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Research paper

The Split Distance 2 Domination in Graphs

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Abstract

A distance - 2 dominating set D ⊆ V of a graph G is a split distance - 2 dominating set if the induced sub graph <V-D> is disconnected. The split distance - 2 domination number $\gamma_{s\leq 2}(G)$ is the minimum cardinality of a split distance - 2 dominating set. In this paper, we defined the notion of split distance - 2 domination in graph. We got many bounds on distance - 2 split domination number. Exact values of this new parameter are obtained for some standard graphs. Nordhaus - Gaddum type results are also obtained for this new parameter.

Key words: *Dominating set, split dominating set, distance -2 dominating set, split distance -2 dominating set, split distance -2 domination Number.*

1. Introduction

All graphs considered here are simple, finite, connected and undirected. Let n and m denote the order and size of a graph G. We use the terminology of [11]. Let $\Delta(G)$ $(\delta(G))$ denote the maximum (minimum) degree and $\lfloor x \rfloor (\lceil x \rceil)$ the greatest (least) integer less (greater) than or equal to x. The independence number β_0(G) is the maximum cardinality among the independent set of vertices of G. The lower independence number i(G) is the minimum cardinality among the maximum independent set of vertices of G. The vertex covering number α_0(G) is the minimum cardinality of vertex covering of G. The girth g (G) of a graph G is the length of the shortest cycle in G. The circumference c (G) is the length of the longest cycle. The radius of G is rad(G) = min{ecc(v): v∈ V} and diam(G) = max{ecc(v):v∈ V}, where ecc(v) = max{dis(u,v): v∈ V }.

A nonempty subset $D \subseteq V(G)$ is said to be a dominating set of G if every vertex not in D is adjacent to at least one vertex in D. A dominating set D ⊆ V of a graph G is a split dominating set if the induced sub-graph < V-D > is disconnected. The split domination number $\gamma_s(G)$ is the minimum cardinality of a split dominating set. A set D of vertices in a graph G is a distance - 2 dominating set if every vertex in V-D is within distance - 2 of at least one

vertex in D. The distance - 2 domination number $\gamma_{\leq 2}(G)$ of G equals the minimum cardinality of a distance 2- dominating set in G.

Kulli V.R. and Janakiram B. introduced the concept of split domination in graph in [13]. The purpose of this paper is to introduce the concept of split distance - 2 domination in graphs.

Definition 1.1

A distance - 2 dominating set $D \subseteq V$ of a graph G is a split distance - 2 dominating set if the induced sub graph < V-D > is disconnected. The split distance - 2 domination number $\gamma_{s\leq 2}(G)$is the minimum cardinality of a minimal split distance - 2 dominating set.

The minimal distance - 2 dominating set in a graph G is a distance - 2 dominating set that contains no distance - 2 dominating set as a proper subset.

The distance - 2 open neighborhood of a vertex $v \in V$ is the set, $N_{\leq 2}(v)$, of vertices within a distance of two from (v).

Example: 1.2

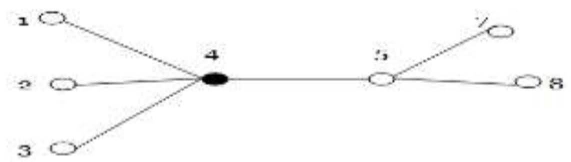

Figure.1

Here $D = \{4\}$, $\gamma_{s\leq 2}(G) = 1$

2. Common Graphs and Exact Values

The split distance - 2 domination number $\gamma_{s\leq 2}(G)$ of some common graphs is given as an observation.

2.1. Observation

1. For any path P_n , for n≥ 2

$$\gamma_{s\leq 2}(P_n) = \left\lceil \frac{n}{5} \right\rceil$$

2. For any cycle C_n, for n≥ 6

$$\gamma_{s\leq 2}(C_n) = \left\lceil \frac{n+1}{5} \right\rceil$$

3. For any complete graph K_n , for n≥ 3

$$\gamma_{s\leq 2}(K_n) = n - 1$$

4. For any star graph $K_{1,m}$, for m ≥1

$$\gamma_{s\leq 2}(K_{1,m}) = 1$$

5. For any complete bipartite graph $K_{n,m}$, for m ≥ n,

$$\gamma_{s\leq 2}(K_{n,m}) = n$$

3. Bounds on the Split Distance -2 Domination Number $\gamma_{s\leq 2}(G)$

Theorem 3.1
For any graph G, $\gamma_{\leq 2}(G) \leq \gamma_{s\leq 2}(G)$.

Proof
Every split distance - 2 dominating set of G is a distance - 2 dominating set of G. We have, $\gamma_{\leq 2}(G) \leq \gamma_{s\leq 2}(G)$.

Theorem 3.2
For any graph G, $\gamma_{s\leq 2}(G) \leq \gamma_s(G)$.

Proof
Every split dominating set of G is a split distance - 2 dominating set of G. Hence $\gamma_{s\leq 2}(G) \leq \gamma_s(G)$.

Theorem 3.3
Let D is a minimal split distance - 2 dominating set if and only if each vertex $v \in D$, satisfies at least one of the following conditions
(i) There exists a vertex $u \in V - D$, such that
Case (a): When D is connected, $N_{\leq 2}(u) \cap D = D$
Case (b): When D is disconnected, $N_{\leq 2}(u) \cap D = \{v\}$
(ii) $v$is an isolated vertex in < D>
(iii) $\{(V - D) \cup \{v\}\}$ is connected.

Proof
Suppose D is a minimal split distance - 2 dominating set. On the contrary, there exists a vertex$v \in D$, such that v does not hold any of the above conditions. Then by conditions (i) and (ii), $D_1 = D - \{v\}$isa distance - 2 dominating set of G**.** Also by (iii), < V-D_1 > is disconnected. Hence D_1 is a split distance - 2 dominating set of G, which is a contradiction. Therefore, the condition is necessary.

Sufficiency follows from the given conditions.

Theorem 3.4
For any graph G with a pendant vertex
$\gamma_{\leq 2}(G) = \gamma_{s\leq 2}(G)$,
Furthermore, there exists a split distance - 2 dominating set of G containing all vertices within the distance - 2 pendant vertices.

Note: 3.5
For any graph G, we observe $\gamma_{\leq 2}(G) \leq \gamma(G)$.

Theorem 3.6 (Kulli and Janakiram, 1997)
For any graph G, $\gamma_s(G) \leq \alpha_0(G)$.

Theorem 3.7
For any graph G, $\gamma_{s\leq 2}(G) \leq \alpha_0(G)$.

Proof
Since $\gamma_{s\leq 2}(G) \leq \gamma_s(G)$ and $\gamma_s(G) \leq \alpha_0(G)$ [By Theorem 3.6]
We have $\gamma_{s\leq 2}(G) \leq \alpha_0(G)$.

Theorem 3.8
For any graph G, $\gamma_{\leq 2}(G) + \gamma_{s\leq 2}(G) \leq n$.

Proof
Since $\gamma(G) \leq \beta_0(G)$, $\gamma_{\leq 2}(G) \leq \gamma(G)$ and $\gamma_s(G) \leq \alpha_0(G)$ [By Theorem 3.6]
Thus $\gamma_{\leq 2}(G) + \gamma_{s\leq 2}(G) \leq \alpha_0(G) + \leq \beta_0(G)$,
We have $\gamma_{\leq 2}(G) + \gamma_{s\leq 2}(G) \leq n$.
The sharpness of this equality can be seen with the path P_2 and cycle C_3.

Theorem 3.9
For any graph G, $(G) + \gamma_{s\leq 2}(G) \leq n$.

Proof
Since $(G) \leq \beta_0(G)$, and $\gamma_s(G) \leq \alpha_0(G)$ [By Theorem 3.6]
Thus $i(G) + \gamma_{s\leq 2}(G) \leq \alpha_0(G) + \beta_0(G)$
We have $i(G) + \gamma_{s\leq 2}(G) \leq n$.
The sharpness of this equality can be seen with the path P_2, cycle C_3 , C_4.

Theorem3.10 (Kulli and Janakiram, 1997)
For any graph G, $\gamma_s(G) \leq \frac{n.\Delta(G)}{(\Delta(G)+1)}$.

Theorem 3.11 (Kulli and Janakiram, 1997)
For any graph G, $\gamma_{s\leq 2}(G) \leq \frac{n.\Delta(G)}{(\Delta(G)+1)}$.

Proof
Since $\gamma_{s\leq 2}(G) \leq \gamma_s(G)$and $\gamma_s(G) \leq \frac{n.\Delta(G)}{(\Delta(G)+1)}$ [By Theorem 3.10]
We have, $\gamma_{s\leq 2}(G) \leq \frac{n.\Delta(G)}{(\Delta(G)+1)}$.

Theorem 3.12
For any graph G, $\gamma_{s\leq 2}(G) = \gamma_s(G)$ if and only if G is a wheel W_n.

Theorem 3.13
For any graph G, $\gamma_{s\leq 2}(G) = \gamma_s(G) = \gamma_{\leq 2}(G) = \gamma(G)$ if and only if G is a star $K_{1,m}$, for m>1.

Theorem 3.14
For any graph G, $\gamma_{s\leq 2}(G) = \gamma_s(G) = \gamma(G)$ if and only if G is a friendship graph F_n.

Theorem 3.15
For any graph G, $\gamma_{s\leq 2}(G) = \gamma_s(G)$ if and only if G is a bipartite graph $K_{n,m}$, for n< m.

Theorem 3.16
For any graph G, $\gamma_{s\leq 2}(G) = p - \Delta(G)$ if and only if G is a star graph $K_{1,m}$, for m> 1, where $p = 1 + m$ is number of vertices.

Theorem 3.17
For any graph G, which is not a tree then $\gamma_{s\leq 2}(G) \leq c(G)$ where c(G) is circumference of a graph G.

Theorem 3.18
For any graph G, which is not a tree then $\gamma_{s\leq 2}(G) \leq g(G)$ where g(G) is girth of a graph G.

Theorem 3.19
For any graph G, $\gamma_{s\leq 2}(G) \leq n - \Delta(G)$.

Note 3.20
For any graph G, $\gamma_{s\leq 2}(G) \leq \Delta(G)$.

Theorem 3.21
For any tree T_n, $\gamma_{s\leq 2}(G) \leq p$where p is number of end vertices.

Note 3.22
(i) For any tree T_n, which is a star graph $\gamma_{s\leq 2}(G) = n - p$ where n is number of vertices and p is number of end vertices.
(ii) $\gamma_{s\leq 2}(T_n) = \gamma_{\leq 2}(T_n)$.

Theorem 3.23
If H is a connected spanning sub graph of G, then $\gamma_{s\leq 2}(G) \leq \gamma_{s\leq 2}(H)$.

Theorem 3.24
Let G be any connected graph of order greater than 3, then $\gamma_{s\leq 2}(G) \leq n-1$, where n is the number of vertices.

Nordhas - Gaddum Type results

Theorem 3.25
Let G be a graph such that both G and$\bar{G}$ have no isolates. Then,
(i) $\gamma_{s\leq 2}(G) + \gamma_{s\leq 2}(\bar{G}) \leq 2(n-1)$
(ii) $\gamma_{s\leq 2}(G) + \gamma_{s\leq 2}(\bar{G}) \leq (n-1)^2$

Lemma 3.26
For$k \geq 1$, every connected graph G has a spanning tree T such that $\gamma_k(G) = \gamma_k(H)$ in [23].

Lemma 3.27
For$k \geq 1$, every connected graph G has a spanning tree T such that $\gamma_{s\leq 2}(G) = \gamma_{s\leq 2}(H)$.

Proof
Since $\gamma_{\leq 2}(G) \leq \gamma_{s\leq 2}(G)$ and $\gamma_{\leq 2}(T) = \gamma_{\leq 2}(G)$ [By Lemma 3.26].
We have $\gamma_{s\leq 2}(G) = \gamma_{s\leq 2}(H)$.

Theorem 3.28
For $k \geq 1$, if G is a connected graph with diameter d, then $\gamma_k(G) \geq \frac{d+1}{2k+1}$ in [23].

Theorem 3.29
For any graph G is a connected graph with diameter d, then $\gamma_{s\leq 2}(G) \geq \frac{d+1}{2k+1}$.

Proof
Since $\gamma_{\leq 2}(G) \leq \gamma_{s\leq 2}(G)$ and $\gamma_{\leq 2}(G) \geq \frac{d+1}{2k+1}$ [By Theorem 3.28].
Thus we have, $\gamma_{s\leq 2}(G) \geq \frac{d+1}{2k+1}$.

Theorem 3.30
If G = P_n where $n \equiv 0 \ mod(2k+1)$, then $\gamma_k(G) = \frac{diam(G)+1}{2k+1}$ in [23].

Theorem 3.31
If G = P_n where$n \equiv 0 \ mod(2k+1)$, then $\gamma_{s\leq 2}(G) = \frac{diam(G)+1}{2k+1}$.
Proof
Since $\gamma_{\leq 2}(G) \leq \gamma_{s\leq 2}(G)$ and $\gamma_{\leq 2}(G) = \frac{diam(G)+1}{2k+1}$ [By Theorem 3.30]
We have $\gamma_{s\leq 2}(G) \geq \frac{diam(G)+1}{2k+1}$

Theorem 3.32
For any graph G is a connected graph with radius r, then$b\gamma_{s\leq 2}(G) \geq \frac{2r}{2k+1}$ in [23].

Theorem 3.33
For any graph G is a connected graph with radius r, then $\gamma_k(G) \geq \frac{2r}{2k+1}$.

Proof
Since $\gamma_{\leq 2}(G) \leq \gamma_{s\leq 2}(G)$ and $\gamma_{\leq 2}(G) \geq \frac{2r}{2k+1}$ [By Theorem 3.32]
We have $\gamma_{s\leq 2}(G) \geq \frac{2r}{2k+1}$

Theorem 3.34
For any graph G is a connected graph with girth g, then $\gamma_k(G) \geq \frac{g}{2k+1}$ in [23].

Theorem 3.35
For any graph G is a connected graph with girth g, then $\gamma_{s\leq 2}(G) \geq \frac{g}{2k+1}$.

Proof
Since $\gamma_{\leq 2}(G) \leq \gamma_{s\leq 2}(G)$ and $\gamma_{\leq 2}(G) \geq \frac{g}{2k+1}$ [By Theorem 3.34]
We have $\gamma_{s\leq 2}(G) \geq \frac{g}{2k+1}$.

4. Conclusion

In general, the concept of dominating sets in graph theory finds wide applications in different types of communication networks. Identification of a dominating set, which is fault tolerant, will be useful in the central location theory.
Thus in this paper, we defined the notion of split distance - 2 domination in graphs. We obtained many bounds on distance - 2 split domination number. Exact values of this new parameter are obtained for some standard graphs. Nordhaus - Gaddum type results are also obtained for this new parameter. They play very vital role in Coding theory, Computer science, Operations Research, Switching Circuits, and Signal Processing Electrical Networks etc.

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