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Research paper

Prime Labeling of Jahangir Graphs

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Abstract

The paper investigates prime labeling of Jahangir graph $J_{n,m}$ for $n \geq 2,\ m \geq 3$ provided that *nm* is even. We discuss prime labeling of some graph operations *viz*. Fusion, Switching and Duplication to prove that the Fusion of two vertices v_1 and v_k where *k* is odd in a Jahangir graph $J_{n,m}$ results to prime graph provided that the product *nm* is even and is relatively prime to *k*. The Fusion of two vertices v_{nm+1} and v_k for any *k* in $J_{n,m}$ is prime. The switching of v_k in the cycle C_{nm} of the Jahangir graph $J_{n,m}$ is a prime graph provided that *nm+1* is a prime number and the switching of v_{nm+1} in $J_{n,m}$ is also a prime graph .Duplicating of v_k, where *k* is odd integer and *nm + 2* is relatively prime to *k*,*k+2* in $J_{n,m}$ is a prime graph.

Keywords: *Prime labeling; Jahangir graph; Fusion; Switching and duplication.*

1. Introduction

The paper considers only finite simple undirected graph throughout. Prime labeling of a graph *G* is a bijection f: $V(G) \to \{1, 2,\ldots |V|\}$ such that gcd $(f(u),f(v))=1$ for each edge *uv* . A graph is called prime graph if it admits a prime labeling. The graph *G* has vertex set $V=V(G)$ and the edge set $E=E(G)$. The set of vertices adjacent to a vertex *u* of *G* is denoted as $N(u)$. For notation and terminology reference to Bondy and Murthy [1] has been made .
Prime labeling is a concept that has been introduced by Roger Entringer .Since then many researchers have studied prime labeling for different types graphs. The cycle C_n on n vertices is a prime graph was proved by Dertsky [2]. Later Fu [3] considered path P_n on n vertices to show that such graphs are prime graphs.
Roger surmised during the period 1980s that all trees possess prime labeling, and what he surmised could not be confirmed as a fact till now. Sundaram [8] is one of the exponents who studied the prime labeling for planner grid. Further investigations included the development of prime labelings by authors such as Ganesan and Balamurugan [4] who developed prime labellings for Theta graphs and Meena and Vaithilingam [6] for graphs related to Helm. In addition, Prime Labeling for several fan related graphs [7] have been proved by them. As far as cycle related graphs are concerned it was Vaidhya and K.K. Kanmani [9] who proved their prime labeling. Lee [5] has been attributed with establishing the fact that wheel Wn is a prime graph iff n is even.

Definition 1.1.
Under specific conditions, when the vertices of the graphs have been demarcated with values then such phenomenon is termed as (vertex) graph labeling.

Definition 1.2.
Suppose $G = (V(G), E(G))$ is a graph possessing n vertices. A bijection $f: V(G) \to \{1, 2, \ldots n\}$ is termed as Prime labeling, when $e = uv$, hcf(f (u) ,f (v)) = 1 for each edge. When prime labeling occurs a graph is considered as prime graph.

Definition 1.3.
In a graph G, the vertices which are an independent set are a set of interdependent vertices that are nonadjacent.

Definition 1.4.
Consider u and v are two separate vertices meant for a graph G. G_1 is a novel graph that has been designed by fusing (identifying) the two vertices u and v by a sole vertex x in G_1 in such a way that each edge that has been incident with either u (or) v in G is at present incident with x in G_1.

Definition 1.5.
A vertex switching G_v of graphs G has been procured by considering a vertex v of G, deleting the total edges which are incident with v and accumulating edges combining v to each vertex that are not neighboring to$v$ in G.

Definition 1.6.
Duplication of a vertex v of a graph G produces a new graph G' by adding a vertex v′ with N (v)= N (v′).
To define the other way a vertex v′ is told to have been in duplication of v under condition that all the vertices that are beside v are at present neighbored to v′ In G′ .

Definition 1.7.
Jahangir graph $J_{n,m}$ for $n \geq 2,\ m \geq 3$ is a graph with on nm + 1 vertices comprising a certain cycle C_{nm} possessing single vertex that is additional and is beside m vertices of C_{nm} placed at a distance n between the C_{nm}. Jahangir graph $J_{2,8}$ that is visible on his tomb which is located in his mausoleum.

2. Main Results of Prime Labeling on Jahangir Graph

Theorem 2. 1.

If nm is even then the Jahangir graph $J_{n,\,m}$ *for* $n \geq 2,\ m \geq 3$ is a prime graph.

Proof:

Let $J_{n,m}$ be a Jahangir graph. $V(J_{n,m})=\{v_1, v_2, \dots, v_{nm+1}\}$ and $E(J_{n,m}) = \{v_i\ v_{i+1}\ /\ 1\leq i \leq nm-1\}\cup v_{nm}v_1\cup\{v_{1+jn}v_{nm+1}\ /\ 0\leq j\leq m-1\}$ then $|V(J_{n,m})| = nm+1$ and $|E(J_{n,m})| = (n+1)m$. Here the set $\{v_i\ v_{i+1}\ /\ 1\leq i \leq nm-1\}\cup v_{nm}v_1$ represent as edges of the cycle and the set $v_{1+jn}v_{nm+1}\ /\ 0\leq j\leq m-1\}$ represent the set of edges adjacent to the vertex v_{nm+1}.

The vertex labeling of $J_{n,\,m}$ is $f{:}V(J_{n,\,m})\rightarrow\{1,2,\dots, nm+1\}$ such that $f(v_i) = i+1$ for $1\leq i\leq nm$ and $f(v_{nm+1}) = 1$. It is to be noted that with $nm+1$ vertices and $nm+1$ labelings f is bijection. As '1' is relatively prime to each natural number and any two successive natural numbers are relatively prime. Therefore, for each edge $e = uv\in E(J_{n,m})$ and gcd $(f(u), f(v))$ =1.Hence, $J_{n,m}$ proves to have undergone prime labeling. Therefore, $J_{n,m}$ is a prime graph.

Illustration 2.2.

The following graphs 1&2 indicates the Prime labeling of $J_{2,3}$ and $J_{3,4}$.

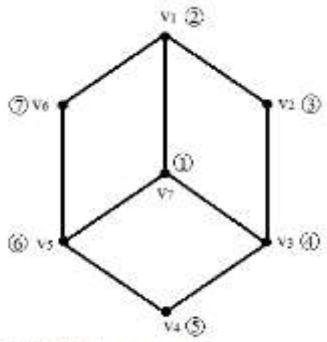

Figure 1. Jahangir graph $J_{2,3}$

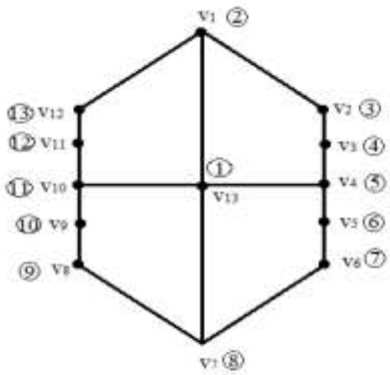

Figure 2. Jahangir graph $J_{3,4}$

Programme 2.3.

Pseudo code for the prime labeling of $J_{n,m}$ is written in 'C' programme

```
#include<stdio.h>
#include<conio.h>
int f(int);
intgcd(int,int);
intn,m;
void main()
{
inth,g,v[1000],e[1000],f1[1000],flag=1,i,j,flag1=1;;
clrscr();
printf("Enter n,m:");
scanf("%d%d",&n,&m);
for(i=1;i<=n*m;i++)
{
f1[i]=f(i);
printf("%d\n",f1[i]);
 }
for(i=1;i<n*m;i++)
 {
 g=gcd(f1[i],f1[i+1]);
printf("GCD=%d",g);
if(g!=1)
 {
flag=0;
break;
 }
 }
 h=gcd(f(n*m),f(1));
printf("h=%d",h);
for(j=0;j<=m-1;j++)
 {
 g=gcd(f(1+j*n),f(n*m+1));
printf("GCD1=%d",g);
if(g!=1)
 {
  flag1=0;
break;
 }
 }
printf("flag1=%d",flag1);
if(flag==1&&h==1&&flag1==1)
printf("\n Prime Graph");
else
printf("\n Not a Prime Graph");
getch();
}
int f(int i)
{
if(i==n*m+1)
return 1;
else
return i+1;
}
intgcd(inta,int b)
{
int r;
while(b!=0)
{
r=a%b;
a=b;
b=r;
}
return a;
}
```

Theorem 2.4.

When nm happens to be an odd number then $J_{n,m}$ will cease to be a prime graph.

Proof:

Note that the order of $J_{n,m}$ is $nm+1$. Hence, one has to use from 1 to $nm+1$ integers while labeling the vertices.In this way we have $\frac{nm+1}{2}$ odd integers. For the moment, one can allocate odd numbers to at most $\frac{nm+1}{2}$ (as nm is odd) vertices from among the said nm vertices in the cycle C_{nm}. Next one must assign a prime number to the center of the graph $J_{n,m}$ and each prime number is odd. Therefore, one has to assign at most places $\frac{nm+3}{2}$ odd numbers to the vertices. However, because we are having $\frac{nm+1}{2}$ odd numbers with us, it is not possible. Finally, $J_{n,m}$ is not considered to be a prime graph for odd *nm*.

3. Main Results on Fusion of Vertices in the Jahangir Graph $J_{n,M}$

Theorem 3.1.
The Fusion of two vertices v_1 and v_k in a Jahangir graph $J_{n,m}$ $n\geq 2, m\geq 3$ such that nm is even and nm is relatively prime to k where k is an odd number is a prime graph**.**

Proof:
Suppose G is a graph resulting from fusion of two vertices v_1 and v_k where k is an odd number in the cycle of $J_{n,m}$ then $|V(G)| = nm$ and $|E(G)| = (n+1)m$. The set of edges are the edges which are incident on v_1 and v_k are incident with the new vertex '$v_1 = v_k$' and the remaining are same.
We define the labeling $f:V(G)\to\{1,2,3,\ldots\ldots\ldots.nm\}$ such that $f(v_1 = v_k) = k$ and $f(v_i) = i$ for all I and $f(v_{nm+1}) = 1$. As 'k' is an odd number so, the gcd(2,k)=1and k is relatively prime to nm and each pair of successive natural numbers are relatively prime. Therefore, hcf(f(u),f(v)) =1 for each edge e=uv∈E (G). Hence, G complies prime labeling. Therefore G is a prime graph.

Illustration 3.2.
The following graphs represent the fusion of $v_1$and v_3 in $J_{2,5}$ and v_1 and v_5 in $J_{3,4}$ respectively.

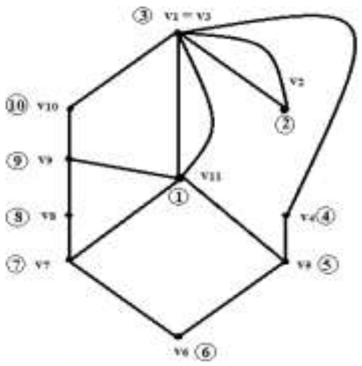

Figure 3. Fusion of v_1 & v_3 in $J_{2,5}$

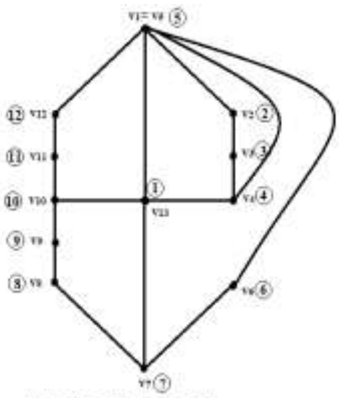

Figure 4. Fusion of v_1& v_5 in $J_{3,4}$

Remark 3.3.
The fusion of $v_1$and v_3 in $J_{2,3}$ is prime even though 6 is not relatively prime to 3.The labeling of fusion of $v_1$and v_3 in $J_{2,3}$ is shown in figure 5.

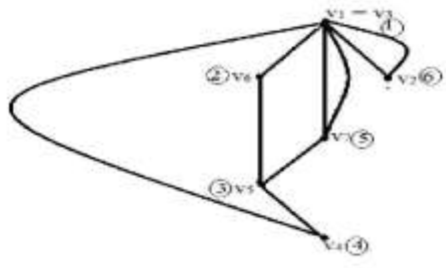

Figure 5. Fusion of v_1 and v_3 in $J_{2,3}$

Theorem 3.4.
The Fusion of two vertices v_{nm+1} and v_k for any k in a Jahangir graph $J_{n,m}$ for $n\geq 2$ and $m\geq 3$ such that nm is even is a prime graph**.**

Proof:
Suppose G is a graph resulting from fusion of two vertices v_{nm+1} and v_k in $J_{n,m}$ then $|V(G)| = nm$ and $|E(G)| = (n+1)m$. The set of edges in G are the set of all edges which are in the cycle C_{nm} and the set of all edges which are adjacent to v_{nm+1}. Define the labeling $f:V(G)\to\{1,2,3,\ldots, nm\}$ such that $f(v_{nm+1} = v_k) = 1$ and $f(v_{k-j}) = nm + 1 - j$ for $1 \leq j \leq k - 1$; $f(v_{k+i}) = i + 1$ for $1 \leq i \leq nm - k$. Note that f(u),f(v) are co-prime numbers for each edge e=uv∈E(G). Hence, G complies prime labeling. Therefore, G is a prime graph.

Illustration 3.5.
The fusion of v_{11} and v_2, fusion of v_{11} and v_4 in $J_{2,5}$ shown in the figures 6 and 7

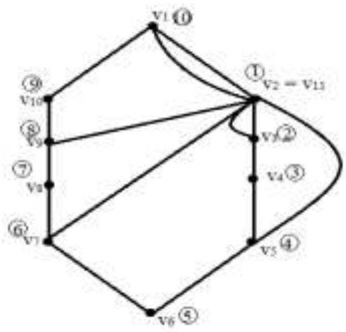

Figure 6.The fusion of v_{11} and v_2 in $J_{2,5}$

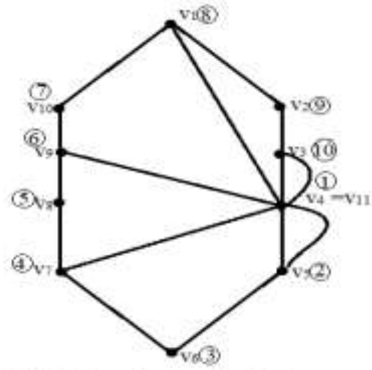

Figure 7.The fusion of v_{11} and v_4 in $J_{2,5}$

4. Main Results on Switching of Vertices in the Jahangir Graph $J_{n,M}$.

Theorem 4.1.
The switching of v_{nm+1}in the Jahangir graph $J_{n,m}$, *for* $n \geq 2$, $m \geq 3$ such that *nm* is even is a prime graph.

Proof:

Suppose G is a graph resulting from switching v_{nm+1}in $J_{n,m}$ then $|V(G)| = nm +1$ and $|E (G) | = (2n-1)m$. The set of edges in G are the set of edges in the cycle C_{nm} and the set of edges which are not adjacent to v_{nm+1}.We define the labeling $f:V(G)\rightarrow\{1,2,3,\ldots, nm+1\}$ such that $f(v_{nm+1})=1$ and $f(v_i) = i+1$ for $1\leq i\leq nm$. Note that $f(u),f(v)$ are co- prime numbers for each edge $e=uv\in E(G)$.Hence, G complies prime labeling. Therefore, G is a prime graph

Illustration 4.2.

Switching of v_{13}in $J_{3,4}$ is shown in the figure 8.

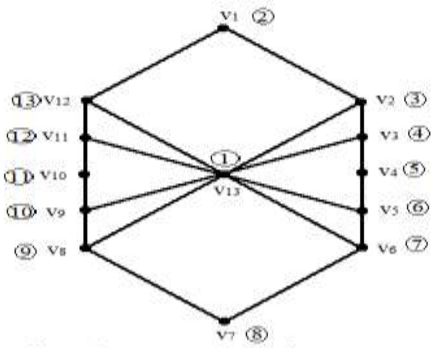

Figure 8. Switching of v13 in $J_{3,4}$

Theorem 4.3.

The switching of v_k $k \geq 1$ in the cycle C_{nm} of the Jahangir graph $J_{n,m}$ *for* $n \geq 2$, $m \geq 3$ such that $nm+1$ is a prime number, is a prime graph.

Proof:

Suppose G *is* a graph obtained by switching v_k in $J_{n,m}$ then $|V(G)| = nm +1$. The set of edges in G are the set of edges in the cycle C_{nm} which are not incident on v_k in $J_{n,m}$ are now incident on v_k and the rest of edges will remain same in $J_{n,m}$ .The required labeling $f:V(G)\rightarrow\{1,2,3,\ldots, nm+1\}$ such that $f(v_k)=nm+1,f(v_{nm+1}) = 1$ and $f(v_{k+i}) = i+1$ for $1\leq i\leq nm-k$ *and* $f(v_{k-i}) = nm+1-i$ for $1 \leq i\leq k-1$. Note that $f(u),f(v)$ are co-prime numbers for each edge $e=uv\in E(G)$.Hence, G complies prime labeling. Therefore, G is a prime graph

Illustration 4.4.

The switching of v_1 and switching ofv_5 in $J_{3,4}$ are shown in the figures 9,10.

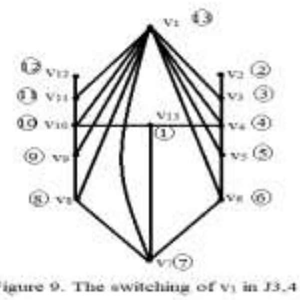

Figure 9. The switching of v_1 in J3.4

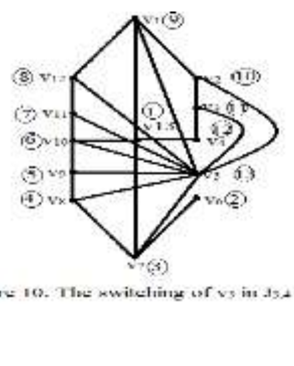

Figure 10. The switching of v_5 in $J_{3,4}$

5. Main Results on Duplication of a Vertex in the Jahangir Graph $J_{n,M}$.

Theorem 5.1.

The Duplication of v_k where k is odd integer and $nm+2$ is relatively prime to $k,k +2$ in the Jahangir graph $J_{n,m}$,for$n\geq 2,m\geq 3$ such that nm is even is a prime graph.

Proof:

Let G be a graph obtained by duplicating v_k by v_k' in $J_{n\,,\,m}$.
We define the labeling $f :V(G) \rightarrow \{1, 2, 3, \ldots, nm + 2\}$ such that $f(v_{nm+1}) = 1$ and $f(v_i) = i + 1$ for $1 \leq i \leq nm$, $f(v_k') = nm + 2$.As mentioned in the theorem 2.1, $hcf\,(f(u),f(v)) =1$ for each edge $e = uv\in E\ (G)$.Hence,G complies prime labeling. Therefore, G is a prime graph.

Illustration 5.2.

Duplication of v_3 in $J_{3,4}$ is shown in the figure 11.

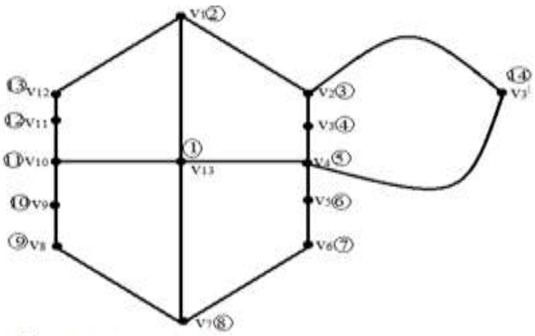

Figure 11. Duplication of v_3 in $J_{3,4}$

6. Conclusion

In this paper we checked the prime labeling of Jahangir graph $J_{n,m}$ by using 'C' Program for different n, m values which satisfy the condition nm is even. Similarly we can apply different languages for checking the labelings of different families of the graph.

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