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Research paper

Direct Product of Finite Interval-Valued Intuitionistic Fuzzy Ideals in BF-Algebra

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Abstract

The present paper gives direct product of finite interval-valued intuitionistic fuzzy ideals. Furthermore, we add more useful results and also prove that, let $A=(\tilde{\mu}_A,\tilde{\lambda}_A)$ be interval-valued intuitionistic fuzzy ideal of BF-algebra X. If $\tilde{\mu}_A(x*y)\geq\tilde{\mu}_A(x)$ and $\tilde{\lambda}_A(x*y)\leq\tilde{\lambda}_A(x)$ for any $x,y\in X$, then $A=(\tilde{\mu}_A,\tilde{\lambda}_A)$ is an interval-valued intuitionistic fuzzy H-ideal of BF-algebra X.

Keywords: *BF-algebras, interval-valued intuitionistic fuzzy sets, interval-valued intuitionistic fuzzy ideals, interval-valued intuitionistic fuzzy closed ideal.*

1 Introduction and Preliminaries

The notion of interval-valued fuzzy sets was first introduced by Zadeh [17] as an extension of fuzzy sets. An interval-valued fuzzy sets is a fuzzy set whose membership function is many-valued and from an interval in the membership scale. This idea gives the simplest method to capture the imprecision of the membership grade for a fuzzy set. On the other hand, Atanassov [5] introduced the notion of intuitionistic fuzzy sets as an extension of fuzzy set which not only a membership degree is given, but also a non-membership degree is involved. Atanassov and Gargov [6] introduced the notion of interval-valued intuitionistic fuzzy sets which is a generalization of both intuitionistic fuzzy sets and interval-valued fuzzy sets(IVIF-sets). In [15] Satyanarayana with others applied the concept of interval-valued intuitionistic fuzzy ideals of BF-algebras. In this paper, direct product of finite interval-valued intuitionistic fuzzy ideals (DPOFIVIF-ideals). Furthermore, we add more useful results and also prove that, let $A=(\tilde{\mu}_A,\tilde{\lambda}_A)$ be IVIF-ideal of BF-algebra X. If $\tilde{\mu}_A(x*y)\geq\tilde{\mu}_A(x)$ and $\tilde{\lambda}_A(x*y)\leq\tilde{\lambda}_A(x)$ for any $x,y\in X$, then $A=(\tilde{\mu}_A,\tilde{\lambda}_A)$ is an interval-valued intuitionistic fuzzy H-ideal (IVIF H-ideal) of BF-algebra X. Also, we have proved for direct product of finite interval-valued intuitionistic fuzzy set (DPOFIVIF-set) of the above theorem.

By a BF-algebra we mean an algebra satisfying the axioms:

(1). $x*x=0$,

(2). $x*0=x$,

(3). $0*(x*y)=y*x$, for all $x,y\in X$

Throughout this paper, X is a BF-algebra.

Example 1.1

Let R be the set of real number and let $A=(R,*,0)$ be the algebra with the operation $*$ defined by

$$x*y=\begin{cases}x, \text{if } y=0\\ y, \text{if } x=0\\ 0, \text{otherwise}\end{cases}$$

Definition 1.2:

The subset I of X is said to be an ideal of X, if

(i) $0\in I$ and (ii) $x*y\in I$ and $y\in I\Rightarrow x\in I$.

Definition 1.3:

A non-empty subset I of BF-algebra X is called an H-ideal of X, if

(1) $0\in I$,

(2) $x*(y*z)\in I$ and $y\in I\Rightarrow x*z\in I$.

The following interval-valued intuitionistic fuzzy concept is taken from [15].

Definition 1.4:

An interval-valued fuzzy subset $\tilde{\mu}_A$ of a BF-algebra X is called a interval-valued fuzzy H-ideal of X, if

(1) $\tilde{\mu}_A(0)\geq\tilde{\mu}_A(x)$

(2) $\tilde{\mu}_A(x*z) \geq \min\{\tilde{\mu}_A(x*(y*z)), \tilde{\mu}_A(y)\}$, for all $x, y, z \in X$.

Definition 1.5:

An IVIF-subset $A = (\tilde{\mu}_A, \tilde{\lambda}_A)$ of a BF-algebra X is called an IVIF-H-ideal of X, if

(IFH1) $\tilde{\mu}_A(0) \geq \tilde{\mu}_A(x)$ and $\tilde{\lambda}_A(0) \leq \tilde{\lambda}_A(x)$,

(IFH2) $\tilde{\mu}_A(x*z) \geq \min\{\tilde{\mu}_A(x*(y*z)), \tilde{\mu}_A(y)\}$,

(IFH3) $\tilde{\lambda}_A(x*z) \leq \max\{\tilde{\lambda}_A(x*(y*z)), \tilde{\lambda}_A(y)\}$,

for all $x, y, z \in X$.

Definition 1.6 9

2. Generalized Product of Interval-Valued Intuitionistic Fuzzy Ideals

Definition 2.1:

Let $A_i = (\tilde{\mu}_{A_i}, \tilde{\lambda}_{A_i})$ be n i-v IFS of BF- algebras X_i, respectively $i = 1, 2, 3, ..., n$. Then $\prod_{i=1}^{n} A_i$ is called direct product of finite interval-valued intuitionistic fuzzy subalgebra (DPFIVIF-Subalgebra) of $\prod_{i=1}^{n} X_i$

$$\prod_{i=1}^{n}\tilde{\mu}_{A_i}((x_1, x_2 .., x_n) * (y_1, y_2 ,.., y_n))$$

$$\geq \min\left\{\prod_{i=1}^{n}\tilde{\mu}_{A_i}(x_1, x_2 .., x_n), \prod_{i=1}^{n}\tilde{\mu}_{A_i}(y_1, y_2 ,.., y_n)\right\}$$

$$\prod_{i=1}^{n}\tilde{\lambda}_{A_i}((x_1, x_2 .., x_n) * (y_1, y_2 ,.., y_n))$$

$$\leq \max\left\{\prod_{i=1}^{n}\tilde{\lambda}_{A_i}(x_1, x_2 .., x_n), \prod_{i=1}^{n}\tilde{\lambda}_{A_i}(y_1, y_2 ,.., y_n)\right\}$$ for all

$(x_1,..,x_n), (y_1,..,y_n) \in \prod_{i=1}^{n} X_i$.

Theorem 2.2

Let $A_i = (\tilde{\mu}_{A_i}, \tilde{\lambda}_{A_i})$ be n IVIF-subalgebras of BF-algebras X_i, respectively $i = 1, 2, 3, ..., n$. $\prod_{i=1}^{n} A_i$ is an IVIF-subalgebra of $\prod_{i=1}^{n} X_i$.

Definition 2.3

Let $A_i = (\tilde{\mu}_{A_i}, \tilde{\lambda}_{A_i})$ be n IVIFS of BF- algebras X_i, respectively $i = 1, 2, 3, ..., n$. Then $\prod_{i=1}^{n} A_i$ is called DPFIVIF-Subalgebra of $\prod_{i=1}^{n} X_i$ if

$\prod_{i=1}^{n}\tilde{\mu}_{A_i}(0,..,0) \geq \prod_{i=1}^{n}\tilde{\mu}_{A_i}(x_1,..,x_n)$ and

$\prod_{i=1}^{n}\tilde{\lambda}_{A_i}(0,..,0) \leq \prod_{i=1}^{n}\tilde{\lambda}_{A_i}(x_1,..,x_n)$

$$\prod_{i=1}^{n}\tilde{\mu}_{A_i}(x_1, x_2 .., x_n)$$

$$\geq \min\{\prod_{i=1}^{n}\tilde{\mu}_{A_i}((x_1, x_2 .., x_n) * (y_1, y_2 ,.., y_n)), \prod_{i=1}^{n}\tilde{\mu}_{A_i}(y_1, y_2 ,.., y_n)\},$$

$$\prod_{i=1}^{n}\tilde{\lambda}_{A_i}(x_1, x_2 .., x_n)$$

$$\leq \max\{\prod_{i=1}^{n}\tilde{\lambda}_{A_i}((x_1, x_2 .., x_n) * (y_1, y_2 ,.., y_n)), \prod_{i=1}^{n}\tilde{\lambda}_{A_i}(y_1, y_2 ,.., y_n)\},$$

for all $(x_1,..,x_n),(y_1,..,y_n) \in \prod_{i=1}^{n} X_i$.

Example 2.4

Let $X_1 = \{0, 1\}$ and $X_2 = \{0, 1, 2\}$ are BF-algebras by the following tables:

*	0	1
0	0	1
1	1	0

*	0	1	2
0	0	2	1
1	1	0	2
2	2	1	0

Then $X_1 \times X_2 = \{(0,0),(0,1),(0,2),(1,0),(1,1),(1,2)\}$ is an BF-algebra. We define i-v IFS $A_1 = (\tilde{\mu}_{A_1}, \tilde{\lambda}_{A_1})$ on X_1 as $\tilde{\mu}_{A_1} : X_1 \to [0,1]$

by $\tilde{\mu}_{A_1}(0) = [0.5, 0.6]$, $\tilde{\mu}_{A_1}(1) = [0.3, 0.4]$

and $\tilde{\lambda}_{A_1} : X_1 \to [0,1]$ by $\tilde{\lambda}_{A_1}(0) = [0.3, 0.4]$, $\tilde{\lambda}_{A_1}(1) = [0.5, 0.6]$. Now we define i-v IFS $A_2 = (\tilde{\mu}_{A_2}, \tilde{\lambda}_{A_2})$ on X_2 as $\tilde{\mu}_{A_2} : X_2 \to [0,1]$ by $\tilde{\mu}_{A_2}(0) = [0.5, 0.6]$, $\tilde{\mu}_{A_2}(1) = [0.2, 0.35]$ and $\tilde{\mu}_{A_2}(2) = [0.2, 0.3]$ $\tilde{\lambda}_{A_2} : X_2 \to [0,1]$ by $\tilde{\lambda}_{A_2}(0) = [0.3, 0.35]$, $\tilde{\lambda}_{A_2}(1) = [0.5, 0.6]$, $\tilde{\lambda}_{A_2}(2) = [0.6, 0.7]$. By routine calculation $A_1 \times A_2 = <\tilde{\mu}_{A_2 \times A_2}, \tilde{\lambda}_{A_2 \times A_2}>$ is an IVIF-ideal of BF-algebra.

Example 2.5

Let $X_1 = \{0, 1\}$, $X_2 = \{0, 1, 2\}$ and $X_3 = \{0, 1, 2, 3\}$ be three BF-algebras by the following table:

*	0	1
0	0	1
1	1	0

*	0	1	2
0	0	2	1
1	1	0	2
2	2	1	0

*	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

Then $X_1 \times X_2 \times X_3$ is BF-algebra. We define IVIFS $A_1 = (\tilde{\mu}_{A_1}, \tilde{\lambda}_{A_1})$ on X_1 as $\tilde{\mu}_{A_1} : X_1 \to [0,1]$ by $\tilde{\mu}_{A_1}(0) = [0.4, 0.5]$, $\tilde{\mu}_{A_1}(1) = [0.2, 0.3]$ and $\tilde{\lambda}_{A_1} : X_1 \to [0,1]$ by $\tilde{\lambda}_{A_1}(0) = [0.3, 0.4]$, $\tilde{\lambda}_{A_1}(1) = [0.5, 0.6]$. Now we define IVIFS $A_2 = (\tilde{\mu}_{A_2}, \tilde{\lambda}_{A_2})$ on X_2 as $\tilde{\mu}_{A_2} : X_2 \to [0,1]$ by $\tilde{\mu}_{A_2}(0) = [0.6, 0.7]$, $\tilde{\mu}_{A_2}(1) = [0.4, 0.45]$ and $\tilde{\mu}_{A_2}(2) = [0.3, 0.4]$ $\tilde{\lambda}_{A_2} : X_2 \to [0,1]$ by $\tilde{\lambda}_{A_2}(0) = [0.2, 0.25]$, $\tilde{\lambda}_{A_2}(1) = [0.3, 0.4]$, $\tilde{\lambda}_{A_2}(2) = [0.5, 0.6]$.

Now we define $\tilde{\mu}_{A_3} : X_3 \to [0,1]$ by $\tilde{\mu}_{A_3}(0) = \tilde{\mu}_{A_3}(1) = [0.7, 0.8]$, $\tilde{\mu}_{A_3}(2) = [0.5, 0.6]$, $\tilde{\mu}_{A_3}(3) = [0.4, 0.5]$ and $\tilde{\lambda}_{A_3} : X_3 \to [0,1]$ by $\tilde{\lambda}_{A_3}(0) = \tilde{\lambda}_{A_3}(1) = [0.1, 0.2]$, $\tilde{\lambda}_{A_3}(2) = [0.3, 0.4]$, $\tilde{\lambda}_{A_3}(3) = [0.45, 0.5]$. By routine calculation $A_1 \times A_2 \times A_2 = <\tilde{\mu}_{A_2 \times A_2 \times A_3}, \tilde{\lambda}_{A_2 \times A_2 \times A_3}>$ is an IVIF-ideal of BF-algebra.

Definition 2.6

Let $A_i = (\tilde{\mu}_{A_i}, \tilde{\lambda}_{A_i})$ be n IVIFS of BF-algebras X_i, respectively. Then:

(i) $\Box(\prod_{i=1}^{n} A_i) = <\prod_{i=1}^{n} \tilde{\mu}_{A_i}, \prod_{i=1}^{n} \tilde{\mu}^{c}_{A_i}>$,

(ii) $\Diamond(\prod_{i=1}^{n} A_i) = <\prod_{i=1}^{n} \tilde{\lambda}^{c}_{A_i}, \prod_{i=1}^{n} \tilde{\lambda}_{A_i}>$

The operators "necessity" and "possibility" over IVIF-sets and they are introduced for the first time in the first paper on these sets.

Theorem 2.7

Let $A_i = (\tilde{\mu}_{A_i}, \tilde{\lambda}_{A_i})$ be n IVIF-subalgebras of BF-algebras X_i, respectively for $i = 1, 2, 3, ..., n$. Then $\Box(\prod_{i=1}^{n} A_i) = <\prod_{i=1}^{n} \tilde{\mu}_{A_i}, \prod_{i=1}^{n} \tilde{\mu}^{c}_{A_i}>$ and $\Diamond(\prod_{i=1}^{n} A_i) = <\prod_{i=1}^{n} \tilde{\lambda}^{c}_{A_i}, \prod_{i=1}^{n} \tilde{\lambda}_{A_i}>$ are fan IVIF-subalgebras of BF-algebras $\prod_{i=1}^{n} X_i$.

Theorem 2.8

Let $A_i = (\tilde{\mu}_{A_i}, \tilde{\lambda}_{A_i})$ be n IVIF-ideals of BF-algebras $\prod_{i=1}^{n} X_i$, for $i = 1, 2, 3, ..., n$ respectively. Then fuzzy sets $\prod_{i=1}^{n} \tilde{\mu}_{A_i}$ and $\prod_{i=1}^{n} \tilde{\mu}^{c}_{A_i}$ are interval-valued fuzzy ideals of $\prod_{i=1}^{n} X_i$.

Theorem 2.9

Let $A_i = (\tilde{\mu}_{A_i}, \tilde{\lambda}_{A_i})$ be n IVIF-ideals of BF-algebras $\prod_{i=1}^{n} X_i$, for $i = 1, 2, 3, ..., n$ respectively. Then interval-valued fuzzy sets $\prod_{i=1}^{n} \tilde{\mu}^c_{A_i}$ and $\prod_{i=1}^{n} \tilde{\lambda}_{A_i}$ are interval-valued fuzzy anti ideals of $\prod_{i=1}^{n} X_i$.

Theorem 2.10

Let $A_i = (\tilde{\mu}_{A_i}, \tilde{\lambda}_{A_i})$ be IVIF closed ideals of BF-algebras $X_1, X_2, ..., X_n$ respectively, then interval-valued fuzzy sets $\prod_{i=1}^{n} \tilde{\mu}_{A_i}$ and $\prod_{i=1}^{n} \tilde{\lambda}^c{}_{A_i}$ are interval-valued fuzzy closed ideals of $\prod_{i=1}^{n} X_i$.

Theorem 2.11

Let $A_i = (\tilde{\mu}_{A_i}, \tilde{\lambda}_{A_i})$ be n IVIF closed ideals of BF-algebras X_i, for $i=1, 2, ..., n$ respectively. Then interval-valued fuzzy sets $\prod_{i=1}^{n} \tilde{\mu}^c_{A_i}$ and $\prod_{i=1}^{n} \tilde{\lambda}_{A_i}$ are interval-valued anti fuzzy closed ideals of $\prod_{i=1}^{n} X_i$.

Theorem 2.12

Let $A = (\tilde{\mu}_A, \tilde{\lambda}_A)$ be an IVIF-ideal of BF-algebra X. If $\tilde{\mu}_A(x * y) \geq \tilde{\mu}_A(x)$ and $\tilde{\lambda}_A(x * y) \leq \tilde{\lambda}_A(x)$ for any $x, y \in X$, then $A = (\tilde{\mu}_A, \tilde{\lambda}_A)$ is an IVIF H-ideal of BF-algebra X.

Proof:

Let $A = (\tilde{\mu}_A, \tilde{\lambda}_A)$ be an IVIF-ideal of BF-algebra X. If $\tilde{\mu}_A(x * y) \geq \tilde{\mu}_A(x)$ and $\tilde{\lambda}_A(x * y) \leq \tilde{\lambda}_A(x)$ for $x, y, z \in X$. We have, by hypothesis

$$\min\{\tilde{\mu}_A(x*(y*z)), \tilde{\mu}_A(y)\}$$
$$\leq \min\{\tilde{\mu}_A((x*z)*(y*z)), \tilde{\mu}_A(y*z)\}$$
$$\leq \tilde{\mu}_A(y*z)$$
$$\tilde{\mu}_A(y*z) \geq \min\{\tilde{\mu}_A(x*(y*z)), \tilde{\mu}_A(y)\},$$ and

$$\max\{\tilde{\lambda}_A(x*(y*z)), \tilde{\lambda}_A(y)\}$$
$$\geq \max\{\tilde{\lambda}_A((x*z)*(y*z)), \tilde{\lambda}_A(y*z)\}$$
$$\geq \tilde{\lambda}_A(y*z)$$
$$\tilde{\lambda}_A(y*z) \leq \max\{\tilde{\lambda}_A(x*(y*z)), \tilde{\lambda}_A(y)\}.$$

Hence $A = (\tilde{\mu}_A, \tilde{\lambda}_A)$ is an IVIF H-ideal of BF-algebra X.

Proposition 2.13

Let $A = (\tilde{\mu}_A, \tilde{\lambda}_A)$ be IVIF closed ideal of BF-algebra X. If $\tilde{\mu}_A(x * y) \geq \tilde{\mu}_A(x)$ and $\tilde{\lambda}_A(x * y) \leq \tilde{\lambda}_A(x)$ for any $x, y \in X$, then $A = (\tilde{\mu}_A, \tilde{\lambda}_A)$ is an IVIF closed H-ideal of BF-algebra X.

Theorem 2.14

Let $A_i = (\tilde{\mu}_{A_i}, \tilde{\lambda}_{A_i})$ be n an IVIF-ideals of BF-algebras $\prod_{i=1}^{n} X_i$, respectively. If $\tilde{\mu}_{A_i}(x * y) \geq \tilde{\mu}_{A_i}(x)$ and $\tilde{\lambda}_{A_i}(x_i * y_i) \leq \tilde{\lambda}_{A_i}(x_i)$ where $i = 1, 2, 3, ..., n$, then $\prod_{i=1}^{n} A_i$ is an IVIF H-ideal of $\prod_{i=1}^{n} X_i$.

Theorem 2.15

Let $A_i = (\tilde{\mu}_{A_i}, \tilde{\lambda}_{A_i})$ be n an IVIF closed ideals of BF-algebras $\prod_{i=1}^{n} X_i$,

respectively. If $\tilde{\mu}_{A_i}(x * y) \geq \tilde{\mu}_{A_i}(x)$ and $\tilde{\lambda}_{A_i}(x * y) \leq \tilde{\lambda}_{A_i}(x)$ where $i = 1, 2, 3, ..., n$, then $\prod_{i=1}^{n} A_i$ is an IVIF closed H-ideal of $\prod_{i=1}^{n} X_i$.

Theorem 2.16

Let $A_i = (\tilde{\mu}_{A_i}, \tilde{\lambda}_{A_i})$ be n an IVIF-ideals of BF-algebras X_i, respectively. If the inequality $(x_i * y_i) \leq z_i$ holds in X_i for $i = 1, 2, 3, ..., n$, then

$$\prod_{i=1}^{n} \tilde{\mu}_{A_i}(x_1, x_2 .., x_n)$$
$$\geq \min\left\{\prod_{i=1}^{n} \tilde{\mu}_{A_i}(y_1, y_2, .., y_n), \prod_{i=1}^{n} \tilde{\mu}_{A_i}(z_1, z_2, .., z_n)\right\}$$

$$\prod_{i=1}^{n} \tilde{\lambda}_{A_i}(x_1, x_2 .., x_n)$$
$$\leq \max\left\{\prod_{i=1}^{n} \tilde{\lambda}_{A_i}(y_1, y_2, .., y_n), \prod_{i=1}^{n} \tilde{\lambda}_{A_i}(z_1, z_2, .., z_n)\right\}$$

Definition 2.17

Let $A = (\tilde{\mu}_A, \tilde{\lambda}_A)$ be an IVIF-set of BF-algebras X. For any $[s_1, s_2], [t_1, t_2] \in D[0,1]$, then the set $U(\tilde{\mu}_A; [s_1, s_2]) = \{x \in X / \tilde{\mu}_A(x) \geq [s_1, s_2]\}$ is called interval-valued upper level of $\tilde{\mu}_A$ and $L(\tilde{\lambda}_A; [t_1, t_2]) = \{x \in X / \tilde{\lambda}_A(x) \leq [t_1, t_2]\}$ is called interval-valued lower level of $\tilde{\lambda}_A$.

Definition 2.18

$A_1 = (\tilde{\mu}_{A_1}, \tilde{\lambda}_{A_1})$,

$A_2 = (\tilde{\mu}_{A_1}, \tilde{\lambda}_{A_1}), \ldots, A_n = (\tilde{\mu}_{A_n}, \tilde{\lambda}_{A_n})$ be n IVIF-sets of BF-algebras $X_1, X_2, \ldots, X_n$ respectively. Then for any $[s_1, s_2], [t_1, t_2] \in D[0,1]$, the set

$$U(\prod_{i=1}^{n} \tilde{\mu}_{A_i}, [t_1, t_2]) = \{(x_1, x_2 .., x_n) \in \prod_{i=1}^{n} X_i : \prod_{i=1}^{n} \tilde{\mu}_{A_i}(x_1, x_2 .., x_n) \geq [t_1, t_2]\}$$

is called interval-valued upper level of $\prod_{i=1}^{n} \tilde{\mu}_{A_i}(x_1, x_2 .., x_n)$ and

$$L(\prod_{i=1}^{n} \tilde{\lambda}_{A_i}, [s_1, s_2]) = \{(x_1, x_2 .., x_n) \in \prod_{i=1}^{n} X_i : \prod_{i=1}^{n} \tilde{\lambda}_{A_i}(x_1, x_2 .., x_n) \leq [s_1, s_2]$$

is called interval-valued lower level of $\prod_{i=1}^{n} \tilde{\lambda}_{A_i}(x_1, x_2 .., x_n)$.

Theorem 2.19

Let $A_i = (\tilde{\mu}_{A_i}, \tilde{\lambda}_{A_i})$ be n IVIFS of BF- algebras X_i, respectively $i = 1, 2, 3, \ldots, n$. Then $\prod_{i=1}^{n} A_i$ is called DPOIVIF-subalgebra of $\prod_{i=1}^{n} X_i$ if and only if $U(\prod_{i=1}^{n} \tilde{\mu}_{A_i}, [t_1, t_2])$ and $L(\prod_{i=1}^{n} \tilde{\lambda}_{A_i}, [s_1, s_2])$ are subalgebras of BF-algebra $\prod_{i=1}^{n} X_i$.

Proof:
Straight forward.

Theorem 6.20

Let $A_i = (\tilde{\mu}_{A_i}, \tilde{\lambda}_{A_i})$ be n an IVIF-ideals of BF-algebras X_i, respectively. If $x_i \leq y_i$, where $i = 1, 2, 3, \ldots, n$, Then

$$\prod_{i=1}^{n} \tilde{\mu}_{A_i}(x_1, x_2 .., x_n) \geq \prod_{i=1}^{n} \tilde{\mu}_{A_i}(y_1, y_2 .., y_n)$$ and

$$\prod_{i=1}^{n} \tilde{\lambda}_{A_i}(x_1, x_2 .., x_n) \leq \prod_{i=1}^{n} \tilde{\lambda}_{A_i}(y_1, y_2 .., y_n).$$

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