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Research paper

A Discourse on Modified Likelihood Ratio (LR), Wald and Lagrange Multipliers (LM) Tests for Testing General Linear Hypothesis in Stochastic Linear Regression Model

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Abstract

In this research paper various new advanced inferential tools namely modified likelihood ratio (LR), Ward and Lagrange Multiplier test statistics have been proposed for testing general linear hypothesis in stochastic linear regression model. In this process internally studentized residuals have been used. This research study has brought out some new advance tools for analysing inferential aspects of stochastic linear regression models by using internally studentized residuals. Miguel Fonseca et.al [1] developed statistical inference in linear models dealing with the theory of maximum likelihood estimates and likelihood ratio tests under some linear inequality restrictions on the regression coefficients. Tim Coelli [2] used Monte carlo experimentation to investigate the finite sample properties of maximum likelihood (ML) and correct ordinary least squares (COLS) estimators of the half –normal stochastic frontier production function. In 2011, p. Bala siddamuni et.al [3] have developed advanced tools for mathematical and stochastic modelling.

Keywords: *Information matrix, OLS residual sum of squares, LR test, Ward test, LM test, stochastic linear regression models, internally stduntized residuals.*

1. Introduction

In spite of the availability of highly innovative tools in Mathematics, the main tool of the Applied Mathematician remains the stochastic regression model in the form of either linear or nonlinear model. More importantly, mastery of the stochastic linear regression model is prerequisite to work with advanced mathematical and statistical tools because most advanced tools are generalizations of the stochastic linear regression model. The various inferential problems of stochastic modelling are considered to be essential to both theoretical and applied mathematicians and statisticians. The selection between alternative models is an important problem in stochastic modelling. Specification of the stochastic regression model is an important stage in any stochastic linear regression analysis. It includes specifying both the expectation function and the characteristics of the error. The various Mis-specification tests and testing general linear hypothesis in the stochastic linear regression models were studied by many mathematicians and statisticians. Most of these people have proposed their tests in stochastic linear regression models by using some inferential criteria. A cursory glance at the recent literature on Model Building clearly suggests a significant shift in the level of mathematical and stochastic rigor brought at research efforts concerning Model Building. A more careful inspection shows that this trend has not been uniform across in the literature. In particular, while mathematical and stochastic modeling efforts in certain fields of science and technology have been appreciable, other research fields of science remain under developed. Successful Mathematical and Stochastic Model buildings are not a collection of simple mechanistic and routine techniques but more of an art requiring wide-ranging knowledge and judgement. In the stochastic model building, the most difficult problem is the specification of the stochastic model. Under the problem of mis-specification of the stochastic regression model, first task is that what set of regressors have to be included in the model; and the second task is that in which mathematical form of the regressors are to be included in the model.

2. Likelihood Ratio Test for Testing General Linear Hypothesis using Internally Studentized Residuals

Consider the general linear hypothesis about β in the standard stochastic linear regression model $y = x\beta + \in$ with $\in \sim N\left[0, \sigma^2 I_n\right]$, as

$$H_0 : R_{m\times k}\, \beta_{k\times 1} = r_{m\times 1}$$

Where R is a (mxk), (m<k) matrix of known constants and r is a (mx1) vector of known constants.

The multivariate probability density function of error vector $\in$ is given by

$$f(\in)=\frac{1}{\left(2\pi\sigma^2\right)^{n/2}}\exp\left\{-\frac{\in^1\in}{2\sigma^2}\right\} \tag{2.1}$$

The multivariate probability density function Y conditional on X is given by

$$f(Y/X)=f(\in)\left|\frac{\partial t}{\partial Y}\right| \tag{2.2}$$

Where

$$\left|\frac{\partial\in}{\partial Y}\right|=\begin{vmatrix}\frac{\partial\in_1}{\partial Y_1} & \frac{\partial\in_1}{\partial Y_2} & \cdots & \frac{\partial\in_1}{\partial Y_n}\\ \frac{\partial\in_2}{\partial Y_1} & \frac{\partial\in_2}{\partial Y_2} & \cdots & \frac{\partial\in_2}{\partial Y_n}\\ \vdots & \vdots & & \vdots\\ \frac{\partial\in_n}{\partial Y_1} & \frac{\partial\in_n}{\partial Y_2} & \cdots & \frac{\partial\in_n}{\partial Y_n}\end{vmatrix} \tag{2.3}$$

is known as the Jacobian of transformation for $\in$ to Y. Here, $\left|\frac{\partial\in}{\partial Y}\right|$ is the absolute value of the determinant obtained from (nxn) matrix of partial derivatives of elements of $\in$ with respect to the elements of y. Further the matrix $\left(\left(\frac{\partial\in}{\partial Y}\right)\right)$ is identify matrix,

i.e., $\left|\frac{\partial\in}{\partial Y}\right|=1$.

Now, the Log_e - likelihood function is given by

$$\ln L\left(\beta,\sigma^2\right)=\ln f\left(Y/X\right)=\ln f(\in)=-\frac{n}{2}\ln 2\pi-\frac{n}{2}\ln\sigma^2-\frac{1}{2\sigma^2}\in'\in$$

$$=-\frac{n}{2}\ln 2\pi-\frac{n}{2}\ln\sigma^2-\frac{1}{2\sigma^2}\left(Y-X\beta\right)'\left(Y-X\beta\right)$$

By the maximum likelihood method of estimation, the first order conditions give the maximum likelihood estimates of β and σ^2 as

$$\hat{\beta}=\left(X'X\right)^{-1}X'Y \tag{2.4}$$

$$\hat{\sigma}^2=\frac{\left(Y-X\hat{\beta}\right)'\left(Y-X\hat{\beta}\right)}{n}=\frac{e^1e}{n} \tag{2.5}$$

Here, $e=\left[Y-X\hat{\beta}\right]$ is the OLS residual vector. The unrestricted maximum likelihood function can be expressed as

$$L\left(\hat{\beta},\hat{\sigma}^2\right)=\left(2\pi e\right)^{-\frac{n}{2}}\left(\hat{\sigma}^2\right)^{-\frac{n}{2}}=\left(2\pi e\right)^{-\frac{n}{2}}\left(\frac{e'e}{n}\right)^{-\frac{n}{2}}$$

$$\Rightarrow L\left(\hat{\beta},\hat{\sigma}^2\right)=C\left(e'e\right)^{-\frac{n}{2}} \tag{2.6}$$

where $C=\left(\frac{2\pi e}{n}\right)^{-\frac{n}{2}}$ is constant

Here, $e'e$ is the unrestricted OLS residual sum of squares, under general linear hypothesis $H_0:R\beta=r$, the restricted maximum likelihood estimates of β and σ^2 can be obtained as β^*_{RLS} and $\tilde{\sigma}^2$ respectively. Write the Restricted Maximum likelihood function as $L\left(\beta^*_{RLS},\tilde{\sigma}^2\right)$, which can be expressed as

$L\left(\beta^*_{RLS},\tilde{\sigma}^2\right)=C\left(e^{*'}e^*\right)^{-\frac{n}{2}}$, C is constant. Where $e^*=Y-X\beta^*_{RLS}$ is the Restricted Least Squares residual vector.

Now, the likelihood Ratio test statistic for testing $H_0:R\beta=r$ is given by

$$LR=-2\ln\lambda$$

$$=-2\ln\left\{\frac{L\left(\beta^*_{RLS},\tilde{\sigma}^2\right)}{L\left(\hat{\beta},\hat{\sigma}^2\right)}\right\} \tag{2.7}$$

$$\Rightarrow LR=2\left[\ln L\left(\hat{\beta},\hat{\sigma}^2\right)-\ln L\left(\beta^*_{RLS},\tilde{\sigma}^2\right)\right] \tag{2.8}$$

By substituting $L\left(\hat{\beta},\hat{\sigma}^2\right)$ and $L\left(\beta^*_{RLS},\tilde{\sigma}^2\right)$ in (2.8), one may obtain,

$$LR=n\left[\ln e^{*'}e^*-\ln e'e\right] \tag{2.9}$$

$$\Rightarrow LR=n\ln\left[1+\left(\frac{e^{*'}e^*-e'e}{e'e}\right)\right]=n\ln\left[\frac{1}{1-\left(\frac{e^{*'}e^*-e'e}{e^{*'}e^*}\right)}\right]$$

or

$$\Rightarrow LR=-n\ln\left[1-\left(\frac{e^{*'}e^*-e'e}{e^{*'}e^*}\right)\right]\sim\chi^2_m \tag{2.10}$$

By replacing unrestricted and restricted least squares residuals with their corresponding unrestricted and restricted Internally Stundentized residuals, one may obtain Likelihood Ratio test statistic for testing $H_0:R\beta=r$ as

$$\left[LR\right]^*=-n\ln\left[1-\left(\frac{q'_Rq_R-q'_{UR}q_{UR}}{q'_Rq_R}\right)\right]\sim\chi^2_m \tag{2.11}$$

Where, q'_Rq_R = Restricted Internally Stundentized Residual sum of Squares

$q'_{UR}q_{UR}$ = Unrestricted Internally Stundentized Residual sum of Squares

3. Wald Test for Testing General Linear Hypothesis using Internally Studentized Residuals

In testing general linear hypothesis, under $H_0:R\beta=r$, it can be shown that $\left(R\hat{\beta}-r\right)$ follows asymptotically multivariate normal distribution with zero mean vector and covariance matrix $\left[R\sigma^2\left(X'X\right)^{-1}R'\right]$.

Now, the Wald test statistic for testing $H_0:R\beta=r$ is given by

$$W=\frac{\left(R\hat{\beta}-r\right)\left[R\left(X'X\right)^{-1}R'\right]^{-1}\left(R\hat{\beta}-r\right)}{\hat{\sigma}^2}\overset{asy}{\sim}\chi^2_m \tag{3.1}$$

Since, the numerator in the test statistic can be expressed as the difference between the restricted and unrestricted Least squares residual sum of squares and $\hat{\sigma}^2$ can be replaced with $\frac{e'e}{n}$; Wald test statistic can be modified by using restricted and unrestricted Internally studentized residual sum of squares, which is given by

$$W^*=\frac{n\left(q'_Rq_R-q'_{UR}q_{UR}\right)}{q'_Rq_R}\overset{asy}{\sim}\chi^2_m \tag{3.2}$$

4. Lagrange Multiplier (LM) Test for Testing General Linear Hypothesis using internally Studentized Residuals

The Lagrange Multiplier test is based on the Gradient vector or score vector

$$G(\theta) = G(\beta, \sigma^2) = \frac{\partial \ln L(\theta)}{\partial \theta}$$

One may obtain unrestricted estimator $\hat{\theta} = (\hat{\beta}, \hat{\sigma}^2)$ by solving $G(\hat{\theta}) = 0$. Here $G(\hat{\theta})$ shows the score vector evaluated at $\hat{\theta}$. The score vector is given by

$$G(\theta) = \begin{bmatrix} \dfrac{\partial \ln L(\beta, \sigma^2)}{\partial \beta} \\ \dfrac{\partial \ln L(\beta, \sigma^2)}{\partial \sigma^2} \end{bmatrix} = \begin{bmatrix} \dfrac{X' \in}{\sigma^2} \\ -\dfrac{n}{2\sigma^2} + \dfrac{\in' \in}{2\sigma^2} \end{bmatrix} \tag{4.1}$$

Under $H_0 : R\beta = r$, the score vector at the restricted estimator θ^*_{RLS} is given by

$$G(\theta^*_{RLS}) = \begin{bmatrix} \dfrac{X' e^*}{\sigma^2 \sigma^{2^*}} \\ 0 \end{bmatrix}$$ and the inverse of the information matrix is given by

$$I^{-1}(\theta^*_{RLS}) = \begin{bmatrix} \sigma^2 (X'X)^{-1} & 0 \\ 0 & \dfrac{2\sigma^4}{n} \end{bmatrix} \tag{4.2}$$

Where $e^*_{RLS} = Y - X\beta^*_{RLS}$ = Restricted Least Squares residual vector.

The LM test statistic for testing $H_0 : R\beta = r$ is given by

$$LM = \left[G'(\theta^*_{RLS}) I^{-1}(\theta^*_{RLS}) G(\theta^*_{RLS}) \right] \overset{asy}{\sim} \chi^2_m \tag{4.3}$$

$$\Rightarrow LM = \frac{e^{*\prime}_{RLS} X (X'X)^{-1} X^1 e^*_{RLS}}{\overset{*}{\sigma^2}} \overset{asy}{\sim} \chi^2_m \tag{4.4}$$

or

$$LM = \frac{n\left[e^{*\prime}_{RLS} X (X'X)^{-1} X^1 e^*_{RLS} \right]}{e^{*\prime}_{RLS} e^*_{RLS}} \tag{4.5}$$

By using restricted and unrestricted Internally Studentized residual sum of squares, the LM test statistic for testing $H_0 : R\beta = r$ is given by

$$(LM)^* = \frac{n(q'_R q_R - q'_{UR} q_{UR})}{q'_R q_R} \overset{asy}{\sim} \chi^2_m \tag{4.6}$$

5. Conclusion

In this monograph LR, Wald and LM test statistics for testing general linear hypothesis in stochastic linear regression model using internally studentized residuals have been developed This work can be extended by developing advanced tools analyze random coefficients regression models by using different types of residuals other than studentized residuals.

References

[1] Miguel Fonseca, Jrau Tiazo Mexia, Bimal K Sinha, Roman Zmyslony, "Likelihood ratio tests in linear models with linear equality restrictions on regression coefficients", REVSTAT-Statistical journal, Vol.(13), (2015), Pp: 103-118.

[2] Tim Coelli, "Estimators and hypothesis test for a stochastic frontier function: A Monte Carlo analysis, Journal of productivity analysis, Vol. (6), (1995), Pp: 247- 268.

[3] Balasiddamuni, P.et.al. "Advanced Tools for Mathematical and Stochastic modeling", Proceedings of the International Conference on Stochastic modeling and Simulation, Allied Publishers, (2011).

[4] Byron J.T. Morgan, (2008), "Applied stochastic Modelling", CRC Press, (2008), 978-1- 58488- 666-2.

[5] Berry L. Nelson, (1995), "Stochastic Modeling, Analysis and Simulation", McGraw-Hill, (1995), 978-0070462137.

[6] Nelson, B.L. "Stochastic Modeling", McGraw-Hill, New York, (1995), 0-486-47770-3.

[7] Taylor, H.M. and Samuel karlin, "An Introduction to Stochastic Modeling", Academic Press, London, (1998), 978-0-12-684887-87.

[8] Nafeez Umar, S. and Balasiddamuni, P. (2013), "Statistical Inference on Model Specification in Econometrics", LAMBERT Academic Publishing, Germany,(2013).