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Research paper

D-distance Magic Labeling on Some Class of Graphs

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Abstract

In this paper we study $\left(0,\frac{n}{2}\right)$, $(0,\frac{n+1}{2})$ -distance magic labeling on path with n vertices, D-distance magic labeling on trees when D= (0,1) and D=(d) where d is a diameter of tree and (0,1)-distance magic labeling of complete graph..

Keywords: *distance magic labeling, centre, path, diameter, complete graph.*

1. Introduction

In 1963, Sedlacek introduced magic labeling of graph G=G(V,E).

Definition 1.1:
[4] A bijection f : E → set of +ve integers (E is a edge set) such that $f(e_i) \neq f(e_j)$ for all distinct $e_i, e_j \in E$ and $\sum_{e\in N_E(x)} f(e)$ is same for all $x \in V$, where $N_E(x)$ is the set contains all the edges incident on X .Such a labelling is called magic labelling.
In 1994, Vilfred and in 2003 Miller.et.al separately introduced distance magic labeling.
Definition 1.2: [6] A bijection $f: V \rightarrow \{1,2,\dots v\}$ such that at any vertex x, $\sum_{y\in N(x)} f(y) = k$, where N(x) is the set of vertices which are adjacent to x and k is a magic constant. This labelling is called distance magic labelling.
Later Jinah introduced variations of distance magic labeling. Here instead of open neighbourhood he took closed neighbourhood. The concept was independently studied by Simanjuntak, Rodgers and Miller. In particular properties of such labeling for a distance set D.

Definition1.3:
[6]A bijection $f: V \rightarrow \{1,2,\dots,v\}$ such that for any vertex x, $w(x) = \sum_{y\in N_D(x)} f(y) = k$
Where $N_D(x)$ is the set containing all the vertices $y \in V$ such that $d(x,y) \in D$.This labelling is D-distance magic labelling.

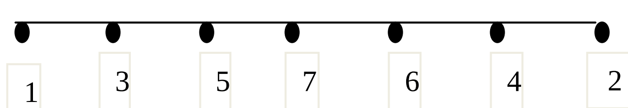
2. Main Results

Theorem 2.1:
A path P_n with n vertices is $\left(0,\frac{n}{2}\right)$ -distance magic graph for even n and $(0,\frac{n+1}{2})$-distance magic graph for odd n.

Proof:
Let n be even.
Then P_n has two central points
Label the vertices as 1,3,5,...(n-1),n,(n-2)…4,2 and name it as $v_1, v_3, v_5 \dots, v_{n-1}, v_n, v_{n-2}, \dots v_4 v_2$.
Then central points v_{n-1} and v_n gets labeled as (n-1) and n.
Since $D = \left(0,\frac{n}{2}\right)$
$N_D(v_1) = v_1, v_n$ & $N_D(v_n) = v_1, v_n$
So $w(v_1) = n+1$ & $w(v_n) = n+1$
$N_D(v_3) = v_3, v_{n-2}$ & $N_D(v_{n-2}) = v_{n-2}, v_3$
So $w(v_3) = n+1$ & $w(v_{n-2}) = n+1$
Continuing the process we get
$N_D(v_{n-1}) = v_{n-1}, v_2$ & $N_D(v_2) = v_2, v_{n-1}$
So $w(v_{n-1}) = n+1$ & $w(v_2) = n+1$
Observe that weight of each vertex is n+1
That implies P_n is $\left(0,\frac{n}{2}\right)$ -distance magic graph when n is even.
Similarly we can prove for odd n.

Observation:
When P_n is $\left(0,\frac{n}{2}\right)$ -distance magic graph, magic constant is n+1
When P_n is $(0,\frac{n+1}{2})$-distance magic graph, magic constant is n.

Example 2.1:
P_7 is a (0, 4)-distance magic graph with magic constant 7.Graph shown below



Example 2.1:
P_7 Is a (0, 4)-distance magic graph (k=7)

Theorem 2.2:
A tree with diameter 2 is not (0,1)-distance magic graph.

Proof:
Let V be the vertex set of a tree T with diameter 2.
Since diameter of T is 2, there exists exactly one vertex $v \in T$ such that v is adjacent to all other vertices.
Then $N_{(0,1)}(v)$=V that implies $w(v) = \sum_{u \in V} f(u)$
But $N_{(0,1)}(u) \subseteq V$ for every $u \in V, u \neq v$
That implies $w(v) \neq w(u)$, for every $u \in V, u \neq v$
This proves that T is not a (0, 1)-distance magic.

Conjecture:
[6] A Graph G is not (d)-distance magic, where d is a diameter of G.
We prove the conjecture for trees with diameter d.

Theorem 2.3:
A tree T with diameter d is not (d)-distance magic.

Proof:
Consider a tree T with diameter d.
Let x_1 be a central point of T.
Then $N_d\,(x_1)=\emptyset$
that implies $w(x_1) = 0$
But $w(x_i) \neq 0$ for some i, $i \neq 1, i = 1,2,3, \ldots$
If $w(x_i) = 0$ for all i then diameter of T will be less than d, which is a contradiction.
Therefore $w(x_1) \neq w(x_i)$ for some i, $i \neq 1$
This proves the theorem.

Theorem 2.4:
Complete graphs are (0, 1)-distance magic graph.

Proof:
For any vertex x of a complete graph G,$N_{(0,1)}(x) = V$ where V is a vertex set.
That implies w(x) =k for any $x \in V$.
Hence the proof.

3. Conclusions

It is very interesting to see d-distance magic labeling of other class of graphs such as regular graph, multipartite graph.

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