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Research paper

Δ^*-Locally Continuous Functions and Δ^*-Locally Irresolute Maps in Topological Spaces

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Abstract

The objective of the paper is to introduce a new types of continuous maps and irresolute functions called Δ^*-locally continuous functions and Δ^*-irresolute maps in topological spaces. The comparative study between these functions with other existing maps is discussed in this paper. Some significant results are also proved as an application of new spaces namely, Δ^*-submaximal space and $_{\Delta^*}T_\delta$ – space. Further the characteristics of these maps under composition maps are exhibited.

Keywords*: Δ^*-closed set , Δ^*-continuous functions , Δ^*-irresolute maps , $_{\Delta^*}T_\delta$ -space and Δ^* -submaximal space.*

1. Introduction

The locally closed sets idea in a topological space was pioneered by Bourbaki [2]. Initially it was defined as the intersection of an open set and a closed set. Using locally closed sets, Ganster and Reilly [3] defined locally continuous functions and locally irresolute maps. Further the concepts of generalized locally closed (briefly, glc) sets, generalised locally continuous and generalised locally irresolute maps were introduced by Balachandran et.al.,[1]. Since then several topologists contributed their study to the development of generalizations of locally closed sets and locally continuous maps. In this paper, two types of new generalised maps namely, Δ^*-locally continuous and Δ^*-locally irresolute maps are defined. Their properties using $_{\Delta^*}T_\delta$ -space and Δ^*-submaximal space are analysed in this paper. Some important results under composition of mappings are also analyzed in this paper.

Remark:

The notation $\delta(\delta g)^*$ was used instead of the notation Δ^*-closed sets at the time of introducing the definition.[4]

2. Preliminaries

The notations (M, μ), (N, ν) and (T, λ) represent topological spaces where μ, ν and λ denote topologies defined on the non empty sets M, N and T respectively.

Definition 2.1

A subset S of a topological space (M, μ) is called a
i) Δ^*-closed set if $\delta cl(S) \subseteq B$ whenever $S \subseteq B$ where B is δg-open in (M, μ). The complement of a Δ^*-closed set is called Δ^*-open set. [4]
ii) δ-open set if it is the union of regular open sets. The complement of a δ-open set is called a δ-closed set.[9]

Definition 2.2

A mapping g : (M, μ) ⟶ (Y, ν) is said to be
i) δ-continuous if the inverse image of every closed set in (Y, ν) is δ-closed in (M, μ). [8]
ii) Δ^*-continuous if the inverse image of every closed set in (Y, ν) is Δ^*-closed in (M, μ). [5]
iii) Δ^*-irresolute if the inverse image of every Δ^*-open set in (Y, ν) is Δ^*-open in (M, μ). [6]
iv) Contra Δ^*-irresolute if the inverse image of every Δ^*-closed set in (Y, ν) is Δ^*-open in (M, μ). [6]

Definition 2.3

A subset S of a topological space (M, μ) is called a [7]
i) **Δ^*-locally closed set** which is denoted as Δ^*lc-set if there exists a Δ^*-open set B and a Δ^*-closed set D of (M, μ) such that $S = B \cap D$.
ii) **Δ^*lc^*-set** if there exists a Δ^*-open set B and a δ-closed set D of (M, μ) such that $S = B \cap D$.
iii) **Δ^*lc^{**}-set** if there exists a δ-open set B and a Δ^*-closed set D of (M, μ) such that $S = B \cap D$.
The collection of all Δ^*lc (resp., Δ^*lc^*-sets, Δ^*lc^{**}) sets of (M, μ) is denoted by $\Delta^*LC(M, \mu)$ (resp., $\Delta^*LC^*(M, \mu)$, $\Delta^*LC^{**}(M, \mu)$).

Remark 2.4

For a topological space (M, μ), the following inclusions are true. [7]
a) $\delta LC(M, \mu) \subseteq \Delta^*LC(M, \mu)$
b) $\delta LC(M, \mu) \subseteq \Delta^*LC^{**}(M, \mu) \subseteq \Delta^*LC(M, \mu)$
c) $\delta LC(M, \mu) \subseteq \Delta^*LC^*(M, \mu) \subseteq \Delta^*LC(M, \mu)$

Remark 2.5

Let g : (M, μ) ⟶ (Y, ν) be a Δ^*-irresolute map. Then the following statements are true. [7]

a) If $B \in \Delta^*LC(Y, \nu)$ then $f^{-1}(B) \in \Delta^*LC(M, \mu)$.

b) If $B \in \Delta^*LC(Y, \nu)$ then $f^{-1}(B) \in g\delta LC(M, \mu)$.

c) If $B \in g\delta LC(Y, \nu)$ and (Y, ν) is a $g\delta T_{\Delta^*}$ -space then $f^{-1}(B) \in \Delta^*LC(M, \mu)$.

Remark 2.6 Let

(M, τ) be a $_{\Delta^*}T_\delta$ -space. Then the following results hold. [7]

a) $\Delta^*LC(M, \mu) = \delta LC(M, \mu) = \Delta^*LC^*(M, \mu) = \Delta^*LC^{**}(M, \mu)$

b) $\Delta^*LC(M, \mu) \subseteq GLC(X, \tau)$

c) $\Delta^*LC(M, \mu) \subseteq GL\delta C(X, \tau)$

3. Δ^*-Locally Continuous Functions and Locally Irresolute Maps

Definition 3.1
Let $g : (M, \mu) \to (Y, \nu)$ be a map. Then f is called
a) Δ^*LC-continuous if $g^{-1}(D) \in \Delta^*LC(M, \mu)$ for each $D \in \nu$.
b) Δ^*LC^*-continuous if $g^{-1}(D) \in \Delta^*LC^*(M, \mu)$ for each $D \in \nu$.
c) Δ^*LC^{**}-continuous if $g^{-1}(D) \in \Delta^*LC^{**}(M, \mu)$ for each $D \in \nu$.

Definition 3.2
Let $g : (M, \mu) \to (Y, \nu)$ be a map. Then f is called

a) Δ^*LC-irresolute if $g^{-1}(D) \in \Delta^*LC(M, \mu)$ for each $D \in \Delta^*LC(Y, \nu)$.

b) Δ^*LC^*-irresolute if $g^{-1}(D) \in \Delta^*LC^*(M, \mu)$ for each $D \in \Delta^*LC^*(Y, \nu)$.

c) Δ^*LC^{**}-irresolute if $g^{-1}(D) \in \Delta^*LC^{**}(M, \mu)$ for each $D \in \Delta^*LC^{**}(Y, \nu)$.

Proposition 3.3
Let $g : (M, \mu) \to (Y, \nu)$ be a map. Then the following statements are true.
a) If g is δLC-continuous then it is Δ^*LC-continuous, Δ^*LC^*-continuous and Δ^*LC^{**}continuous.
b) If g is Δ^*LC^*-continuous or Δ^*LC^{**}-continuous then it is Δ^*LC-continuous.

Proof:
a) Follows from remark 2.4(a) and by the fact that every δlc-set is Δ^*lc-set, Δ^*lc^*-set and Δ^*lc^{**}-set.[7]
b) Since every Δ^*lc^*-set is Δ^*lc-set and every Δ^*lc^{**}-set is Δ^*lc-set, the proof follows.
The converse is not possible that can be viewed by the counter example given below.

Counter Example 3.4
a) Let $M = \{e1, e2, e3\} = Y$ and $\mu = \{\phi, M, \{e1\}, \{e1, e2\}, \{e1, e3\}\}$ and $\nu = \{\phi, Y, \{e1, e2\}\}$.
Let $g : (M, \mu) \to (Y, \nu)$ be the identity map. Then f is Δ^*LC-continuous, Δ^*LC^*-continuous and Δ^*LC^{**}-continuous but not δLC-continuous since for the open set $\{e1, e2\} \in (Y, \nu)$, $g^{-1}(D)\ \{e1, e2\} = \{e1, e2\} \notin \delta LC(M, \mu)$.
b) Let $M = \{e1, e2, e3\} = Y$ and $\mu = \{\phi, M, \{e1\}, \{e1, e2\}, \{e1, e3\}\}$ and $\nu = \{\phi, Y, \{e1\}, \{e2, e3\}\}$. Let $g : (M, \mu) \to (Y, \nu)$ be the identity map. Then f is Δ^*LC-continuous but it is neither Δ^*LC^*-continuous nor Δ^*LC^{**}-continuous, since for the open set $\{e2, e3\} \in f^{-1}(Y, \nu)$, $\{e2, e3\} = \{e2, e3\} \notin \Delta^*LC^*(M, \mu)$ and for the open set $\{e2\} \in (Y, \nu)$, $g^{-1}\{e2\} = \{e2\} \notin \Delta^*LC^{**}(M, \mu)$.

Proposition 3.5
Let $g : (M, \mu) \to (Y, \nu)$ be a Δ^*-irresolute map. Then f is Δ^*LC-irresolute but not conversely.

Proof:
It follows by the definitions of Δ^*-irresolute map, Δ^*LC-irresolute map and by the remark 2.5 (a).

Counter Example 3.6
Let $M = \{e1, e2, e3\} = Y$ and $\mu = \{\phi, M, \{e1\}, \{e2\}, \{e1, e2\}, \{e1, e3\}\}$ and $\nu = \{\phi, Y, \{e1, e2\}\}$. Let $g : (M, \mu) \to (Y, \nu)$ be a map defined by $g(e1) = e1$, $g(e2) = e3$, $g(e3) = e2$. Then g is Δ^*LC-irresolute but not Δ^*-irresolute since for the Δ^*-open set $\{e2\} \in (Y, \nu)$, $g^{-1}\{e2\} = \{e3\}$ is not Δ^*-open in (M, μ).

Proposition 3.7
If $g : (M, \mu) \to (Y, \nu)$ is Δ^*LC-continuous (Δ^*LC^*-continuous (or) Δ^*LC^{**}-continuous) and (M, μ) is a $_{\Delta^*}T_\delta$ -space then g is δLC-continuous.

Proof :
Let D be an open set in (Y, ν). Since g is Δ^*LC- continuous (Δ^*LC^*-continuous (or) Δ^*LC^{**}-continuous), $g^{-1}(D)$ is Δ^*lc-set (Δ^*lc^*-set, Δ^*lc^{**}-set) in (M, μ). Since (M, μ) is $_{\Delta^*}T_\delta$ -space, by the remark 2.6 (a), $g^{-1}(D)$ is δlc-set in (M, μ). Hence g is δLC-continuous.

Remark 3.8
The independency of Δ^*LC-irresolute maps and Δ^*LC-continuous maps are proved by the following examples.

Counter example 3.9
Let $M = \{e1, e2, e3\} = Y$ and $\mu = \{\phi, M, \{a\}, \{b, c\}\}$ and $\nu = \{\phi, Y, \{a, b\}\}$. Let $g : (M, \mu) \to (Y, \nu)$ be a map defined by $f(a) = c$, $f(b) = b$, $f(c) = a$. Then g is Δ^*LC-continuous but not Δ^*LC-irresolute since for the set $\{e2\} \in \Delta^*LC(Y, \nu)$, $f^{-1}\{e2\} = \{e3\}$ is not Δ^*-open in $\Delta^*LC(M, \mu)$.

Counter example 3.10
Let $M = \{e1, e2, e3\} = Y$ and $\mu = \{\phi, M, \{a\}, \{b, c\}\}$ and $\nu = \{\phi, Y, \{a\}, \{a, b\}, \{a, c\}\}$. Let $g : (M, \mu) \to (Y, \nu)$ be the identity map. Then g is Δ^*LC-irresolute but not Δ^*LC-continuous since for the open set $\{e1, e2\} \in (Y, \nu)$, $g^{-1}\{e1, e2\} = \{e1, e2\}$ is not in $\Delta^*LC(M, \mu)$.

Proposition 3.11
Any map defined on a δ-door space is Δ^*LC-irresolute.

Proof :
Let $g : (M, \mu) \to (Y, \nu)$ be a map where (M, μ) is a δ-door space and (Y, ν) is any space. Let $S \in \Delta^*LC(Y, \nu)$. Then by the assumption on (M, μ), $f^{-1}(S)$ is either δ-open or δ-closed. Since every δ-closed is Δ^*-closed [4], $f^{-1}(S) \in \Delta^*LC(M, \mu)$. Hence f is Δ^*LC-irresolute.

Proposition 3.12
If $g : (M, \mu) \to (Y, \nu)$ is Δ^*LC-continuous and contra δ-continuous where (N, ν) is a $_{\Delta^*}T_\delta$ -space then g is a Δ^*LC-irresolute map.

Proof :
Let $g : (M, \mu) \to (Y, \nu)$ be Δ^*LC-continuous and Contra δ-continuous map. Let (Y, ν) be a $_{\Delta^*}T_\delta$ -space. Let $P \in \Delta^*LC(Y, \nu)$. Then there exists a Δ^*-open set B and a Δ^*-closed set Q of (Y, ν)

such that P = B ∩ Q. Since (Y, ν) is a $_{\Delta^*}T_\delta$ -space, B is δ-open and Q is δ-closed. As every δ-open set is open, B is open. Since g is Δ^*LC-continuous, g^{-1} (B) is Δ^*lc-set. Since g is Contra δ-continuous, g^{-1} (Q) is δ-open and hence Δ^*-open. Also g^{-1} (P) = g^{-1} (B) ∩ g^{-1} (Q). So g^{-1} (P) is a Δ^*lc-set in (M, μ). Therefore g is Δ^*LC-irresolute.

Remark 3.13

In the above proposition 3.12 if the mapping g is considered as Δ^*LC-continuous and contra Δ^*-irresolute then the same result follows.

Proposition 3.14

Let g : (M, μ) → (N, ν) be a map. If g is Δ^*LC^*-continuous and contra δ-continuous where (N, ν) is a $_{\Delta^*}T_\delta$ -space then g is a Δ^*LC^*-irresolute map.

Proof :

Let g : (M, μ) → (N, ν) be a Δ^*LC^*-continuous and contra δ-continuous in which (N, ν) is a $_{\Delta^*}T_\delta$ -space. Let P ∈ Δ^*LC^*(N, ν). Then there exists a Δ^*-open set B and a δ-closed set Q in (N, ν) such that P = B ∩ Q. Since (N, ν) is a $_{\Delta^*}T_\delta$ -space, B is δ-open in (N, ν). Since g is contra δ-continuous, g^{-1} (Q) is δ-open in (M, μ) and hence g^{-1} (Q) is Δ^*-open in (M, μ). Therefore by remark 2.5 (c), g^{-1} (P) is a Δ^*lc^*-set in (M, μ). Hence g is Δ^*lc^*-irresolute.

Proposition 3.15

Let g : (M, μ) → (N, ν) and h : (N, ν) → (T, η) be any two maps. Then

a) (h ∘ g) : (M, μ) → (T, η) is Δ^*LC-irresolute (resp. Δ^*LC^*-irresolute, Δ^*LC^{**}-irresolute) if g is Δ^*LC-irresolute (resp. Δ^*LC^*-irresolute, Δ^*LC^{**}-irresolute) and h is also Δ^*LC-irresolute.(resp., Δ^*LC^*-irresolute, Δ^*LC^{**}-irresolute).

b) (h ∘ g) : (M, μ) → (T, η) is Δ^*LC- continuous if g is Δ^*LC-irresolute and h is Δ^*LC-continuous.

Proof : a) Let D ∈ Δ^*LC(T, η) (resp., D ∈ Δ^*LC^*(T, η), D ∈ Δ^*LC^{**}(T, η)). Since h is Δ^*LC-irresolute (resp., Δ^*LC^*-irresolute, Δ^*LC^{**}-irresolute), h^{-1}(D) ∈ Δ^*LC(N, ν) (resp., h^{-1}(D) ∈ Δ^*LC^*(N, ν), h^{-1}(D) ∈ Δ^*LC^{**}(N, ν)). Since g is Δ^*LC-irresolute (resp., Δ^*LC^*-irresolute, Δ^*LC^{**}-irresolute), $g^{-1}[h^{-1}(D)]$ = $(h \circ g)^{-1}$ ∈ Δ^*LC(M, μ) (resp., $(h \circ g)^{-1}$ ∈ Δ^*LC^*(M, μ), $(h \circ g)^{-1}$ ∈ Δ^*LC^{**}(M, μ)). Therefore (h ∘ g) is Δ^*LC-irresolute (resp., Δ^*LC^*-irresolute, Δ^*LC^{**}-irresolute).

b) Let D be any open set in (T, η). Since h is Δ^*LC-continuous, h ∈ Δ^*LC(N, ν). Since g is Δ^*LC-irresolute, $g^{-1}[h^{-1}(D)]$ = $(h \circ g)^{-1}$ ∈ Δ^*LC(M, μ). Therefore (h ∘ g) is Δ^*LC-continuous.

Remark 3.16

The composition of continuity is not preserved by Δ^*LC-continuous which is justified by the following counter examples.

Counter Example 3.17

Let g : (M, μ) → (N, ν) be a map defined by g(a) = c, g(b) = b, g(c) = a and h : (N, ν) → (T, η) be a map defined by h(a) = c, h(b) = b, h(c) = a where μ = { ϕ, M, {a}, {b, c}}, ν = { ϕ, N, {a, b}} and η = { ϕ, T, {a}, {a, b}, {a, c}}. Let (h ∘ g) : (M, μ) → (T, η) be the composition map defined by (h ∘ g)(a) = a, (h ∘ g)(b) = b and (h ∘ g)(c) = c. Then both g and h are Δ^*LC- continuous but (h ∘ g) is not Δ^*LC- continuous, because for the subset {e1, e2} ∈ (T, η), $(h \circ g)^{-1}$\{e1, e2\} = {e1, e2} ∉ Δ^*LC(M, μ).

Proposition 3.18

For any two maps g : (M, μ) → (N, ν) and h: (N, ν)→(T, λ) the results given below are true.

a) (h ∘ g) : (M, μ) → (T, λ) is $\Delta*$LC- continuous if g is Δ^*LC irresolute and h is δLC-continuous (resp., Δ^*LC^*-continuous, Δ^*LC^{**}-continuous).

b) (h ∘ g) is Δ^*LC- continuous (resp. Δ^*LC^*-continuous) if g is Δ^*LC-irresolute (resp. Δ^*LC^*-irresolute) and h is Δ^*-continuous.

c) (h ∘ g) is Δ^*LC is Δ^*LC- continuous (resp., Δ^*LC^*-continuous, Δ^*LC^{**}-continuous) if g is Δ^*LC-continuous and h is δ-continuous.

Proof:

Let D be any open set in (T, λ).

a) Since h is δLC-continuous (resp., Δ^*LC^*-continuous, Δ^*LC^{**} continuous), h^{-1}(D) ∈ δLC(N, ν) ⇒ h^{-1}(D) ∈ Δ^*LC(N,ν) [7]. Since g is Δ^*LC -irresolute, $g^{-1}[h^{-1}(D)]$ = $(h \circ g)^{-1}$∈ Δ^*LC (M,μ). Hence (h ∘ g) is Δ^*LC- continuous.

b) Since h is Δ^*- continuous, h^{-1}(D) is Δ^*-open in (N, ν) and hence h^{-1}(D) is Δ^*lc-set (resp., $\Delta*lc*$-set) in (n, ν). Since g is Δ^*LC -irresolute (resp., Δ^*LC^*-irresolute), $g^{-1}[h^{-1}(D)]$ = $(h \circ g)^{-1}$ ∈ Δ^*LC (M, μ). Hence (h ∘ g) is Δ^*LC continuous.(resp., Δ^*LC^*-continuous).

c) Since h is δ-continuous, h^{-1}(D) is δ-open in (N, ν). Since every δ-open is Δ^*-open, h^{-1}(D) is Δ^*-open in (N, ν) and hence h^{-1}(D) is Δ^*lc-set(resp., Δ^*lc*-set, Δ^*lc**-set) in (N, ν). Since g is Δ^*LC -continuous.(resp.,Δ^*LC*-continuous,$\Delta*LC**$continuous), $g^{-1}[h^{-1}(D)]$ = $(h \circ g)^{-1}$ ∈ Δ^*LC(M,μ) (resp., ∈ Δ^*LC^*(M, μ), and Δ^*LC^{**}(M,μ)). Hence (h ∘ g) is Δ^*LC - continuous. (resp., Δ^*LC^*-continuous, Δ^*LC^{**}-continuous).

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