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# New Soliton Solutions of the (2+1)-Dimensional System Davey-Stewartson Equation

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**Abstract**

In this paper, we derive several soliton solutions of the generalized Davey-Stewartson equation with the complex coefficients. First we use the travelling wave transformation to reduce the initial system to ODE. The equivalent ODE is then solved, giving several classes of solutions, depending on the values of the parameters. Finally, the Extended Tanh-Coth method and Modified simple equation method.

***Keywords***: *Solitary wave solutions, Davey-Stewartson equation, Tanh-Coth method, modified simple equation method.*

## 1. Introduction

The analysis of some physical phenomena is investigated by the exact solutions of nonlinear evolution equations (NLEEs). Partial differential equations which arise in real-world physical problems are often too complicated to be solved exactly [1]-[10].
In this study, we try to solve the Davey–Stewartson (DS) equations in the following [2]

$$i q_t + \frac{1}{2}\sigma^2\left(q_{xx} + \sigma^2 q_{yy}\right) + \lambda\,|q|^2 q\ - \Phi_x q = 0 \qquad (1)$$

$$\left(\Phi_{xx} - \sigma^2 \Phi_{yy}\right) - 2\,\lambda\,(|q|^2)_x\ = 0 \qquad (2)$$

where $q(x, y, t)$ and $\Phi(x, y, t)$ are unknown functions.
DS equation arises in fluid dynamics. In fact, this equation particularly studies the long-wave-short-wave resonances and other patterns of propagating waves.This equation describes the evolution of a 2-dimensional wave-packet on water of finite depth too [5-8].
The system of equations (1) and (2), when $\sigma = 1$ the system is called the DS-I system, and on the other hand for $\sigma = i$ the system is called DS-II system. The focusing or defocusing case is characterized by parameter $\lambda$ [9].
Recent analytical methods of solving NLEEs have been developed which will be applied to obtain the soliton solutions to the equation. Some of these methods are (G'/G )-expansion method [6, 10, 11], Exp-function method [12,13], the tanh method [14], homogeneous balance method [15], the extended Weierstrass transformation method [16-19] the trial equation method [20-23].
In this paper, we find several traveling wave solutions of (2 + 1)-dimensional DS equation. We first reduce the general form of the equation to the corresponding ODE (Ordinary Differential Equation) using the traveling wave transformation. After that, the solutions are obtained from the ODE. We distinguish total 5 cases, depending on the solution and the equation parameters. Finally, we use extended Tanh-Coth method and Modified simple equation method to obtain additional soliton solutions, which are also valid under certain conditions on the parameters of the input equation.

## 2. Traveling Wave Transformation and Reduction to ODE

Consider the nonlinear partial differential equation in the form

$$F\left(u, u_t, u_x, u_y, u_{tt}, u_{xx}, u_{xy}, u_{yy}, \dots\dots\dots\right) = 0 \qquad (3)$$

where $u(x, y, t)$ is a traveling wave solution of nonlinear partial differential equation Eq.(3). We use the transformations,

$$u(x, y, t) = f(\xi) \qquad (4)$$

where $\xi = k(x + ly - \lambda t)$ This enables us to use the following changes:

$$\frac{\partial}{\partial t}(.) = -\lambda k \frac{d}{d\xi}(.)\ ,\ \frac{\partial}{\partial x}(.) = k\ \frac{d}{d\xi}(.)\ ,\ \frac{\partial}{\partial y}(.) = k\ l \frac{d}{d\xi}(.) \qquad (5)$$

Using Eq. (5) to transfer the nonlinear partial differential equation Eq. (1) to nonlinear ordinary differential equation

$$Q(f, f', f'', f''', \dots\dots\dots\dots.) = 0 \qquad (6)$$

The ordinary differential equation (6) is then integrated as long as all terms contain derivatives, where we neglect the integration constants.
Let us consider the (2+1)-dimensional system Davey-Stewartson Equation DS in Eqs. (1-2). this equation studied by [2]. With the transformations

$$q(x, y, t) =\ e^{i\,\theta}.u(\xi), \qquad \Phi(x, y, t) =\ V(\xi) \qquad (7)$$

Where:

$$\theta = k_1\,x + k_2\,y + k_3\,t\,, \xi = k\,(x + l\,y - \lambda\,t) \qquad (8)$$

Substituting (7-8) into (5) and (6) with
$q_t\ = [-\lambda\,k\,.u'(\xi) + i\,k_3\,.u(\xi)]e^{i\,\theta}$

$q_{xx} .= [k^2 u''(\xi) + 2\, i\, k\, k_1 . u'(\xi) - \, {k_1}^2\, u(\xi)] e^{i\,\theta}$
$q_{yy} = [k^2\, l^2 . u''(\xi) + 2\, i\, k\, k_2\, l . u'(\xi) - \, {k_2}^2 . u(\xi)] e^{i\,\theta}$
$\Phi_x = \, V'(\xi)\, k$

Replacing the previous expressions into (1) and decomposing into real and imaginary parts, yields to

$$\sigma^2 k^2 (\, 1 \, + \sigma^2 l^2)\, u'' + 2\lambda\, u^3 - \{\sigma^2 (\, {k_1}^2 + \sigma^2\, {k_2}^2\, ) + 2\, \, k_3\} u - 2k\, V'\, u = 0 \quad (9)$$

$$\lambda = \, \sigma^2 (\, k_1 + \sigma^2 l\, k_2) \quad (10)$$

Eq.(2) transform to

$$k\, (\sigma^2\, l^2 - 1) V'' + 2\, \lambda\, (\, u^2\, )' \, = 0 \quad (11)$$

Integrating (11) with zero constant then

$$k\, (\sigma^2\, l^2 - 1\, ) V' + 2\, \lambda\, u^2 \, = 0 \quad (12)$$

Assume :
$c_0 = \sigma^2 k^2 (\, 1 \, + \sigma^2 l^2)$ ,
$c_1 = [\sigma^2 (\, {k_1}^2 + \sigma^2\, {k_2}^2\, ) + \, 2\, k_3]$ ,

$$c_2 = k\, (\sigma^2\, l^2 - 1\, ) \quad (13)$$

Then Eq. (9) and (12) can be written as

$$c_0\, u'' + 2\lambda\, u^3 - \, c_1\, u - 2\, k\, V'\, u = 0 \quad (14)$$

$$c_2\, V' + 2\, \lambda\, u^2 \, = 0 \quad (15)$$

From eq.(15) $V' = -\frac{2\,\lambda}{c_2} u^2$ substitute in Eq.(14), then

$$c_0 c_2\, u'' + 2\, \lambda\, [\, c_2 + 2\, k]\, u^3 - \, c_1 c_2\, u = 0 \quad (16)$$

multiplying both sides of Eq.(16) by $u'$ and integrating with respect to $\xi$ , we get

$$c_0 c_2\, {u'}^2 + \lambda\, [\, c_2 + 2\, k]\, u^4 - c_1 c_2\, u^2 = 0 \quad (17)$$

We first obtain the travelling wave solutions of equation (17) by direct integration. Total five cases are distinguished, depending on values of the input parameters. That is done in Section 4. After that, additional soliton solutions of the equation (16) are found in Section 5 and Section 6 using the extended Tanh-Coth method, and the Modified simple equation method.

## 3. Traveling wave solutions

Both $c_2 = 0$ and $c_0 = 0$ lead to the trivial solutions for u (u = 0 and u = const.). Therefore, we assume that $c_2 c_0 \neq 0$ Due to the fact that $u \geq 0$ (since$u = |q|$) and the continuity of $u = u(\xi)$ and its derivatives, we can write the equation (17) in the following way

$$u' = \mp u \sqrt{\frac{c_1}{c_0} - \frac{\lambda\, (c_2 + 2\, k)}{c_2 c_0}\, u^{\,2}} \quad (18)$$

The further analysis of the equation (18) will be divided into the following 3 cases. Recall that all obtained solutions are valid under the assumption that
$\lambda = \, \sigma^2 (\, k_1 + \sigma^2 l\, k_2)$

### 3.1 Case 1

Assume that $c_0 c_1 > 0$ and $\frac{\lambda\, (c_2 + 2\, k)}{c_1 c_2} > 0$ . Equation (18) becomes

$$u' = \mp \mu\, . u \sqrt{1 - \frac{\lambda\, (c_2 + 2\, k)}{c_1 c_2}\, u^{\,2}} \quad , \, \mu = \sqrt{\frac{c_1}{c_0}} \quad (19)$$

i.e

$$u' = \mp \mu\, . u \sqrt{1 - \alpha^{-2}\, u^{\,2}} \quad , \, \alpha = \sqrt{\frac{c_1 c_2}{\lambda\, (c_2 + 2\, k)}} \quad (20)$$

Equation (20) has the closed-form solution given by

$$u(\xi) = \, \alpha \, \, sech(\mp \mu \xi + \varphi) \quad (21)$$

where $\varphi$ is an arbitrary real constant. Now replacing $\mu$ and $\alpha$ given by (19) and (20) into (21), we get

$$u(\xi) = \sqrt{\frac{c_1 c_2}{\lambda\, (c_2 + 2\, k)}}\, \, sech(\mp \sqrt{\frac{c_1}{c_0}}\, \xi + \varphi) \quad (22)$$

Furthermore, equation (15) implies

$$V(\xi) = - \int \frac{2\, \lambda}{c_2} u^2(\xi)\, \, d\xi$$

$$V(\xi) = \frac{2\, c_1}{(c_2 + 2\, k)} \tanh(\mp \sqrt{\frac{c_1}{c_0}}\, \xi + \varphi) \quad (23)$$

Now by using (7)-(8), we obtain the following closed-form soliton solutions for $q(x, y, t)$ and $\Phi(x, y, t)$.

$$q(x, y, t) = \, e^{i\, [k_1\, x + k_2\, y + k_3\, t]} . \sqrt{\frac{c_1 c_2}{\lambda\, (c_2 + 2\, k)}}\, \, sech(\mp \sqrt{\frac{c_1}{c_0}}\, \xi + \varphi),$$

$$\Phi(x, y, t) = \frac{2\, c_1}{(c_2 + 2\, k)} \tanh(\mp \sqrt{\frac{c_1}{c_0}}\, \xi + \varphi) \quad (24)$$

Where $c_0, c_1$, and $c_2$ are defined in (13), and $\xi$ is defined in (8).

### 3.2 Case 2

Assume that $c_0 c_1 > 0$ and $\frac{\lambda\, (c_2 + 2\, k)}{c_1 c_2} < 0$ . Equation (18) becomes

$$u' = \mp \mu\, . u \sqrt{1 + \alpha^{-2}\, u^{\,2}} \quad , \, \alpha = \sqrt{-\frac{c_1 c_2}{\lambda\, (c_2 + 2\, k)}} \quad (25)$$

Equation (25) has the closed-form solution given by

$$u(\xi) = \, \alpha\, \, csch(\mp \mu \xi + \varphi) \quad (26)$$

where $\varphi$ is an arbitrary real constant.
Therefore

$$u(\xi) = \sqrt{-\frac{c_1 c_2}{\lambda\, (c_2 + 2\, k)}}\, \, csch(\mp \sqrt{\frac{c_1}{c_0}}\, \xi + \varphi) \quad (27)$$

$$V(\xi) = - \int \frac{2\, \lambda}{c_2} u^2(\xi)\, \, d\xi$$

$$V(\xi) = \mp \frac{2\, c_1}{(c_2 + 2\, k)} \coth\left(\mp \sqrt{\frac{c_1}{c_0}}\, \xi + \varphi\right) + C$$

Hence we obtain the following closed-form soliton solutions for $q(x, y, t)$ and $\Phi(x, y, t)$.

$$q(x, y, t) = e^{i\, [k_1\, x + k_2\, y + k_3\, t]} \times \sqrt{-\frac{c_1 c_2}{\lambda\, (c_2 + 2\, k)}}\, \, csch(\mp \sqrt{\frac{c_1}{c_0}}\, \xi + \varphi)$$

$$\Phi(x, y, t) = \mp \frac{2\, c_1}{(c_2 + 2\, k)} \coth\left(\mp \sqrt{\frac{c_1}{c_0}}\, \xi + \varphi\right) + C \quad (28)$$

In fact, solution (28) has the similar form as (24), where sech function is replaced by csch, and tanh by coth.

### 3.3 Case 3

Assume that $c_0 c_1 < 0$ and $\frac{\lambda\,(c_2+2\,k)}{c_1 c_2} < 0$ . Equation (18) becomes

$$u' = \mp\mu\,.\,u\sqrt{\alpha^{-2}\,u^{\,2} - 1}\;,\; \alpha = \sqrt{-\frac{c_1 c_2}{\lambda\,(c_2+2\,k)}}\,,\; \mu = \sqrt{-\frac{c_1}{c_0}} \qquad (29)$$

Its solution is given by:

$$u(\xi) = \frac{\alpha(\alpha+b)}{\sqrt{b(2\alpha+b)}\sin(\mu\xi)+\alpha\cos(\mu\xi)} \qquad (30)$$

where $b > 0$ is an arbitrary real constant. The corresponding function $V(\xi)$ is now equal to

$$V(\xi) = -\int \frac{2\,\lambda}{c_2} u^2(\xi)\;d\xi$$

$$V(\xi) = -\frac{2\,\alpha^2\lambda\left((a+b)^2\sin(2\mu\xi) - 2\alpha\sqrt{b(2\alpha+b)}\right)}{c_2\,\mu\left(\alpha^2 - b^2 - 2\alpha b + (a+b)^2\cos(2\mu\xi)\right)} + C \qquad (31)$$

Note that $u(0) = \alpha + b > 0$ and hence the expression (30) for $u(\xi)$ is valid for $[\xi] < \arctan\left(\alpha\sqrt{b(2\alpha+b)}\right)$

Now the closed-form expressions for $q(x,y,t)$ and $\Phi(x,y,t)$are obtained by replacing (29) and (13) into (30)-(31) and using $q(x,y,t) = e^{i\,[k_1 x + k_2 y + k_3 t]}\,u(k(x - ly - \lambda\,t)$ and $\Phi(x,y,t) = V(k(x - ly - \lambda\,t)$. The exact expressions are too clumsy, so we omit them.

### 3.4 Case 4

Let $c_1 = 0$ Then equation (17) is now reduced to

$$u' = \frac{u^2}{\gamma}\,,\quad \gamma = \sqrt{-\frac{c_0 c_2}{\lambda\,[\,c_2+2\,k]}} = |k|\sqrt{\frac{(1-\sigma^2\,l^2\,)}{(\,k_1+\sigma^2 l\,k_2)}} \qquad (32)$$

under the assumption that $c_0 c_2 < 0$ (otherwise, equation does not have real solutions). The solution is then given by:

$$u(\xi) = \frac{\gamma}{b-\xi} = \sqrt{-\frac{c_0 c_2}{\lambda\,[\,c_2+2\,k]}}\,\frac{1}{b-\xi} = |k|\sqrt{\frac{(1-\sigma^2\,l^2\,)}{(\,k_1+\sigma^2 l\,k_2)}}\,\frac{1}{b-\xi} \qquad (33)$$

where b is arbitrary real constant. In that case, we have

$$V(\xi) = -\int \frac{2\,\lambda}{c_2} u^2(\xi)\;d\xi = \frac{2\,c_0}{(c_2+2\,k)}\frac{1}{b-\xi} + C = \frac{2\sigma^2 k}{b-\xi} + C \qquad (34)$$

where C is arbitrary real constant. Finally, the expressions for $q(x,y,t)$ and $\Phi(x,y,t)$are

$$q(x,y,t) = e^{i\,[k_1 x + k_2 y + k_3 t]}\,|k|\sqrt{\frac{(1-\sigma^2\,l^2\,)}{(\,k_1+\sigma^2 l\,k_2)}}\,\frac{1}{b-\xi}$$

$$\Phi(x,y,t) = \frac{2\sigma^2 k}{b-\xi} + C \qquad (35)$$

Recall that they are valid under the assumptions $\lambda = \sigma^2(\,k_1 + \sigma^2 l\,k_2)$, $c_1 = 0$ and $\frac{c_0 c_2}{\lambda\,[\,c_2+2\,k]} < 0$. The second one and the third one are equivalent with $k_3 = -\frac{\sigma^2}{2}\left(\,{k_1}^2 + \sigma^2\,{k_2}^2\,\right)$ , $\lambda\sigma^2(\sigma^2\,l^2 - 1\,) < 0$ respectively.

### 3.5 Case 5

Let $[\,c_2 + 2\,k] = 0$ , i.e. $\sigma^2\,l^2 = -1$. But then $c_0 = \sigma^2 k^2(\,1 + \sigma^2 l^2) = 0$, so we do not have non-trivial solutions in this case.

Similarly, under the assumption $c_0 c_1 < 0$ and $\frac{\lambda\,[\,c_2+2\,k]}{c_1 c_2} > 0$, the expression in (18), under the square root is always negative (since $u = |q| \ge 0$ ) and hence the equation (18) has no solutions.

## 4. The Extended Tanh-Coth Method

Now we use the extended Tanh-Coth method. It is more suitable to use it on the (more general) equation (16) than the (36). Hence, it will be applied it on the equation (16) which is here restated:

$$c_0 c_2\,u'' + 2\,\lambda\,[\,c_2 + 2\,k]\,u^3 - c_1 c_2\,u = 0 \qquad (36)$$

The method consists of using the new independent variable $Y = \tanh(\xi)$, that leads to the following changes of variables:

$$\frac{du}{d\xi} = (1 - Y^2)\frac{du}{dY}$$

$$\frac{d^2u}{d\xi^2} = -2Y(1 - Y^2)\frac{du}{dY} + (1 - Y^2)^2\frac{d^2u}{dY^2} \qquad (37)$$

Assume that the solution is expressed in the form

$$u(\xi) = \sum_{i=0}^{m} a_i\,Y^i + \sum_{i=1}^{m} b_i\,Y^{-i} \qquad (38)$$

where the parameter $m$ can be found by balancing the highest-order linear term with the nonlinear terms in Eq.(17), we balance $U^3$ with $\left(\frac{d^2U}{dY^2}\right)$ , to obtain: $3\,m = (m + 2)$ , then $m = 1$ . The Tanh-Coth method admits the use of the finite expansion for :

$$u = (a_0 + a_1 Y + b_1\,Y^{-1}) \qquad (39)$$

$$\frac{du}{dY} = a_1 - b_1\,Y^{-2} \qquad (40)$$

Coefficients $a_0, a_1$, and $b_1$ are to be determined. Substituting (39)-(41) into (36) yields to:

$$-2c_0c_2a_1Y + 2c_0c_2a_1Y^3 + 2\,c_0c_2b_1Y^{-3} - 2c_0c_2\,b_1Y^{-1} + 2\,\lambda\,[\,c_2 + 2\,k]\left({a_0}^3 + 3{a_0}^2\,a_1Y + 3a_0\,{a_1}^2Y^2 + {a_1}^3Y^3 + 3b_1[{a_0}^2Y^{-1} + 2a_0\,a_1 + {a_1}^2\,Y] + 3{b_1}^2[a_0Y^{-2} + a_1Y^{-1}] + {b_1}^3\,Y^{-3}\right) - c_1c_2\,(a_0 + a_1Y + b_1\,Y^{-1}) = 0 \qquad (41)$$

Equating expressions at $Y^i$, $(i = -3, -2, -1, 0, 1, 2, 3)$ to zero we have the following system of equations:

$$\begin{aligned}
&\left[2\,c_0c_2 + 2\,\lambda\,(c_2 + 2\,k)\,{b_1}^2\right]b_1 = 0\\
&6\,\lambda\,(c_2 + 2\,k)\,{b_1}^2 a_0 = 0\\
&[6\,\lambda\,(c_2 + 2\,k)\,({a_0}^2 + b_1a_1\,) - (c_1 + 2c_0)c_2]\,b_1 = 0\\
&[2\,\lambda\,(c_2 + 2\,k)\,({a_0}^2 + 6b_1a_1) - c_1c_2]\,a_0 = 0\\
&[6\,\lambda\,(c_2 + 2\,k)\,({a_0}^2 + b_1\,a_1\,) - (c_1 + 2c_0)c_2]a_1 = 0\\
&6\,a_0\,{a_1}^2\,\lambda\,(c_2 + 2\,k)\; = 0\\
&[2c_0c_2 + 2\,\lambda\,(c_2 + 2\,k){a_1}^2\;]\;a_1 = 0
\end{aligned} \qquad (42)$$

The only non-trivial real solution of the system (42) is given by

$$a_1 = b_1 = \sqrt{-\frac{c_0 c_2}{\lambda\,(c_2+2\,k)}} = |k|\sqrt{\frac{(1-\sigma^2\,l^2\,)}{(\,k_1+\sigma^2 l\,k_2)}}\;, \qquad (43)$$

and is valid under the assumption $c_1 = -8c_0$ and $\frac{c_0 c_2}{\lambda\,(c_2+2\,k)} < 0$ . In that case, the expressions for $u(\xi)$ and $V(\xi)$are given by

$$u(\xi) = |k|\sqrt{\frac{(1-\sigma^2\,l^2\,)}{(\,k_1+\sigma^2 l\,k_2)}}\;(\tanh(\xi) + \coth(\xi))$$

$$v(\xi) = -\frac{2\,\lambda}{c_2}\int u^{\,2}\;d\xi$$

$$v(\xi) = \;2\,\sigma^2 k\,[4\xi - \tanh(\xi) - \coth(\xi)] + C \qquad (44)$$

where C is an arbitrary real constant. Therefore, the corresponding expressions for $q(x,y,t)$ and $\Phi(x,y,t)$ are

$q(x,y,t) = e^{i\,[k_1 x + k_2 y + k_3 t]}\,|k|\sqrt{\frac{(1-\sigma^2 l^2)}{(k_1+\sigma^2 l\, k_2)}}\,(\tanh(\xi) + \coth(\xi))$
$\Phi(x,y,t) = 2\,\sigma^2 k\,[4\xi - \tanh(\xi) - \coth(\xi)] + C$ (45)

Where $\xi = k(x + ly - \sigma^2(k_1 + \sigma^2 l\, k_2)t$, Recall that this solution is valid under the assumptions $\lambda = \sigma^2(k_1 + \sigma^2 l\, k_2)$ , $c_1 = -8c_0$ and $\frac{c_0 c_2}{\lambda\,(c_2+2\,k)} < 0$. The second and the third one are equivalent with

$$k = \frac{1}{2}\sqrt{\frac{\sigma^2(k_1{}^2 + \sigma^2 k_2{}^2) + 2\,k_3}{-2\sigma^2(1+\sigma^2 l^2)}},\quad \lambda\,\sigma^2\,(\sigma^2 l^2 - 1) < 0 \qquad (46)$$

## 5. The Modified Simple Equation Method

Finally, we apply the Modified Simple Equation Method. We also apply it on the more general equation (16), i.e. equation (36). The idea is to consider the function $u(\xi)$ in the following form:

$$u(\xi) = A_0 + A_1 \frac{\psi_\xi}{\psi} \qquad (47)$$

Where $\psi_\xi$ is the unknown function and $\psi_\xi \neq 0$. Then, the first and second derivative of $u(\xi)$ are given
By

$$u_\xi = A_1\left(\frac{\psi\,\psi_{\xi\xi} - \psi_\xi{}^2}{\psi^2}\right)$$
$$u_{\xi\xi} = A_1\left(\frac{\psi^2\psi_{\xi\xi\xi} - 3\psi\,\psi_{\xi\xi}\psi_\xi + 2\,\psi_\xi{}^3}{\psi^3}\right) \qquad (48)$$

and (36) becomes

$$A_1\,c_0 c_2\left(\frac{\psi_{\xi\xi\xi}}{\psi} - \frac{3\psi_{\xi\xi}\psi_\xi}{\psi^2} + 2\frac{\psi_\xi{}^3}{\psi^3}\right) + 2\lambda\,[\,c_2 + 2\,k]\,\left(A_0 + A_1\frac{\psi_\xi}{\psi}\right)^3 - c_1 c_2\,\left(A_0 + A_1\frac{\psi_\xi}{\psi}\right) = 0 \qquad (49)$$

Expanding the equation (49), grouping all terms with the same power of  and equating them to zero, we get the following system of equations:

$$2\,A_1[\,A_1{}^2\,\lambda\,(c_2 + 2\,k) + c_0 c_2] = 0$$
$$3\,A_1\psi'(2\,\lambda\,A_0 A_1(c_2 + 2\,k)\,\psi' - c_0 c_2\,\psi'') = 0$$
$$A_1\left(6\lambda A_0{}^2\,(c_2 + 2\,k)\psi' + c_2\left(c_0\,\psi^{(3)} - c_1\psi'\right)\right) = 0$$
$$2A_0{}^3\lambda\,(c_2 + 2\,k) - A_0 c_1 c_2 = 0 \qquad (50)$$

From the last equation of (50) we have two possibilities: $A_0 = 0$ or

$$A_0 = \sqrt{\frac{c_1 c_2}{2\lambda\,[\,c_2 + 2\,k]}} \qquad (51)$$

Assume that $A_0 = 0$ , Then the second equation in (50) becomes
$$-3\,A_1 c_0 c_2\,\psi'\psi'' = 0$$
implying either $\psi' = 0$ or $\psi'' = 0$. The first case leads to the trivial solution u = const. while the second case implies $\psi''' = 0$ and then the third equation also further implies $\psi' = 0$ or $u = cons$
Assume now that $A_0$ is given by (51). From the first equation of (50) we find ,

$$A_1 = \sqrt{-\frac{c_0 c_2}{\lambda\,[\,c_2 + 2\,k]}} \qquad (52)$$

Expressions (51) and (52) are valid under the assumptions that the expressions under square roots are positive. Replacing (51) and (52) into the system (50) we get

$$\psi''' - \mu^2\,\psi' = 0$$
$$\psi'' + \mu\,\psi' = 0 \qquad (53)$$

Where

$$\mu = \sqrt{-\frac{2\,c_1}{c_0}}$$

The solution of (53) is given by

$$\psi(\xi) = -\frac{b}{\mu}\,e^{-\mu\xi} + C \qquad (54)$$

where $b \neq 0$ and $C$ are arbitrary real constants. Replacing the previous expression into (47) and using (51) and (52), we get

$$u(\xi) = \sqrt{\frac{c_1 c_2}{2\lambda\,[\,c_2 + 2\,k]}}\,\left(1 + \frac{2}{C_1\,e^{\mu\xi} - 1}\right) \qquad (55)$$

where $C_1 = C\,b^{-1}\mu \neq 0$ is an arbitrary real constant. In that case, $V(\xi)$ is given by

$$V(\xi) = -\frac{2\,\lambda}{c_2}\int u^2\,d\xi$$
$$V(\xi) = -\frac{c_1}{c_2 + 2\,k}\,\left(\xi - \frac{4}{\mu(C_1\,e^{\mu\xi} - 1)}\right) + C \qquad (56)$$

therefore

$$q(x,y,t) = e^{i\,[k_1 x + k_2 y + k_3 t]}\,\sqrt{\frac{c_1 c_2}{2\lambda\,[\,c_2 + 2\,k]}}\,\left(1 + \frac{2}{C_1\,e^{\mu\xi} - 1}\right)$$

and

$$\Phi(x,y,t) = -\frac{c_1}{c_2 + 2\,k}\,\left(\xi - \frac{4}{\mu(C_1\,e^{\mu\xi} - 1)}\right) + C \qquad (57)$$

where $C_1 \neq 0$ is an arbitrary real constant. $\xi = k\,(x + ly - \sigma^2(k_1 + \sigma^2\,k_2 l\,)t)$, and

$$\mu = \sqrt{-\frac{2[\sigma^2(k_1{}^2 + \sigma^2\,k_2{}^2) + 2\,k_3]}{\sigma^2 k^2(1+\sigma^2 l^2)}} \qquad (58)$$

The required conditions are again $\lambda = \sigma^2(k_1 + \sigma^2 l\, k_2)$ , $\frac{c_0 c_2}{\lambda\,(c_2+2\,k)} < 0$, ( i.e $\lambda\,\sigma^2\,(\sigma^2 l^2 - 1) < 0$,) and $c_0\,c_1 < 0$ (59)

## 6. Conclusion

In this paper, several new soliton solutions of the (2+1)-dimensional DS equation are found. The obtained solutions are very useful in the field of nonlinear science. Shown procedure can be also applied to solve other types of the generalized nonlinear evolution equations with complex coefficients.

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## References

[1] H. Jafari, M. Alipour (2010), Solution of the Davey–Stewartson

equation using homotopy analysis method, *Nonlinear Analysis: Modelling and Control*, Vol.15, No.4, 423–433.

[2] Hassan A. Zedan and Ahmed Al Saedi (2010), Periodic and Solitary Wave Solutions of the Davey-Stewartson Equation, *Applied Mathematics & Information Sciences*, Vol.4, No.2, 253–260

[3] C. Babaoglu (2008), Long-Wave Short-Wave Resonance Case for a Generalized Davey-Stewartson System, *Chaos Soliton. Fract*. Vol.38 , No. 1, 48-54.

[4] K. W. Chow, and S. Y. Lou (2006), Propagating Wave Patterns and 'Peakons' of the Davey-Stewartson System, *Chaos Soliton. Fract*. Vol.27, No. 2, 561-567.

[5] G. Ebadi, E. V. Krishnan, M. Labidi, E. Zerrad, and A. Biswas (2011), Analytical and numerical solutions to the Davey-Stewartson equation with power-law nonlinearity, *Waves Random Media* Vol.21, No. 4, 559-590.

[6] G. Ebadi, and A. Biswas (2011), The (G'/G ) method and 1-soliton solution of the Davey-Stewartson equation, *Math. Comput. Model*. Vol.53 , No. 5-6, 694-698.

[7] Hossein Jafaria, Atefe Soorakia, Yahya Talebia, Anjan Biswas.(2012), The first integral method and traveling wave solutions to Davey–Stewartson equation, *Nonlinear Analysis: Modeling and Control*, Vol. 17, No. 2, 182–193

[8] Ghodrat Ebadia, E.V. Krishnanb, Manel Labidic, Essaid Zerradd and Anjan Biswas (2011), Analytical and numerical solutions to the Davey–Stewartson equation with power-law nonlinearity, *Waves in Random and Complex Media*, Vol. 21, No. 4, 559–590.

A. Davey, K. Stewartson (1974), On three-dimensional packets of surfaces waves, *Proc. R. Soc. Lond., Ser. A.*, Vol.338, 101–110.

[9] Ghodrat Ebadi a, Anjan Biswas (2011), The G′/ G method and 1-soliton solution of the Davey–Stewartson equation, *Mathematical and Computer Modelling* Vol.53, 694–698.

[10] Y. Gurefe, and E. Misirli (2011), New variable separation solutions of two-dimensional Burgers system, *Appl. Math. Comput*. Vol. 217, No. 22, 9189-9197.

[11] E. Misirli, and Y. Gurefe (2010), Exact solutions of the Drinfel'd-Sokolov-Wilson equation using the Exp-function method, *Appl. Math. Comput.* Vol.216, No. 9, 2623-2627.

[12] Y. Gurefe, and E. Misirli (2011), Exp-function method for solving nonlinear evolution equations with higher order nonlinearity, *Comput. Math. Appl*. Vol. 61 , No. 8, 2025-2030.

[13] E. Fan (2000), Extended tanh-function method and its applications to nonlinear equations, *Phys. Lett. A* Vol.277 , No. 4, 212-218.

[14] M. Wang (1995), Solitary wave solutions for variant Boussinesq equations, *Phys. Lett. A* Vol. 199, No. 3, 169-172.

A. V. Porubov, and M. G. Velarde (1999), Exact periodic solutions of the complex Ginzburg-Landau equation, *J. Math. Phys.* Vol.40 , No. 2, 884-896.

[15] Huber (2001), The calculation of novel class of solutions of a non-linear fourth order evolution equation by the Weierstrass transform method, *Appl. Math. Comput*. Vol. 201, No. 1-2, 668-677.

[16] Huber (2006), A novel class of solutions for a non-linear third order wave equation generated by the Weierstrass transformation, *Chaos Soliton. Fract.* Vol.28, No. 4, 972-978.

[17] E. A. Wakil, E. M. Abulwafa, and M. A. Abdou (2010), Extended Weierstrass transformation method for nonlinear evolution equations, *Nonl. Sci. Lett. A: Math. Phys. Mech.* Vol.1, No. 3, 253-262.

[18] S. Liu (2005), Trial equation method and its applications to nonlinear evolution equations, *Acta Phys. Sin.-Ch.* Ed. Vol.54, No. 6, 2505.

[19] S. Liu (2010), Applications of complete discrimination system for polynomial for classifications of traveling wave solutions to nonlinear differential equations, *Comput. Phys. Commun*. Vol.181, No. 2, 317-324.

[20] X. H. Du (2010), An irrational trial equation method and its applications, *Pramana-J. Phys*. Vol.75, No. 3, 415-422.

[21] Y. Jun (2011), Classification of traveling wave solutions to the Vakhnenko equations, *Comput. Math. Appl*. Vol.62, No. 10, 3987-3996.