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Research paper

Three Degree of Freedom Spatial Parallel Manipulator Inverse Kinematic Position Analysis

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Abstract

This paper presents inverse kinematic position analysis of three degree of freedom spatial parallel manipulator, which has three similar kinematic closed loops. Each loop consist of an actuated sliding linkage- rotational joint and spherical joint. The actuated sliding linkage is coupled to inclined limb of fixed base platform and rotational joints are integrated to the linear sliding actuators. The limbs are connected from rotational joints to moving platform by spherical joints. The degree of freedom of a manipulator is obtained by spatial kutzbach criterion. The inverse kinematic position analysis problem solved by using closed loop technique is applied to 3-coupled trigonometric equations which are obtained with side and behaviour constraints of a parallel manipulator. By using MATLAB the three non-linear coupled algebraic equations are solved. The inverse kinematic position analysis procedure is used in the development process of spatial parallel manipulator. The part of kinematic analysis is used to check the required positions-orientations and after kinematic process the obtained positions-orientations of the moving platform of the developed spatial parallel manipulator.

Keywords: *Inverse kinematics; position analysis; 3- dof spatial parallel manipulator*

1. Introduction

The spatial parallel manipulators perform the task of controlling the moving platform with respect to the base platform. The 6-dof parallel manipulators such as Stewart –Gough platform type and Stewart universal type test machines [1,2] suffer the analysis part of position kinematics. For several applications require less than six degree of freedom spatial parallel manipulators. Yangmin Li and QingsongXu[3,4] proposed kinematic analysis of 3-dof translational parallel manipulators which are moving in rectangular coordinate system. Meng-Shiun T sai[5] proposed the analysis of a 3-PRS parallel mechanism which indicate the analysis of mechanism. J.A.carretero [6] proposed analysis and optimization of parallel manipulator in which prismatic joints are actuated. Lee and Shah[7] proposed analysis of three degree of freedom parallel actuated manipulator, in which the motion will be constrained. Yangmin Li and QingsongXu[8] presented the analysis of 3-PRS parallel manipulator in which the sliding joints were actuated. Xin-JunLiu and Farhad Tahmasebi[9,10] performed new parallel manipulators using the Grubler mobility criterion as it may be demonstrated that the mechanism has three degree of freedom. The purpose of this work is to develop an analytical and systematic design procedure to analyze the basic inverse position kinematics of 3-dof spatial parallel manipulator. The three trigonometric coupled algebraic equations generated by closed loop technique, and also considered the side and behaviour constraints of the spatial parallel manipulator. By using MATLAB the three non-linear coupled algebraic kinematic equations are solved.

2. Geometry Description of Three Degree of Freedom Spatial Parallel Manipulator

The 3-dof spatial parallel manipulator consists of a moving platform which is connected to a fixed base by three supporting limbs, actuators and rotational joints in three symmetric loops as shown in Fig 1. In these three loops the number of rotational joints, type of joints and number of linear actuators and number of limbs are same and equal to the degree of freedom of moving platform of the parallel manipulator. The linear actuated joint of each limb is inclined from the fixed link of base platform by an angle α_{ji} for i^{th} position of moving limb. The linear actuator is actuated on a limb of fixed length via a rotational joint and limbs are connected to the moving platform by spherical joints. The fixed platform co-ordinate reference frame O{x, y, z} is considered at the centre of $\Delta^{le} B_1 B_2 B_3$. Similarly the moving platform co-ordinate reference frame P {u, v, w} is considered at the centre of $\Delta^{le} S_1 S_2 S_3$. Consider x-axis in the direction $\overrightarrow{OB_1}$ and u-axis in the direction $\overrightarrow{PS_1}$. For each linear actuation along three symmetric loops the distance between B_j and P_{R_j} is denoted by $\overrightarrow{d_{ji}}\big|_{j=1,2,3 \ and \ i=1,2,3,4 \ etc}$ as in Fig. 1. The Fig.2 and Fig. 3 represent the position vector of fixed platform vertices $B_j\big|_{j=1,2,3}$ and moving platform vertices $S_j\big|_{j=1,2,3}$ with respect to fixed base frame centre O {x ,y, z} and moving platform frame centre P {u, v, w} can be expressed as follows:

$OB_1 = g_1, OB_2 = g_2, OB_3 = g_3, OS_1 = h_1, OS_2 = h_2, OS_3 = h_3, \overrightarrow{OP} = \overrightarrow{p_i}, \overrightarrow{OS_{ji}} = \overrightarrow{q_{ji}}$

$\overrightarrow{P_{R_j}S_j}\Big|_{j=1,2,3} = \overrightarrow{L_j}\Big|_{j=1,2,3}$

2.1. Geometry Diagrams of Three Degree of Freedom Spatial Parallel Manipulator

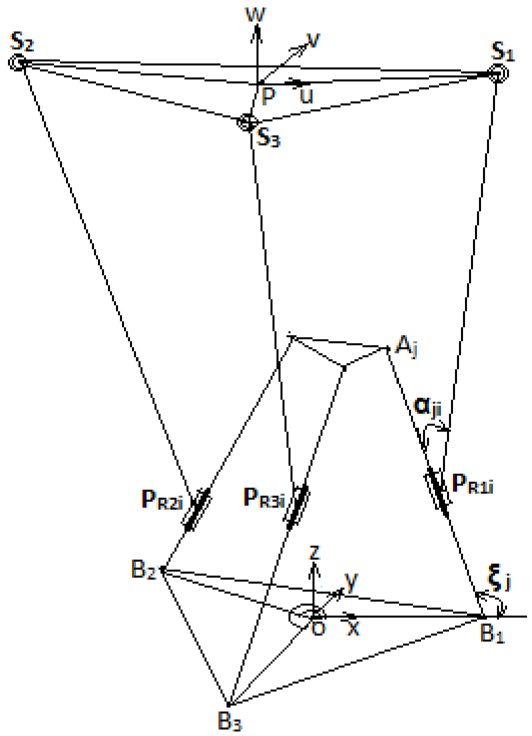

Fig. 1: Three degree of freedom spatial parallel manipulator

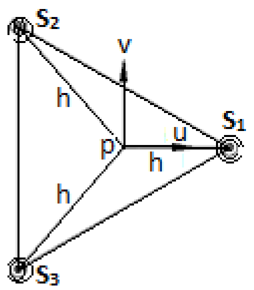

Fig. 2: Geometry of moving platform

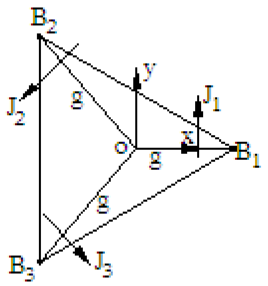

Fig. 3: Geometry of fixed base platform

3. Degree pf Freedom of Spatial Parallel Manipulator

The degree of freedom is the number of independent translations and rotations required for moving platform to specify the configuration of a spatial parallel manipulator. The equation depends on number of limbs, number of joints and type of joints incorporated in the selected manipulator. The parallel manipulator consists of 8-links, 3- rotational joints, 3-sliding joints and 3- spherical joints. The degree of freedom of parallel manipulator can be calculated by using spatial Kutzbach equation.

$$f = 6\,(n-1) - 5j_1 - 4j_2 - 3j_3 - 2j_4 - j_5 = 6*(8-1) - 5*6 - 4*0 - 3*3 - 2*0 - 1*0 = 3 \quad (1)$$

Where f denotes degree of freedom of a spatial manipulator.
n denotes number of links of a spatial manipulator.
$j_{1\ to}$ j_5 indicate the loss of 5-dof or 4-dof or 3-dof or 2-dof or 1-dof respectively of corresponding limbs or joints.

3.1. Position Constraints of Spatial 3-Dof Parallel Manipulator

The position vector of vertices $B_j\big|_{j=1,2,3}$ and $S_j\big|_{j=1,2,3}$ with respect to fixed base frame centre O{x, y, z} and mobile frame centre P{u, v, w} can be expressed as follows:

$$OB_1 = g_1 = [g \quad 0 \quad 0]^T, \qquad OB_2 = g_2 = \left[-\frac{1}{2}g \quad \frac{\sqrt{3}}{2}g \quad 0\right]^T$$

$$OB_3 = g_3 = \left[-\tfrac{1}{2}g \quad -\tfrac{\sqrt{3}}{2}g \quad 0\right]^T, \; OS_1 = h_1 = [h \quad 0 \quad 0]^T$$

$$OS_2 = h_2 = \left[-\frac{1}{2}h \quad \frac{\sqrt{3}}{2}h \quad 0\right]^T$$

$$OS_3 = h_3\left[-\frac{1}{2}h \quad -\frac{\sqrt{3}}{2}h \quad 0\right]^T$$

Let $\vec{u}, \vec{v}, \vec{w}$ be represents three unit vectors along u, v and w-axis of the mobile platform reference coordinate frame P{u, v, w}. Then the rotation matrix can be expressed in terms of direction cosines of $\vec{u}, \vec{v}, \vec{w}$ as

$$O_{R_P} = \begin{bmatrix} u_x & v_x & w_x \\ u_y & v_y & w_y \\ u_z & v_z & w_z \end{bmatrix} = \begin{bmatrix} c\theta_i c\phi_i + s\psi_i s\theta_i s\phi_i & -c\theta_i s\phi_i + s\psi_i s\theta_i c\phi_i & s\theta_i c\psi_i \\ c\psi_i s\phi_i & c\psi_i c\phi_i & -s\psi_i \\ -s\theta_i c\phi_i + s\psi_i c\theta_i s\phi_i & s\theta_i s\phi_i + s\psi_i c\theta_i c\phi_i & c\theta_i c\psi_i \end{bmatrix} \quad (2)$$

Where ϕ_i, θ_i, ψ_i Euler angles are indicate about ZXY fixed axis. The Position vector of mobile frame vertices $S_j\big|_{j=1,2,3}$ with respect to fixed base frame centre O {x, y, z} can be expressed as $\overrightarrow{OS_j} = \overrightarrow{OP} + \overrightarrow{PS_j}$ that means $\overrightarrow{OP_i}\big|_{i=1,2,3,4\ etc}$ and $\overrightarrow{P_iS_j}\big|_{j=1,2,3\ and\ i=1,2,3,4\ etc}$ represent position vector and rotation matrix of a reference point $P_i|_{i=1,2,3,4\ etc}$ on a mobile platform respectively at the i[th] specified position and orientation with respect to the fixed reference frame O { x, y, z}. i.e.,

$$\overrightarrow{OS_{ji}} = \overrightarrow{OP_i} + \overrightarrow{P_iS_j} \Rightarrow \overrightarrow{q_{ji}} = [P_{xi} \quad P_{yi} \quad P_{zi}]^T + o_{R_{Pi}} h_j^{uvw} \quad (3)$$

Where $\overrightarrow{OP_i}$ is the transformation from the moving frame to the fixed frame that can be described by position vector $\overrightarrow{OP_i} = [P_{xi} \quad P_{yi} \quad P_{zi}]^T$ and rotation matrix $O_{R_{Pi}} = \begin{bmatrix} u_{xi} & v_{xi} & w_{xi} \\ u_{yi} & v_{yi} & w_{yi} \\ u_{zi} & v_{zi} & w_{zi} \end{bmatrix}$.

The position kinematic equations of mobile frame vertices with respect to fixed base frame centre O {x, y, z} i.e.,

$$\overrightarrow{q_{1i}} = \begin{bmatrix} P_{xi} + hu_{xi} \\ P_{yi} + hu_{yi} \\ P_{zi} + hu_{zi} \end{bmatrix} \quad (4)$$

$$\overrightarrow{q_{2i}} = \begin{bmatrix} P_{xi} - \frac{1}{2}hu_{xi} + \frac{\sqrt{3}}{2}hv_{xi} \\ P_{yi} - \frac{1}{2}hu_{yi} + \frac{\sqrt{3}}{2}hv_{yi} \\ P_{zi} - \frac{1}{2}hu_{zi} + \frac{\sqrt{3}}{2}hv_{zi} \end{bmatrix} \quad (5)$$

Similarly

$$\overrightarrow{q_{3i}} = \begin{bmatrix} P_{xi} - \frac{1}{2}hu_{xi} - \frac{\sqrt{3}}{2}hv_{xi} \\ P_{yi} - \frac{1}{2}hu_{yi} - \frac{\sqrt{3}}{2}hv_{yi} \\ P_{zi} - \frac{1}{2}hu_{zi} - \frac{\sqrt{3}}{2}hv_{zi} \end{bmatrix} \quad (6)$$

3.2. Revolute Joint Constraints of Spatial 3-Dof Parallel Manipulator

The mechanical constraints are imposed by the revolute joints, so the spherical joints at $S_j\big|_{j=1,2,3}$ can only move in the planes of OB_jS_j. Therefore the constraints from Eq. (4), Eq. (5) and Eq. (6) are

$$[q_{1i}]_y = 0 \Rightarrow P_{yi} + hu_{yi} = 0 \quad (7)$$

$$[q_{2i}]_y = -\sqrt{3}[q_{2i}]_x$$
$$P_{yi} - \tfrac{1}{2}hu_{yi} + \tfrac{\sqrt{3}}{2}hv_{yi} = -\sqrt{3}\left[P_{xi} - \tfrac{1}{2}hu_{xi} + \tfrac{\sqrt{3}}{2}hv_{xi}\right] \quad (8)$$

Similarly
$$[q_{3i}]_y = \sqrt{3}[q_{3i}]_x$$
$$P_{yi} - \tfrac{1}{2}hu_{yi} - \tfrac{\sqrt{3}}{2}hv_{yi} = \sqrt{3}\left[P_{xi} - \tfrac{1}{2}hu_{xi} - \tfrac{\sqrt{3}}{2}hv_{xi}\right] \quad (9)$$

By adding Eq. (7), Eq. (8) and Eq. (9) then
$P_{yi} = -hv_{xi} \Rightarrow u_{yi} = v_{xi}$ therefore
$$P_{xi} = \tfrac{1}{2}h\big(u_{xi} - v_{yi}\big) \quad (10)$$

4. Inverse Kinematic Position Analysis

Inverse kinematic analysis is obtained by locations. If positions and orientations of moving platform are given, then the vector position of the three prismatic actuated joint variables for i^th^ positions of $\overrightarrow{d_{ji}}\big|_{j=1,2,3\ and\ i=1,2,3,4\ etc}$ are to be resolved by inverse kinematic analysis. These actuators are actuated along the inclined fixed limbs which are at an angle of ξ_j with respect to $\overrightarrow{OB_j}\big|_{j=1,2,3}$. Let us consider the three degrees of unconstrained variables θ_i, ψ_i, P_{zi} of the moving platform for i^th^ positions are given, then the three constrained variables that are ϕ_i, P_{xi}, P_{yi} are to be determined from Eq. (2), Eq. (9) and Eq. (10) are

$$\tan\phi_i = \frac{\sin\psi_i \sin\theta_i}{\cos\psi_i + \cos\theta_i} \quad (11)$$

$$P_{xi} = \frac{h}{2\cos\phi_i}(\cos\theta_i - \cos\psi_i\cos 2\phi_i) \quad (12)$$

$$P_{yi} = -h\cos\psi_i \sin\phi_i \quad (13)$$

Then the vectors position of the three prismatic actuated joint variables for i^th^ position of $\overrightarrow{d_{ji}}\big|_{j=1,2,3\ and\ i=1,2,3,4\ etc}$ are to be determined by the geometry

$\overrightarrow{d_{ji}} = \overrightarrow{q_{ji}} - \overrightarrow{g_j} - \overrightarrow{L_j}\ for\ \big|_{j=1,2,3\ and\ i=1,2,3,4\ etc}$, that means
$$\overrightarrow{d_{ji}} = \overrightarrow{p_i} + {}^{o}R_{p_i}\overrightarrow{h_j} - \overrightarrow{g_j} - \overrightarrow{L_j} \quad (14)$$

Let us consider at $\beta_j\big|_{j=1,2,3}\ are\ 0^0, 120^0, 240^0 and\ g_1 = g_2 = g_3 = g, L_1 = L_2 = L_3 = L, h_1 = h_2 = h_3 = h, \xi_1 = \xi_2 = \xi_3 = \xi$
Then the Eq. (14) can be written as

$$\begin{bmatrix} d_{ji}\cos\xi\cos\beta_j \\ d_{ji}\cos\xi\sin\beta_j \\ d_{ji}\sin\xi \end{bmatrix} = \begin{bmatrix} p_{xi} \\ p_{yi} \\ p_{zi} \end{bmatrix} + \begin{bmatrix} u_{xi} & v_{xi} & w_{xi} \\ u_{yi} & v_{yi} & w_{yi} \\ u_{zi} & v_{zi} & w_{zi} \end{bmatrix}\begin{bmatrix} h\cos\beta_j \\ h\sin\beta_j \\ 0 \end{bmatrix} - \begin{bmatrix} g\cos\beta_j \\ g\sin\beta_j \\ 0 \end{bmatrix} - \begin{bmatrix} L\cos(\xi-\alpha_{ji})\cos\beta_j \\ L\cos(\xi-\alpha_{ji})\sin\beta_j \\ L\sin(\xi-\alpha_{ji}) \end{bmatrix} \quad (15)$$

$$\begin{bmatrix} L_{xji} \\ L_{yji} \\ L_{zji} \end{bmatrix} = \begin{bmatrix} L\cos(\xi-\alpha_{ji})\cos\beta_j \\ L\cos(\xi-\alpha_{ji})\sin\beta_j \\ L\sin(\xi-\alpha_{ji}) \end{bmatrix}$$

$$\begin{bmatrix} L_{xji} \\ L_{yji} \\ L_{zji} \end{bmatrix} = \begin{bmatrix} p_{xi} \\ p_{yi} \\ p_{zi} \end{bmatrix} + \begin{bmatrix} c\theta_i c\phi_i + s\psi_i s\theta_i s\phi_i & -c\theta_i s\phi_i + s\psi_i s\theta_i c\phi_i & s\theta_i c\psi_i \\ c\psi_i s\phi_i & c\psi_i c\phi_i & -s\psi_i \\ -s\theta_i c\phi_i + s\psi_i c\theta_i s\phi_i & s\theta_i s\phi_i + s\psi_i c\theta_i c\phi_i & c\theta_i c\psi_i \end{bmatrix}\begin{bmatrix} h\cos\beta_j \\ h\sin\beta_j \\ 0 \end{bmatrix} - \begin{bmatrix} g\cos\beta_j \\ g\sin\beta_j \\ 0 \end{bmatrix} - \begin{bmatrix} d_{ji}\cos\xi\cos\beta_j \\ d_{ji}\cos\xi\sin\beta_j \\ d_{ji}\sin\xi \end{bmatrix} \quad (16)$$

The Eq. (16) is in the form of
$$\begin{bmatrix} L_{xji} \\ L_{yji} \\ L_{zji} \end{bmatrix} = \begin{bmatrix} C_{xji} + C_{xj}d_{ji} \\ C_{yji} + C_{yj}d_{ji} \\ C_{zji} + C_{zj}d_{ji} \end{bmatrix} \quad (17)$$

Where
$C_{xji} = p_{xi} + (\cos\theta_i\ \cos\phi_i + \sin\psi_i \sin\theta_i\ \sin\phi_i)h\cos\beta_j + (-\cos\theta_i\ \sin\phi_i + \sin\psi_i \sin\theta_i\ \cos\phi_i)h\sin\beta_j - g\cos\beta_j$

$C_{xj} = -\cos\xi\cos\beta_j$

$C_{yji} = p_{yi} + (\cos\psi_i \sin\phi_i)h\cos\beta_j + (\cos\psi_i\cos\phi_i)h\sin\beta_j - g\sin\beta_j$

$C_{yj} = -\cos\xi\sin\beta_j$

$C_{zji} = p_{zi} + (-\sin\theta_i\ \cos\phi_i + \sin\psi_i\cos\theta_i\ \sin\phi_i)h\cos\beta_j + (\sin\theta_i\ \sin\phi_i + \sin\psi_i\cos\theta_i\ \cos\phi_i)h\sin\beta_j$

$C_{zj} = -\sin\xi$

In Eq. (17) the unconstrained variables θ_i, ψ_i, P_{zi} of the moving platform for i^th^ position are given and the three constrained variables ϕ_i, P_{xi}, P_{yi} are calculated by using Eq. (11), Eq. (12) and Eq. (13). The unconstrained variables θ_i, ψ_i, P_{zi} and the assumed values of $\beta_j\big|_{j=1,2,3}$ are 0°, 120°, 240° and also $g_1 = g_2 = g_3 = g$, $L_1 = L_2 = L_3 = L$, $h_1 = h_2 = h_3 = h$, $\xi_1 = \xi_2 = \xi_3 = \xi$ are the suitable considered values of the structure of the parallel manipulator. But $L_{xji}, L_{yji}, L_{zji}\ and\ d_{ji}$ are the unknown values, that means $\alpha_{ji}\ and\ d_{ji}$ are the unknown values.

From geometry, the Eq. (17) can be written as
$$L^2 = L_{xji}{}^2 + L_{yji}{}^2 + L_{zji}{}^2$$
$$L^2 = C_{xji}{}^2 + C_{yji}{}^2 + C_{zji}{}^2 + \big(C_{xj}{}^2 + C_{yj}{}^2 + C_{zj}{}^2\big)d_{ji}{}^2 + \big(2C_{xji}C_{xj} + 2C_{yji}C_{yj} + 2C_{zji}C_{zj}\big)d_{ji} \quad (18)$$

The Eq. (18) is in the form of
$$A_{1j}d_{ji}{}^2 + A_{2ji}d_{ji} + A_{3ji} = 0 \quad (19)$$

Where
$A_{1j} = C_{xj}{}^2 + C_{yj}{}^2 + C_{zj}{}^2$
$A_{2ji} = 2C_{xji}C_{xj} + 2C_{yji}C_{yj} + 2C_{zji}C_{zj}$
$A_{3ji} = C_{xji}{}^2 + C_{yji}{}^2 + C_{zji}{}^2 - L^2$

From the Eq. (19) the value of d_{ji} can be calculated as
$$d_{ji} = \frac{-A_{2ji} \pm \sqrt{A_{2ji}{}^2 - 4A_{1j}A_{3ji}}}{2A_{1j}} \quad (20)$$

That means the three prismatic actuated joint variables for i[th] number of positions of $\overrightarrow{d_{ji}}\,\big|_{j=1,2,3\ and\ i=1,2,3,4\ etc}$ can be calculated by using Eq. (20). These actuators of the manipulator are actuated along the inclined fixed limbs which are at an angle of ξ_j with respect to $\overrightarrow{OB_j}\,\big|_{j=1,2,3}$.

Substitute Eq. (20) the calculated values of d_{ji} for i[th] number of positions in the Eq. (17) then the values of L_{xji} , L_{yji}, L_{zji} are to be calculated. The Eq. (17) can be represented as

$$\begin{bmatrix} L_{xji} \\ L_{yji} \\ L_{zji} \end{bmatrix} = \begin{bmatrix} L\cos(\xi - \alpha_{ji})\cos\beta_j \\ L\cos(\xi - \alpha_{ji})\sin\beta_j \\ L\sin(\xi - \alpha_{ji}) \end{bmatrix} = \begin{bmatrix} C_{xji} + C_{xj}d_{ji} \\ C_{yji} + C_{yj}d_{ji} \\ C_{zji} + C_{zj}d_{ji} \end{bmatrix} \qquad (21)$$

Therefore the Eq. (21) can be represented as

$$L\cos(\xi - \alpha_{ji}) = \sqrt{{C_{xji}}^2 + {C_{yji}}^2 + ({C_{xj}}^2 + {C_{yj}}^2){d_{ji}}^2 + (2C_{xji}C_{xj} + 2C_{yji}C_{yj})d_{ji}} \qquad (22)$$

The Eq. (21) and Eq. (22) simplified as

$$\tan(\xi - \alpha_{ji}) = \frac{C_{zji} + C_{zj}d_{ji}}{\sqrt{{C_{xji}}^2 + {C_{yji}}^2 + ({C_{xj}}^2 + {C_{yj}}^2){d_{ji}}^2 + (2C_{xji}C_{xj} + 2C_{yji}C_{yj})d_{ji}}} \qquad (23)$$

Then α_{ji} for i[th] number of positions for three prismatic actuators, that can be calculated from

$$\alpha_{ji} = \xi - \tan^{-1}\left(\frac{C_{zji} + C_{zj}d_{ji}}{\sqrt{{C_{xji}}^2 + {C_{yji}}^2 + ({C_{xj}}^2 + {C_{yj}}^2){d_{ji}}^2 + (2C_{xji}C_{xj} + 2C_{yji}C_{yj})d_{ji}}}\right) \qquad (24)$$

5. Conclusion

In this paper the inverse kinematic position analysis of spatial three degree of freedom parallel manipulator are analyzed. The degree of freedom of manipulator is obtained by using spatial kutzbach criterion. The Euler angle representation is considered along with side and behaviour constraints of manipulator in the plane of spherical joint-actuated sliding joint-limb. The inverse position kinematic problem is solved by using closed loop technique. The purpose of this inverse kinematics is to develop an analytical method and systematic design procedure to analyze the basic kinematics of a spatial parallel manipulator, to check the required positions- orientations and obtained positions-orientations of the moving platform of the developed manipulator.

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