interval was used. Where $\alpha = 0.1$. Implementing the same approach as described by ref [5], the initial population member value ϑ_i is reached as expressed in (13)

$$\vartheta_i = \vartheta_{il} + (\vartheta_{iu} - \vartheta_{il})xrand \tag{13}$$

where $rand$ is a random number of uniform distribution [0,1]. Similarly, the initial velocity v_i was obtained from the (14).

$$v_i = (\vartheta_{il} - \vartheta_{iu}) + 2(\vartheta_{iu} - \vartheta_{il})\ x\ rand \tag{14}$$

3.3. Iteration Update

For each iteration, the velocity, position, "pb" and "gb" are updated. The velocity of the jth particle in the $k+1$ iteration is updated by (15).

$$V_j^{k+1} = w^k x V_j^k + c_1 x randx(pb_j - P_j^k) + c_2 x randx\left(gb_j - P_j^k\right) \tag{15}$$

where k denotes the kth iteration. c_1 and c_2 are constants and their values were set to 2 in this study, w is the weight of the velocity of last iteration, which is computed using (16)

$$w^k = .9999^k \tag{16}$$

The velocity of the jth particle in the $(k+1)th$ iteration is updated by (17).

$$P_j^{k+1} = V_j^{k+1} + P_j^k \tag{17}$$

Given that $F_\vartheta(t)$ is the cumulative distribution function such as Weibull, exponential or lognormal and $\vartheta = (\vartheta_1, \vartheta_2, \vartheta_3, \dots, \vartheta_s)$ is the set of distribution parameters, and s is the number of parameter. For two parameter weibull, $\vartheta = (\beta, \eta)$. let t_i for $i = 1,2, \dots, d$ denote the ordered random reliability time samples. The update of "pb" and "gb" is according to the fitness function. The fitness function was computed using (18).

$$fit(\bar{\vartheta}) = 1 \Big/ \sum_{i=1}^{d} \left(\bar{F}_\vartheta(t_i) - F_{\bar{\vartheta}}(t_i)\right)^2 \quad (18)$$

where $\bar{F}_\vartheta(t_i) = \frac{i}{d}$ and $F_{\bar{\vartheta}}(t_i)$ is the distribution function with parameter to be estimated. $\bar{\vartheta}$ is the estimator of ϑ. This non linear least-square method was implemented to avoid complexity associated with linearization of fitness function. For the jth particle in the $(k+1)th$ iteration, if $f(P_j^{k+1}) < f(pb_j^k)$, then P_j^{k+1} is pb_j^k, else pb_j^k is pb_j^k. The p_j which has the least $f(pb_j^k)$ in the population, is the "gb".

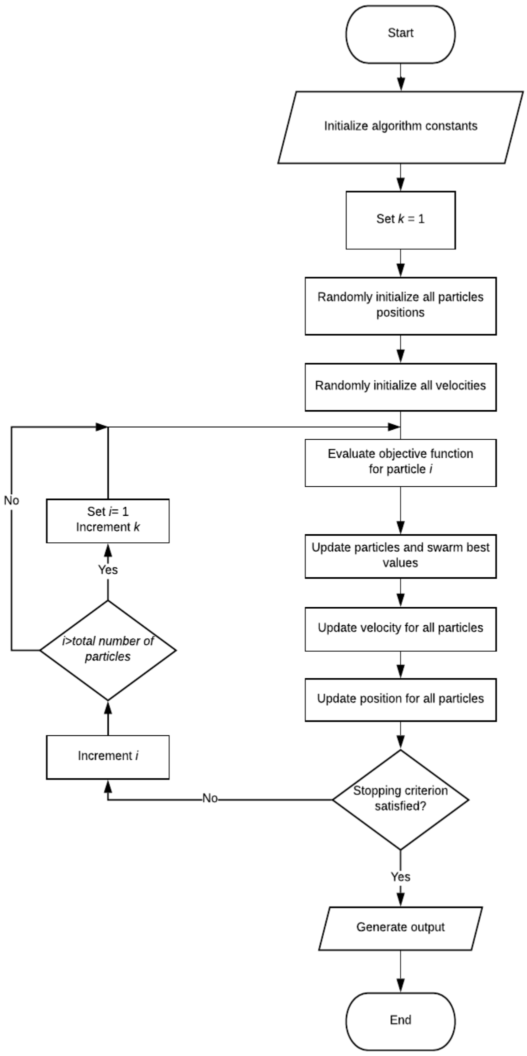

Fig. 1: PSOA implementation flowchart

3.4. Termination Condition of Iteration

The condition for iteration termination adopted in this work is given in eqn. (19).

$$f(gb_l) - f(gb_{l-1}) < \delta \quad (19)$$

From (19), the difference between "gb" in the final iteration and that of the preceding iteration if less thanδ, satisfies the condition for termination. where l represent the last iteration.

4. Parameter Setting and Results

The most widely used distribution in reliability prediction of engineering system is the Weibull distribution. In this work Weibull distribution was used. The probability density and reliability function of Weibull distribution given. Twenty-five (25) randomly generated time series comprising of 1094, 3244, 5242, 5963, 6998, 7995, 8646, 9607, 10241, 11819, 12451, 15017, 16677, 18406 19485 20226, 20987, 22170, 24274, 26596, 29217, 32274, 36013, 40977 and 65048 were generated using $\boldsymbol{\beta = 1.5}$ and $\boldsymbol{\eta = 23500}$. Using the PSO implemented in this work and setting $\boldsymbol{\delta = 0.0001}$, $\boldsymbol{\beta}$ and $\boldsymbol{\eta}$ values were found to be 1.518 and 22578 respectively, while implementing MLE solved using Newton-Raphson method the value of $\boldsymbol{\beta}$ and $\boldsymbol{\eta}$ values were found to be 1.535 and 21307 respectively.

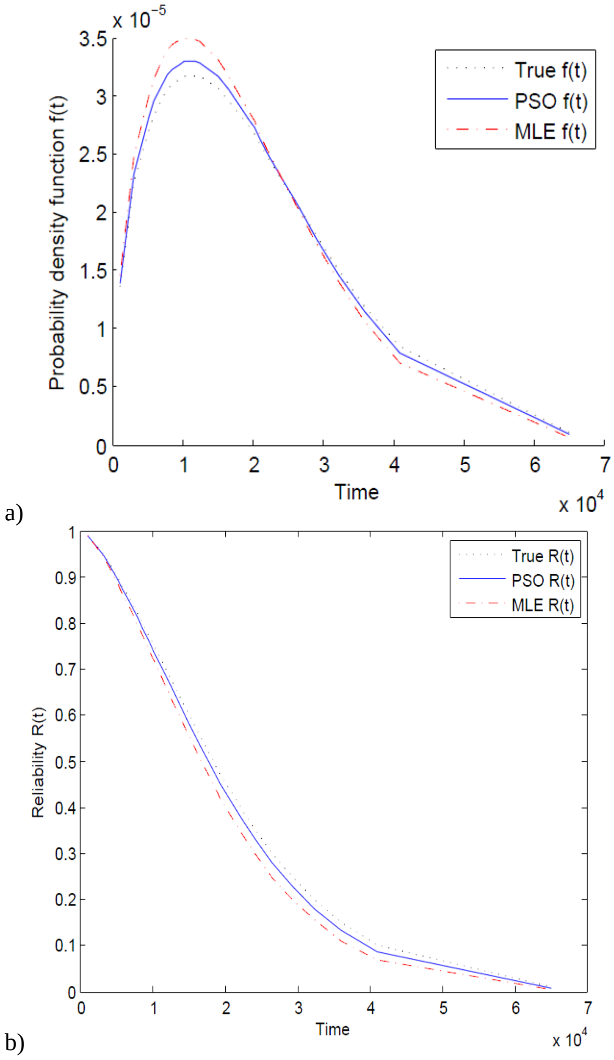

Fig. 2: Plot of probability density function (a) and reliability (b) against time

Figure 2 shows the true probability density curve and reliability curve in comparison with the PSO and MLE discussed in the paper. It is evident PSO performed better than MLE

5. Conclusion

In this work Particle Swarm Optimization (PSO) was used as a Weibull parameter estimation tool in comparison with numerical method (maximum likelihood estimator). Non linear least-squares method was adopted in order to avoid the linearization of the fitness function. The result reveals that the PSO technique outperformed the MLE.

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