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Research paper

Dynamics Model of Chickenpox with Effect of the Mass Media

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Abstract

In this paper, an SIR model (Susceptible – Infectious – Recovered) analyze the Chickenpox transmission by considering the effect of mass media to understand the dynamic of the disease in Thailand. This dynamical model is analyzed using a standard dynamical modeling method. The stability of the model was determined by using Routh-Hurwitz criteria. In this paper, the disease free and endemic state have been found. To determine the basic reproductive number (R_0) which is the threshold parameter, if R_0 <1, the disease free equilibrium point is locally asymptotically stable. The result shows that the mass media significantly cause the reduction in transmission and infection of Chickenpox in Thailand. So, the mass media may be another option to prevent and control the disease.

Keywords: *Mathematical model, Varicella, Mass media,Basic reproductive number*

1. Introduction

Varicella (Chickenpox) caused by the varicella zoster virus is a well-known disease that causes an itchy rash and red spots or blisters (pox) all over the body. This virus has been with human being for many centuries and it can wide-spread easily through the air by coughing, sneezing or sharing foods or drinks with. Signs of these problems can include: a lack of energy, drowsiness, confusion, seizures, vomiting, severe headaches, a stiff neck, behavioural changes problems with walking, balance or speech. Chickenpox usually isn't a serious health problem in healthy children but they need to be quarantined at home. However, it can be serious, even deadly, especially for babies, pregnant women, adolescents, adults, and people with weakened immune systems. In some cases, chickenpox can cause serious problems, such as Skin infections, Dehydration, Pneumonia, Swelling of the brain (WHO (1998), Gershon et al.(2004), Tunbridge et al.(2008) and Brisson et al.(2001)).
The mass media nowadays is a matter that associates with all people lives. It has been accessed from many sources; for example, Television, Internet, Radio, Billboard etc. When people easily to get information anywhere at any time, the media can play important role to form people personality and knowledge. Therefore, it is the effective tool to provide the right information about the Chickenpox in awareness and treatment to prevent widespread and deadly side-effects. There were several researches on investigation of the role of media to infectious diseases. Tchuenche et al.(2011) and Greenhalgh et al.(2015), the authors studied the effect of awareness programs by media on the spreading of infectious disease and they found that an awareness program has a significant effect on disease control.
The Mathematical model is very useful tool to undestand the spread of infectious diseases. The analytical solution, numerical solution and simulation are ones of various mathematical methods used to analyze the harmful diseases without researching in the real environment that may infect the researchers, Ghosh et al.(2006) and Yoo et al.(2010). Several researches mainly studied on the interactions between susceptible and infective but there are other factors, such as media coverage, vaccination or migration of population, which also affect the spread of infectious diseases.
In this paper, the new mathematical model is used to analyze the Chickenpox transmission by considering the effect of mass media to understand the dynamic of the disease in Thailand.

2. Materials and Methods

2.1 Mathematical model

As the SIR model, we consider effect of mass media the reduction of transmission level of this disease, then we separated susceptible class to susceptible class have no disease information and susceptible class have disease information**.** And both classes can be infective class**.** In this paper, applied the structure of transfer diagram of model system from Kaur**(**2014**)** to suitable with the Chickenpox in Thailand**.** The population assumed that the human population is constant and divided into five compartments as follows**:** S, I, R, S_M and M represent the number of susceptible class, infective class, recovered class, susceptible class acknowledge from content of media and the cumulative density of the mass media in that region at time t respectively**.** The transmission dynamics of the disease are described by the following systems of nonlinear ordinary differential equations**:**

$$\frac{dS}{dt} = \Lambda - \left[\beta\left(1-\delta_M\right)I + \beta_1 M + d\right]S \quad (1)$$

$$\frac{dS_M}{dt} = \beta_1 SM - \left[\beta\alpha_M\left(1-\delta_M\right)I + d\right]S_M \quad (2)$$

$$\frac{dI}{dt} = \left[\beta\left(1-\delta_M\right)\left(S+\alpha_M S_M\right) - \left(\gamma+\gamma_M+d\right)\right]I \quad (3)$$

$$\frac{dR}{dt} = \left(\gamma+\gamma_M\right)I - dR \quad (4)$$

$$\frac{dM}{dt} = \mu + \mu_1 I - \mu_0 M \quad (5).$$

Since the population of $N = S + S_M + I + R$; where Λ denote the recruitment rate of human; β the probability of Chickenpox transmit of susceptible, γ the recovery rate of human; μ_0 The natural decay rate constant of media coverage/mass media; μ the rate constant corresponding to regular mass media coverage; d the natural mortality rate; β_1 the dissemination rate of mass media among susceptibles due to mass media; γ_M the probability of recovery rate driven by mass media; μ_1 the rate constant influenced by number of infectives; δ_M the probability of infectives; α_M the probability of aware susceptible class interacts, respectively.

2.2 Equilibrium Analysis

As the SIR model, we consider effect of mass media the reduction. To begin with the disease free equilibrium, the endemic equilibrium and the basic reproductive number, respectively. The disease free equilibrium (DEF) : The system has two equilibrium points; a disease free equilibrium point and an endemic equilibrium point.

$$E_0(S,S_M,I,M) = E_0\left(\frac{\mu_0\Lambda}{\beta_1\mu + d\mu_0}, \frac{\beta_1\mu_0\Lambda\mu}{(\beta_1\mu + d\mu_0)\mu_0 d}, 0, \frac{\mu}{\mu_0}\right)$$

And the endemic equilibrium *:*

$$E_1\left(S^*, S_M^*, I^*, M^*\right) = E_1\left(\begin{array}{l}\dfrac{\mu_0\Lambda}{\left[\beta\mu_0(1-\delta_M)+\beta_1\mu_1\right]I^* + \beta_1\mu + d\mu_0}, \\ \dfrac{H_3I^* + H_4}{H_5I^{2*} + H_6I^* + \mu_0 dH_2}, I^*, \dfrac{\mu + \mu_1 I^*}{\mu_0}\end{array}\right)$$

So, the basic reproductive number obtained by the next generation matrix. By Van*(*2002*)* we start with

$\frac{dX}{dt} = F(x) - V(x)$ where , F is the matrix of new infectious and V is the matrix of the transfers between the compartments in the infective equations. We obtained

$$F(x) = \begin{bmatrix} 0 \\ 0 \\ \beta(1-\delta_M) IS + \alpha_M \beta(1-\delta_M) IS_M \\ 0 \end{bmatrix},$$

$$V(x) = \begin{bmatrix} \beta(1-\delta_M) I\, S + \beta_1 MS + dS - \Lambda \\ \beta\alpha_M (1-\delta_M) IS_M + dS_M - \beta_1 SM \\ (\gamma + \gamma_M + d) I \\ \mu_0 M - \mu - \mu_1 I \end{bmatrix}.$$

Hence, basic reproductive number $(\Re_0)$ is

$$\Re_0 = \sqrt{\frac{\Lambda\beta(1-\delta_M)(d\mu_0 + \alpha_M\beta_1\mu)}{d(\beta_1\mu + d\mu_0)(\gamma + \gamma_M + d)}}.$$

2.3 Stability Analysis

The local stability of an equilibrium points is determine from the jacobian matrix of the ordinary differential equation evaluated at E_0.

$$J_0 = \begin{bmatrix} -[\beta_1 M + d] & 0 & -\beta(1-\delta_M)S & -\beta_1 S \\ \beta_1 M & -d & -\beta\alpha_M(1-\delta_M)S_M & \beta_1 S \\ 0 & 0 & \beta(1-\delta_M)(S + \alpha_M S_M) - (\gamma + \gamma_M + d) & 0 \\ 0 & 0 & \mu_1 & -\mu_0 \end{bmatrix}.$$

The eigenvalues value of J_0 are obtained by solving $\det(J_0 - \lambda I)$. And we obtain the characteristic equation,

$$\left[\beta(1-\delta_M)(S + \alpha_M S_M) - (\gamma + \gamma_M + d) - \lambda\right]\left[-d - \lambda\right]$$
$$\left[-(\beta_1 M + d) - \lambda\right]\left[-\mu_0 - \lambda\right] = 0$$

where;

$$\lambda_1 = -(\gamma + \gamma_M + d) + \beta(1-\delta_M)(S + \alpha_M S_M),$$
$$\lambda_2 = -d,\ \lambda_3 = -(\beta_1 M + d),\ \lambda_4 = -\mu_0.$$

Next, we consider the stability of the endemic equilibrium points E_1, we examine the eigenvalue of Jacobian matrix at E_1, which is

$$J_1 = \begin{bmatrix} -\left[\beta(1-\delta_M)I^* + \beta_1 M^* + d\right] & 0 & -\beta(1-\delta_M)S^* & -\beta_1 S^* \\ \beta_1 M^* & -\left[\beta\alpha_M(1-\delta_M)I^* + d\right] & -\beta\alpha_M(1-\delta_M)S_M^* & \beta_1 S^* \\ \beta(1-\delta_M)I^* & \alpha_M\beta(1-\delta_M)I^* & \beta(1-\delta_M)(S^* + \alpha_M S_M^*) - (\gamma + \gamma_M + d) & 0 \\ 0 & 0 & \mu_1 & -\mu_0 \end{bmatrix}$$

The eigenvalues value of J_0 are obtained by solving $\det(J_0 - \lambda I)$. And we obtain the characteristic equation, $I^4 + C_1 I^3 + C_2 I^2 + C_3 I + C_4 = 0$

where;

$$C_1 = \mu_0 + B_1 B_2 + B_3, \qquad C_2 = (B_1 B_2 + B_3)\mu_0 - B_7,$$
$$C_3 = B_9 - \mu_0 B_7 - B_8, \quad C_4 = B_{10} - \mu_0 B_8$$

By Routh-Hurwitz criteria, equilibrium points are locally asymptotically stable if all conditions are satisfied**:**

1) $C_1 > 0,\ C_3 > 0,\ C_4 > 0$
2) $C_1 C_2 C_3 > C_3^2 + C_1^2 C_4$.

3. Results and Discussion

In this section, we present several scenarios using demonstrated data to validate the performance of the mathematical models using the set of estimated parameter values given in Table 1. The parameters were obtained from literatures of Kaur et al.**(**2014**)**, the parameters that were not available in literatures were estimated**.** The software used for computation is Maple**.** First, we consider stability of disease free state using set of parameter values of study the system of nonlinear ordinary differential equations **(**1-5**).** We found that, the eigenvalues corresponding to the equilibrium point E_0 and basic reproductive number are following**:** $\lambda_1 = -0.05958$ **,** $\lambda_2 = -0.00004$ **,** $\lambda_3 = -1.66671$ **,** $\lambda_4 = -0.03$, $R_0 = 0.40495$ **,** $\Re_0 = 0.2025 < 1$. Since all eigenvalues corresponding to E_0 be negative, thus E_0 is locally asymptotically stable and basic reproductive number less than 1. Further, to illustrate the stability of endemic free state, shown that the results : $\mathrm{Re}(\lambda_{1,2}) = -0.06602$, $\lambda_3 = -0.03$, $\lambda_4 = -0.00002$, $R_0 = 596$, $\Re_0 = 298 > 1$. The same, all eigenvalues to be negative and basic reproductive number is greater than 1, the equilibrium state will be the endemic state, E_1. The disease free equilibrium (E_0) will be local asymptotically stable, as follows;

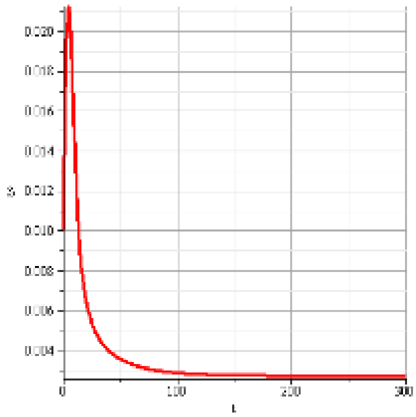

Figure 1:. Time series of susceptible class

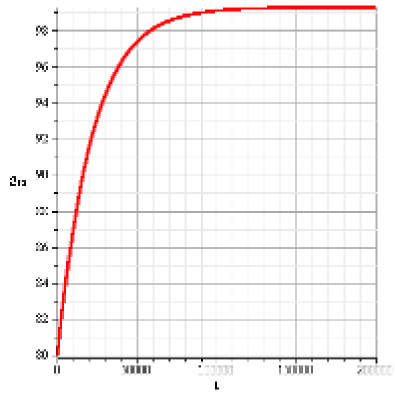

Figure 2:. Time series of susceptible class acknowledge from content of media

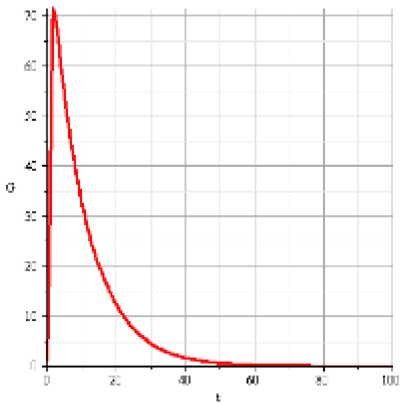

Figure 3:. Time series of infective class

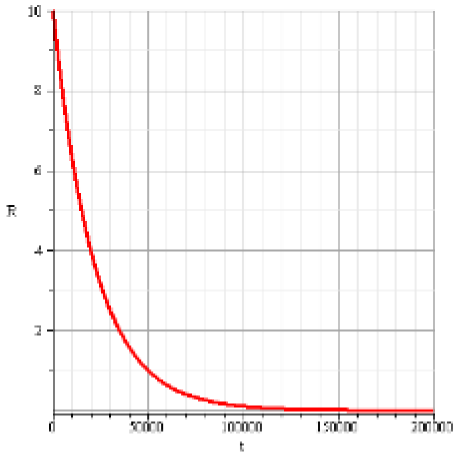

Figure 4:. Time series of recovered class

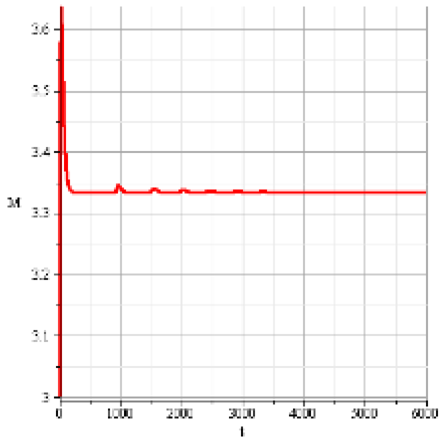

Figure 5:. Time series of the cumulative density of the mass media

Figure 1-5 Time series of susceptible class, susceptible class acknowledge from content of media, infective class, recovered class and the cumulative density of the mass media, respectively**.**

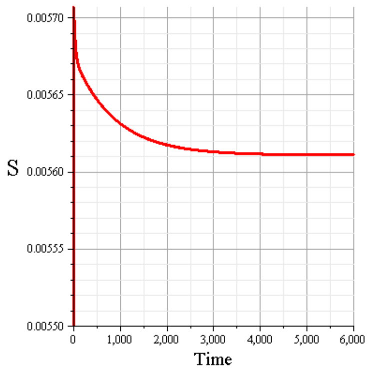

Figure 6. Time series of susceptible class

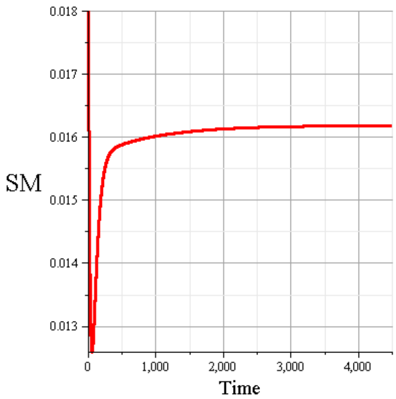

Figure 7:. Time series of susceptible class acknowledge from content of media

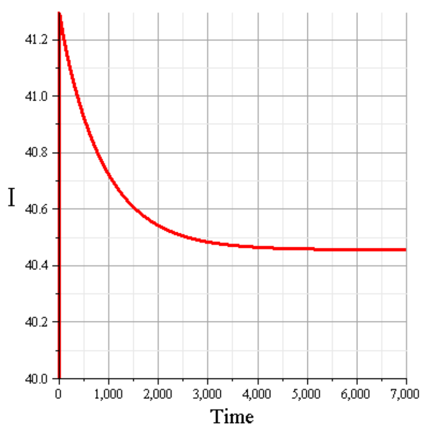

Figure 8:. Time series of infective class

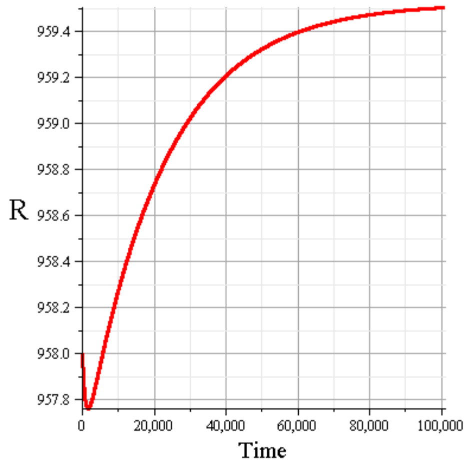

Figure 9:. Time series of recovered class

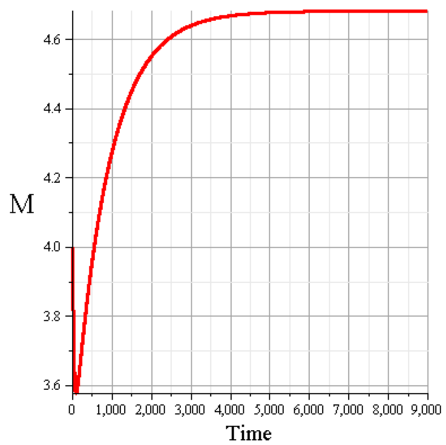

Figure 10:. Time series of the cumulative density of the mass media

From Fig 6-10, we found that the values of parameters are in the text and $R_0>1$. We see that the solutions converge to endemic free state.

4. Conclusion

The objective of this research is to analyze the spread of Chickenpox by considering effect of mass media to the reduction of transmission level of this disease. For numerical illustration we start from analytical solution, numerical solution and simulation. The result are summarized in Fig (2-3) by presenting the important values and the model system shown that the disease free equilibrium is stable until, the basic reproduction number, $R_0<1$. The disease free equilibrium becomes unstable for $R_0>1$, which leads to the existence of an endemic equilibrium, it clearly that the mass media significantly cause the reduction in transmission and infection of Chickenpox in Thailand.

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References

[1] WHO (1998) Varicella vaccines. WHO position paper. Wkly Epidemiol Rec 73: 241–248
[2] Gershon A., Steinberg S., Gelb L., Galasso G., Borkowsky W., LaRussa P., et al. Live attenuated varicella vaccine. Efficacy for children with leukemia in remission. JAMA (1984) 252: 355–362
[3] Tunbridge AJ et al. Chickenpox in adults - Clinical management. Journal of Infection (2008) 57:95e102
[4] Brisson M., Edmunds W., Law B., Gay N., Walld R., Brownell M., et al. Epidemiology of varicella zoster virus infection in Canada and the United Kingdom. Epidemiol Infect (2001) 127: 305–314
[5] Tchuenche J.M., Dube N., Bhunu C.P., Smith R.J., Bauch C.T., The impact of media coverage on the transmission dynamics of human influenza, BMC Public Health 11 (2011).
[6] Greenhalgh D., Rana S, Samanta S, Sardar T, Bhattacharya S, Chattopadhyay J, Awareness programs control infectious disease – Multiple delay induced mathematical model, Applied Mathematics and Computation 251 (2015) 539–563.
[7] Ghosh M., Chandra P., Sinha P., J.B. Shukla, Modelling the spread of bacterial infectious disease with environmental effect in a logisticallygrowing human population, Nonlinear Analysis. Real World Applications 7 (2006) 341–365.
[8] Yoo B.K., Holland M.L., Bhattacharya J., Phelps C.E., Szilagyi P.G., Effects of mass media coverage on timing and annual receipt of influenza vaccination among medicare elderly, Health Serv. Res. 45 (2010) 1287–1309.
[9] Kaur N., Ghosh M., Bhatia S.S., Modeling and analysis of an SIRS epidemic model with effect of awareness programs by media, Int. J. Math. Comput.Phys. Quant. Eng. 8 (2014) 233–239.
[10] Van den Driessche P., Watmough J., Reproduction numbers and sub-threshold endemic equilibria for compartmental models of disease transmission, Mathematical Biosciences 180 (2002) 29–48.