

# Design methodology and simulation study of a FEL amplifier

Ram Gopal<sup>1\*</sup>, Pradip Kumar Jain<sup>2</sup>

<sup>1</sup> Centre of Research in Microwave Tubes, Department of Electronics Engineering, Indian Institute of Technology (Banaras Hindu University), Varanasi-221005, India

<sup>2</sup> Director, National Institute of Technology, Patna-80005, India and Professor, CRMT, Department of Electronics Engineering, Indian Institute of Technology (Banaras Hindu University), Varanasi-221005, India

\*Corresponding author E-mail: ramgopal.rs.eee@itbhu.ac.in

## Abstract

In this paper, the operating behavior and optimization in the  $TM_{01}$  mode of a free electron laser amplifier (FELA) has been discussed using the eigen mode analysis and the simulation techniques and their results has been also analyzed with 3-D particle-in-cell (PIC) codes "CST particle studio". To understand the performance analysis and sensitivity of the FEL amplifier using an intense relativistic electron beams (REBs) on various parameter effects such as power, gain, efficiency, beam current, beam voltage and operating frequencies has been explained. In the modeling of an FEL amplifier, the pattern of the magnetic and electric field in  $TM_{01}$  mode has been showed and performed their RF simulation results in the absent case of electron beams i.e., the cold simulation. The reading of the measurement values has been showed for the large radiation growth rate 2dB/cm approximately with 231 GHz instantaneous band width. The good agreement of the simulation results has been found as reported experimental values with predicted theory of operation in the Collective Raman Regime. Additionally, the result to extraction of the kinetic energy from the electron beams to beat-wave has been observed the power and efficiency largely increases experimentally with the linear tapering of the strong axial magnetic field that produces 20% experimental efficiency in the FEL amplifier.

**Keywords:** FEL Amplifier, Beam wave interaction chamber, Relativistic electron beams (REBs), Wiggler, Whistler, PIC simulation and Tapered magnetic field operation.

## 1. Introduction

The device, free electron laser amplifier (FELA) has been developed using a relativistic electron beams (REBs) with zero percent velocity spreading and 1–2MeV moderating energy [1]-[4]. It is demonstrated for extremely high power, continuous tunable radiation and moderated efficiency of the source in millimeter wave to sub millimeter ranges and the potential application in the field of communication system, nuclear fusion, radio astronomy, military applications, atmospheric studies, chemical and isotope separation, inertial confinement fusion, remote sensing and security identification, plasma accelerator diagnostics, medicine and molecular spectroscopy, biological imaging (general test, measurement and diagnosis), room temperature THz imaging, materials characterization (spectroscopy of solids and liquids), build computer chips and to clean up toxic waste, electronic material processing, nanotubes synthesis and many other curriculums. The operating radiation frequency of FEL amplifier  $\lambda_1$  scales with wiggler period  $\lambda_o$  and beam Lorentz factor or the relativistic gamma factor ( $\gamma_o$ ) of the electron beam,  $\gamma_o = 1 + E_b / mc^2$  as  $\lambda_1 = \lambda_o / 2\gamma_o^2$  where  $E_b$  is the beam kinetic energy,  $m$  is the rest mass of electrons and  $c$  is the light velocity in vacuum and the synchronous bunching and phase matching phenomenon to be satisfied as  $\omega = \omega_1 - \omega_o$  and  $k = k_o + k_1$ . The primary importance of the device is essential for operating behavior and optimization in the  $TM_{01}$  and extraction of the RF output from the electron beams. Although the radiated could be tuned by very small wiggler period and/or higher electron beams

energy or ambient magnetic fields, however, in practically, it is not accessible more easily for high beams energy as well as shortening the wiggler periods and also typically, not all, the magnetic wiggler field are not easily accessible for very small wiggler wavelengths [5]-[9].

The typical layout and 3-D schematic diagram of the FEL amplifier is shown in below Fig.1, which consists electron gun and accelerator, bitter magnets/axial field coils, wiggler magnets/undulator, depressed collector for beam dumped, beam interaction chamber, electron beams and optical mirror for radiated output of the FEL amplifiers. An electron gun is the first component. The accelerator, which generated a suitable voltage  $>500kV$  (it should become very high as 1GV) and current ( $>1A$  to 100kA) for the electron beams but it should have some limitations. It is approximated 17kA [10] [11]. Normally the pulse duration can be used  $\leq 100nsec$  for the beams while  $\approx 1msec$  are also used in some other experiments. The hot or cold cathode provided electrons acceleration through the diode structure together with the strong axial guided magnetic field and/or focusing elements. Here the FEL amplifier frequency radiation underlets on  $\gamma_{ob}^o$  (relativistic gamma factor) while, approximate 0.1% spreaded energy of the electron beams and the radius of the beams 1cm with  $\approx 1-2mm$  beams thickness. Typically, the approximate values of the wiggler strengths is  $\approx 1kG$  and the magnetic wiggler wavelengths  $\approx 3cm$  [11]. The tapered wiggler with magnitude  $B_o$  and the period of the wiggler  $2\pi/k_o$  have adiabatically changes along the FEL amplifier structure with slowing down the beat-wave, hence, the

linear tapered axial magnetic fields leading to improve the efficiency of the amplifier also.

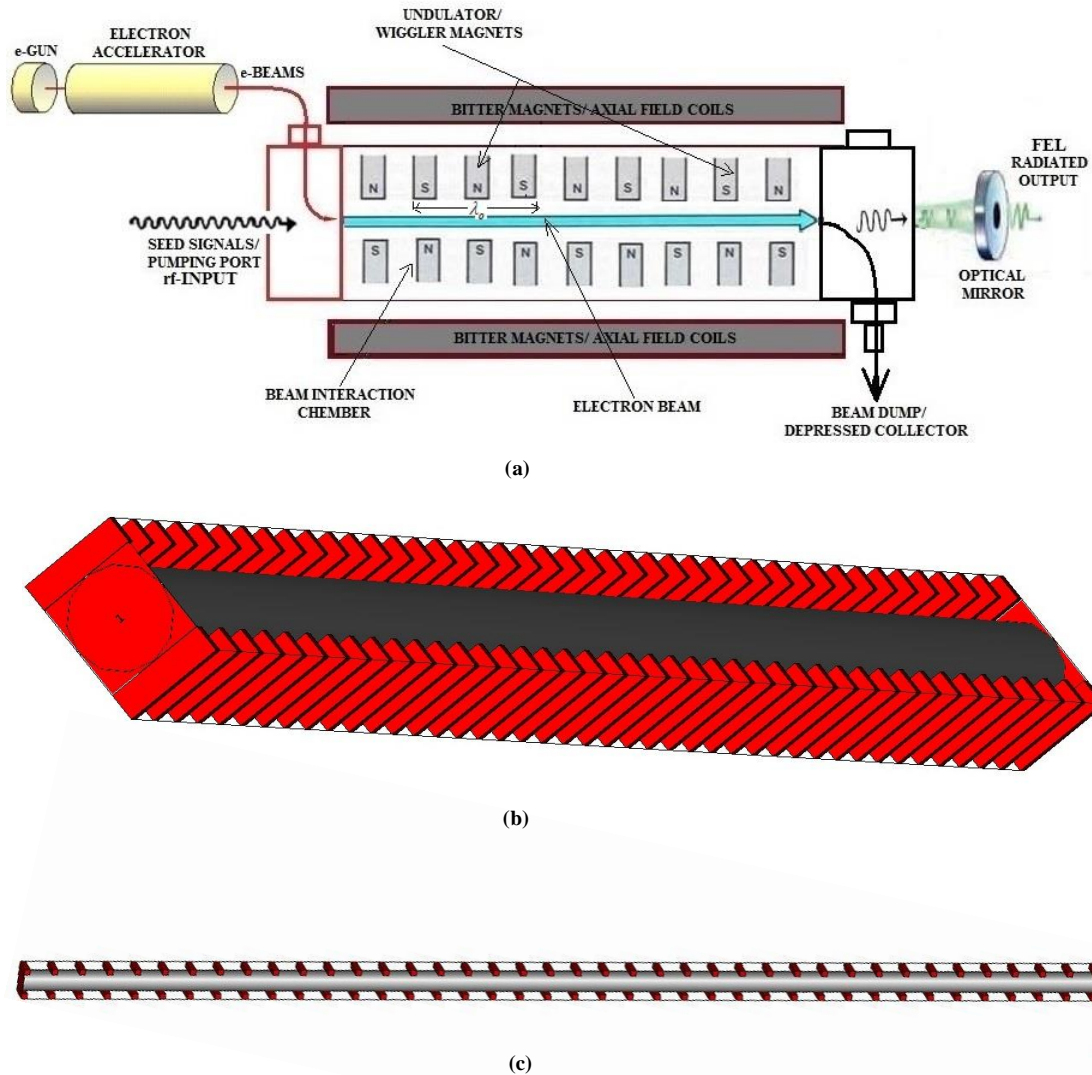


Fig. 1: Typical layout (a) and 3-D Schematic diagram of free electron laser amplifier (FELA) (b & c).

An FEL amplifier would have a high broad band reflecting mirror at output. The recovery of energy is another important issue of the beams spent into the interaction chamber and collected at the output port through the depressed collectors and ignoring here, boundary effects for FEL amplifier growth rate radiated into the interaction chamber [12]-[14].

The facilities of the designation of FEL amplifier (FELA) device and the description of the mechanism between beam-wave interactions, there are several modeling and simulation techniques has been reported. The MAGY, ARGUS and CHICPIC, there are many numerical technique based simulation codes are employed successfully for the electromagnetic behavior of the amplifier, commercially, it is not easily available but only can use for the design optimization and their validation of the device [15]-[17]. The MAGIC, "particle-in-cell (PIC) simulation codes" has been envisaged to examine the FEL amplifier successfully and also applicable to use for commercially [18] [19]. The CST particle studio, PIC simulation codes is another technique to study the finite integration methods and exhibition of the possibilities of main mode temporal growth along with the various competing modes simultaneously [20]-[25].

In the recent and presented work, the simulation and design methodology through particle-in-cell (PIC) simulation codes of the FEL amplifier by emissions of electrons, comprehensive are not reported. Here we examine an explosive derivation to start the

amplifier oscillation and derived the design methodology of the device. The emission process in the FEL amplifier interaction chamber is simulated through the commercially 3-D PIC simulation codes "CST Particle Studio". To find out the excitation of electromagnetic modes, frequencies and EM field patterns inside the interaction region of an amplifier is carried out through the eigen mode simulation (i.e., cold simulation or beams absent simulations). To observed the overall performance of an FEL amplifier, such as frequency of operations, gain, power, efficiency, RF output and the beams present PIC simulation is carried out.

In the present paper, Section II gives an analysis of FEL amplifier to start oscillation. The description of the design methodology of an FEL amplifier is in section III. The device simulation using PIC simulation in "CST Particle Studio" is briefly discussed in section IV and the conclusion is drawn in section V.

## 2. Analysis of FEL amplifier

In a free electron laser, consider the interaction region  $0 < z < L$  and it comprises an electron plasma density  $n_{op}^o$  immersed into a static magnetic field  $B_z(z)$ . Since the magnetic field has a linear taper, therefore,

$$B_z = B_{os} (1 - z/L)$$

and,

$$\vec{B}_o = B_o (\hat{x} + i\hat{y}) e^{ik_o dz} \quad (1)$$

The propagation of a circularly polarized whistler wave through the plasma along  $-\hat{z}$  direction and using the WKB approximation and an electric field is as,

$$\vec{E}_o = A_o (\hat{x} + i\hat{y}) e^{-i(\omega_o t + \int k_o dz)} \quad (2)$$

Where,  $k_o = \frac{\omega_o}{c} \varepsilon^{1/2}$ ,  $\omega_c = \omega_{co} (1 - \frac{z}{L})$ ,  $\omega_{co} = \frac{eB_{os}}{mc}$ ,  $\omega_p = (\frac{n_{op} e^2}{\varepsilon_o m})^{1/2}$

and  $\varepsilon = [1 - \frac{\omega_p^2}{\omega_o(\omega_o - \omega_c)}]$ . Here  $\omega_c$  and  $\omega_{co}$  is electron cyclotron

frequency and initial electron cyclotron frequency,  $\omega_o$  is wiggler wave frequency and the wiggler wave number  $k_o$ ,  $\omega_p$  is plasma frequency of an electron cyclotron and  $n_{op}$  is the plasma electron density of the medium,  $-e$  is the charge of electrons and  $m$  is the electrons rest mass,  $c$  is the light speed in vacuum,  $L$  is the magnetic field scale length,  $\varepsilon_o$  and  $\varepsilon$  are free space and relative permittivity respectively. An electromagnetic wave frequency that produces by wiggler fields is  $\gamma_{ob}^o k_o v_{ob}^o$  and the guided axial field gives the motion of electrons cyclotron at  $\omega_c$  (cyclotron frequency). It acquires an oscillatory velocity in transverse direction due to the whistler wave. Therefore, the relativistic equation of motion one obtains,

$$\vec{v}_{ob} = \frac{e\vec{E}_o (1 + k_o v_{ob}^o / \omega_o)}{imc\gamma_{ob}^o (\omega_o + k_o v_{ob}^o - \omega_c / \gamma_{ob}^o)} \quad (3)$$

Here,

$$\gamma_{ob}^o = [1 - (v_{ob}^o / c)^2]^{-1/2}, \quad \vec{B}_o = \frac{k_o (-\hat{z} \times \vec{E}_o)}{\omega_o}, \quad \vec{v}_{ob} = \frac{\omega_1}{k_1} \quad \text{and}$$

$$\vec{k}_o = -k_o \hat{z}. \quad \text{Where, } \gamma_{ob}^o \text{ is relativistic gamma factor.}$$

We launch a circularly polarized FEL radiation through the length of  $z = 0$  end with electric field,

$$\vec{E}_1 = A_1 (\hat{x} + i\hat{y}) e^{-i(\omega_1 t - k_1 z)} \quad (4)$$

Where wave number of FEL radiation,  $k_1 = \omega_1 / c$  and frequency of radiation,  $\omega_1 \gg \omega_p, \omega_c$ . If radiated electrons move with a whistler wave, then the phase matching condition to be satisfied on synchronism state as  $\omega = \omega_1 - \omega_o$  and  $k = k_o + k_1$ , hence,

$$v_{ob} = \frac{\omega}{k} = \frac{\omega_1 - \omega_o}{k_1 + k_o} \quad (5)$$

or,

$$\lambda_1 = \lambda_o / 2\gamma_{ob}^o{}^2$$

Furthermore, the wiggler magnetic field ( $B_o$ ) as wiggler wavelength ( $\lambda_o$ ) and electric field ( $E_o$ ) is an important parameter to start oscillation and confirmation of the stability of an amplifier and phase bunching should be proper to compute the operating wavelengths  $\lambda_1$  of the FEL amplifiers. The operating radiation

frequency of FEL amplifier  $\lambda_1$  scales with wiggler period  $\lambda_o$  and beam Lorentz factor or the relativistic gamma factor ( $\gamma_o$ ) of the electron beam,  $\gamma_o = 1 + E_b / mc^2$ , where  $E_b$  is the beam kinetic energy,  $m$  is the rest mass of electrons and  $c$  is the light velocity in vacuum. The whistler wave and the FEL radiation wave imparts oscillatory velocities  $v_{1b}$ , and  $v_{1p}$  to beam and plasma electrons, then using equation of motion, one obtain  $\vec{v}_{1b} = \frac{e\vec{E}_1}{im\gamma_{ob}^o \omega_1}$  and  $\vec{v}_{1p} = \frac{e\vec{E}_1}{im\omega_1}$ . Since the Ponderomotive force on electrons beam at  $(\omega_1, k_1)$  exerted by whistler wave and FEL beat signal, assuming here,  $\omega = \omega_1 - \omega_o$  and  $k = |k_o| + k_1$  and  $\vec{\omega}_1 \gg \vec{\omega}_c$ , therefore the total Ponderomotive force is as,

$$\vec{F}_{pb} = -(\frac{e^2 A_o^* A_1}{imc\omega_1 \omega_o \gamma_{ob}^o}) [k_o + \frac{k_1 (\omega_o + k_o v_{ob}^o)}{(\omega_o + k_o v_{ob}^o - \omega_c / \gamma_{ob}^o)}] \hat{z} e^{-i\psi} \quad (6)$$

Under the influence of these Ponderomotive wave, an electric field  $\vec{E}_{pb}$  can be written as,

$$\vec{E}_{pb} = (\frac{eA_o^* A_1}{im\omega_1 \omega_o \gamma_{ob}^o}) [k_o + \frac{k_1 (\omega_o + k_o v_{ob}^o)}{(\omega_o + k_o v_{ob}^o - \omega_c / \gamma_{ob}^o)}] \hat{z} e^{-i\psi} \quad (7)$$

Where,  $\psi = \omega t - \int (k_o + k_1) dz$ ,  $\omega = \omega_1$ , and  $k = |k_o| + k_1$ , and the z-component of single particle beam momentum equation under the beat Ponderomotive force can be written as,  $\frac{dP_{pb}}{dt} = v_{zb} \frac{dP_{pb}}{dz} = mc^2 \frac{d\gamma_e}{dz} = F_{pbz}$  or  $\frac{d\gamma_e}{dz} = \frac{1}{mc^2} F_{pbz}$ . Here  $\gamma_e$  denotes the relativistic energy of an electron at any point. Hence from Eq. (6) and taking real part, we have,

$$\frac{d\gamma_e}{dz} = -\frac{eA_{pb}}{mc^2} \cos \psi \quad (8)$$

Where  $\vec{A}_{pb} = -i\vec{E}_{pb}$ ,

$$E_{pb} = (\frac{a_1 A_o^* c}{\omega_o \gamma_{ob}^o}) [k_o + \frac{k_1 (\omega_o + k_o v_{ob}^o)}{(\omega_o + k_o v_{ob}^o - \omega_c / \gamma_{ob}^o)}] \quad \text{And } a_1 = \frac{eA_1}{m\omega_1 c}$$

This equation represent that the wiggler is in uniform state, means no resonance is possible here. Now from Eq. (8), the momentum of the wave i.e.,  $P = \frac{d\psi}{d\xi}$  one obtains,

$$\frac{d\psi}{d\xi} = \frac{\omega_1 L \Delta \gamma_e}{2c(\gamma_r^2 - 1)^{3/2}} \quad (9)$$

Therefore the constitute the phase momentum and energy evolution equations can be rewritten as,

$$\frac{dP}{d\xi} = -A \cos \psi \quad (10)$$

$$\frac{d\psi}{d\xi} = P$$

Where  $A$  is constant i.e.,  $A = \frac{eA_{pb} L^2 \omega_1}{2mc^3 (\gamma_r^2 - 1)^{3/2}}$ . For the small radi-

ation of single pass amplification, it is worthwhile solving Eq. (10), i.e., an electron can lose the energy to the wave with phase  $\psi$  is between  $-\pi/2$  to  $\pi/2$ .

### 3. Design methodology

There are major rigorous to design of free electron laser amplifier (FELA) at higher frequencies for the various parameters. The process of proper designing of FELA is required to ensure the RF interaction structure with desired operating mode, mode of extraction, frequency operation, electron beam parameter, high output power, moderate efficiency and so on. The beam current and voltage chosen initially through the selection procedure of the desired power output, extraction efficiency and the beams electron space charge effect. Typically, the  $TM_{01}$  mode is selected for the operating behavior of an amplifier and their dimensions of RF interaction structure, the beam parameters (i.e., velocity ratio  $\leq 0.1\%$  and the guiding radius), modulating interaction radius and strong axial magnetic field are estimated to established analysis [22]-[24].

If the FEL amplifier is operating in the  $TM_{mn}$  mode and the radius of an interaction chamber wall is  $R_{felwallradius}$ , then the expression [24] is as-

$$R_{felwallradius} = \frac{\chi'_{mn} c}{2\pi f'_c} \quad (11)$$

Where  $\chi'_{mn}$  is the eigen value of the cylindrical waveguide system and the values are 2.405 for the  $TM_{01}$  mode as the root of the Bessel for  $TM_{mn}$  mode dispersion relation,  $c$  is the light speed in the vacuum and  $f'_c$  is the cut off frequency of the cylindrical waveguide.

Now the guiding beam radius of an electron [24] is given as-

$$r_b = \left( \chi'_{m\pm h}, \frac{iR_{felwallradius}}{\chi'_{mn}} \right) = \left( \chi'_{m\pm h}, \frac{ic}{2\pi f'_c} \right) \quad (12)$$

Where the radius of the electron beams is  $r_b$ ,  $i$  is the beam radial positions (i.e., 1 or 2) and the harmonic number is  $h$ , always  $h \geq 1$ . In the amplifier interaction chamber, the RF wave is extracted out from the beams electron of slow wave space charge through the forward propagation with group velocities in positive direction and leaving out the RF signal at the output port, however, the negative group velocities are also available here with backward waves propagation into the interaction region. To foreclose the negative group velocities propagate in the backward wave direction, the oscillation frequency of the device should be greater than the cut off frequency of the cylindrical wave guide which is used into the amplifier. The detailed design procedure and flow chart of the free electron laser amplifier (FELA) is given below fig. 2 and the listed data given in **Table I**.

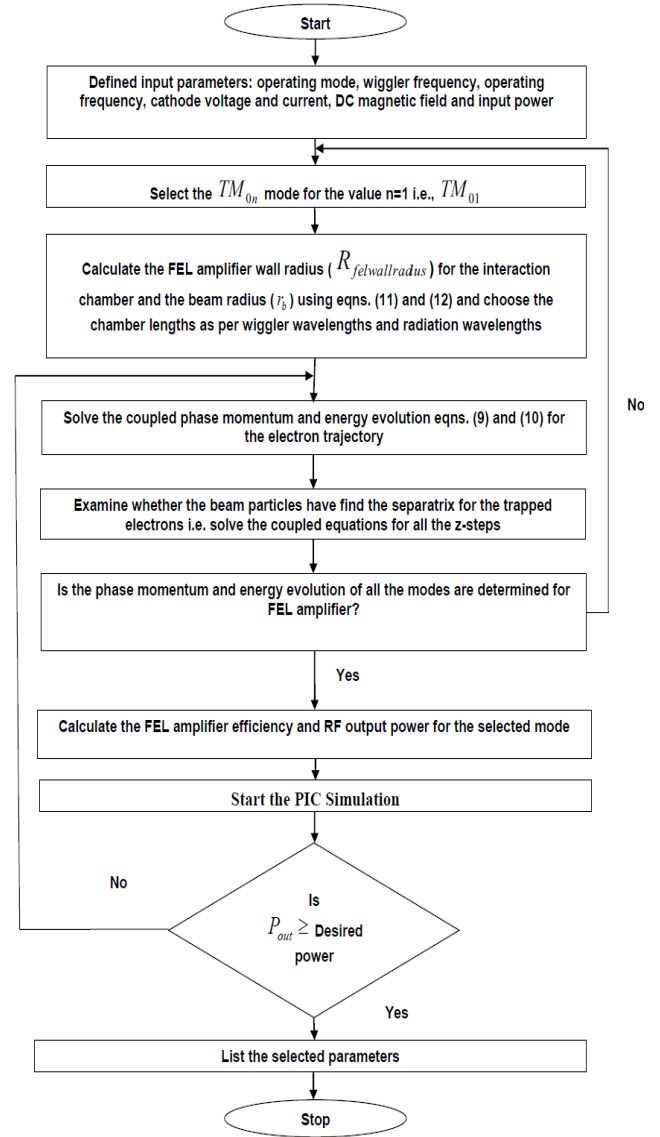


Fig. 2: Flow chart and design procedure of the FEL Amplifier.

TABLE I  
Design parameters of the FEL Amplifier (231 GHz)  
[Gold *et al.* (1984), Pant and Tripathi (1994)]

FEL amplifier parameters	Values
<b>Magnetics wiggler parameters</b>	
Wiggler wavelengths ( $\lambda_o$ )	30mm
Wiggler magnets size	$3 \times 3 \times 1 \text{mm}^3$
Gap size	12mm
Wiggler frequency ( $f_o$ )	10GHz
Wiggler fields ( $B_o$ )	3kG
No. of poles ( $N$ )	21
Interaction Lengths ( $L$ )	630mm
<b>Electron beams parameters</b>	
Wave guide type	Uniform cylindrical
Wave guide radius ( $R_{felwallradius}$ )	5.5mm (outer) 5.4mm (inner)
Transverse mode	$TM_{01}$
Wave guide cut off frequency ( $f'_c$ )	21.27GHz
Beam radius ( $r_b$ )	3mm
Bunch energy ( $E_b$ )	1.25MeV

Beam current ( $I_b$ )	1kA
Pulse duration ( $t_b$ )	50nsec
<b>Radiated output parameters</b>	
Radiation wavelengths ( $\lambda_1$ )	1.2796mm
Radiation frequency ( $f_1$ )	231.2GHz
Relativistic Gamma Factor ( $\gamma_{ob}^o$ )	3.4
Axial guide fields ( $B_s$ )	$U_p$ to 20kG
Longitudinal momentum spreading ( $p_l$ )	0.1%
Output Power ( $P_o$ )	50MW

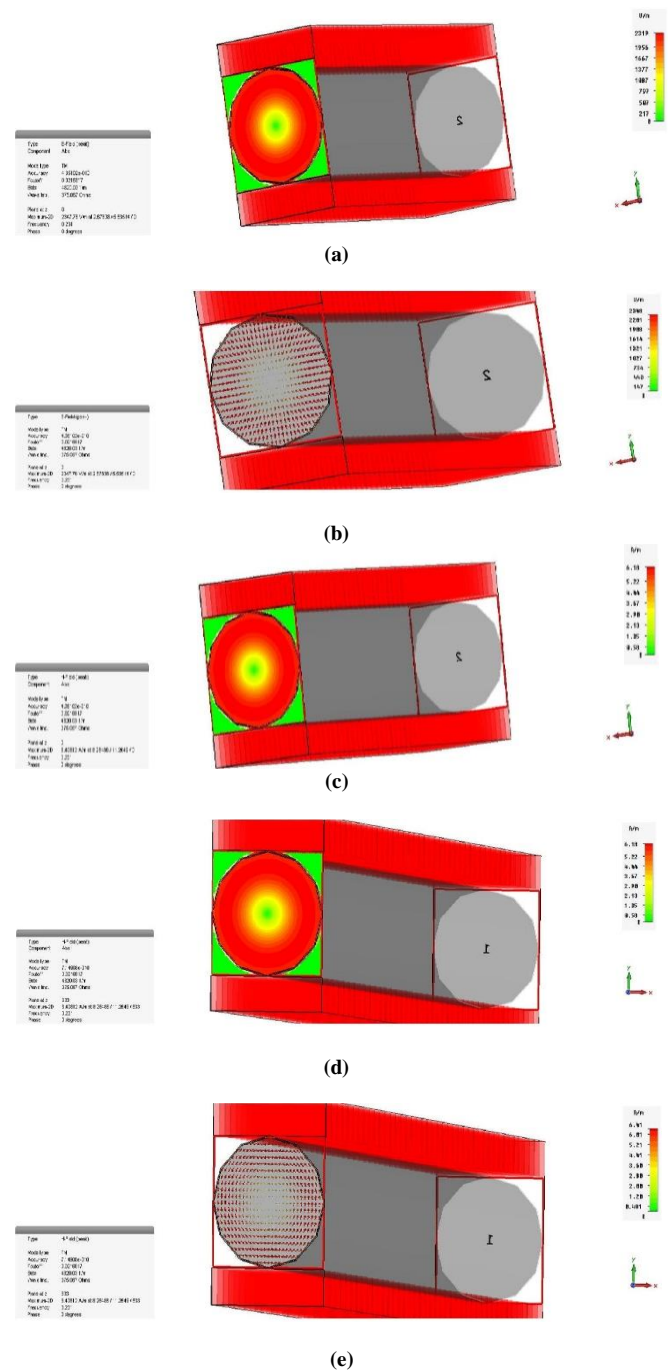
#### 4. Device simulation

The fig. 1 (a) shown a typical lay out in 2-D and fig. 1 (b & c) represented 3-D design schematics of a FEL amplifier which consist of e-gun, cathode, cylindrical waveguide, modulating interaction chamber, undulator/wiggler as magnetic arrangement, axial magnetic fields, depressed collector for beam collection and finally optical mirror to collect the FEL amplifier radiated output. The interaction chamber is mainly used to phase bunching phenomenon and implementation of beam modulation techniques between wiggler magnetic fields to electron beams which produces beat-wave excitation as ponderomotive waves. In order to understanding the mechanism of beam-wave interaction and performance analysis of the FEL amplifier is designed and simulated through the commercial PIC simulation using ‘‘CST particle studio’’ [21] [22]. The FEL amplifier is modeled as fig. 1, as per the material property, magnetic field, electric field and designed parameters as listed in **Table I**.

The performance analysis of EM simulation in eigen mode (i.e., beam-absent analysis or cold analysis) is examined to secure the operating mode field stimulation and oscillating frequency and so on. In the last the device is excited in the supervision of beams electron (i.e., hot analysis) to examine the frequency of operation, extraction efficiency, RF power output and gain. The amplifier design methodology and simulation techniques using PIC simulation codes ‘‘CST particle studio’’ is given below in details.

##### A. RF interaction modeling

The FEL amplifier is designed undulator/wiggler to performing wiggler array of the RF interaction behavior in CST particle studio for particle-in-cell (PIC) simulation as shown in fig. 2, (flow chart), as designed parameters described in **Table I**. The modeling and designing of RF interaction chamber, there are various parameters used as undulator/wiggler and their dimensions, cylindrical waveguide with specific radius, drift tube, beams and beams radius, undulator/wiggler magnetic fields, gap size between magnets of undulator/wiggler, lengths of interaction chamber, wiggler periods, operating frequency and S-parameters and so on. The undulator/wiggler structure is modeled with iron magnetic materials (NdFeB) and vacuum background set. The drift tube of the interaction chamber is loaded with lossy dielectric materials to achieving the optimized s-parameters and allows adjusting of the amplifier behavior. Also the electric field (E-field) monitor is adjusting to ensure the electric fields into the interaction chamber through the input port and the observation of desired mode and their operating frequency is examined. On other hand, the magnetic field (H-field) monitor is set to observe the calculation of s-parameters which provide the complete isolation between the interactions chambers of both ends by observing the field leaked inside the interaction chamber. The phenomenon of the energy transferred from electrons to beat-wave ensured by setting the bunch formation of electrons through the phase space monitor.



**Fig. 3:** Fields pattern of the FEL Amplifier for the desired mode  $TM_{01}(i)$  Electric field patterns at the input port (a) contour plot (b) vector plot (c) Magnetic field patterns at the input port, contour plot, and (ii) Magnetic field patterns at the output port (d) contour plot and (e) vector plot.

##### B. Eigen mode simulation

The eigen mode simulation (i.e., beam absent simulation) is performed to assure the observation of the device in the desired mode and frequency using listed data in **Table I**. The eigen mode solver in ‘‘CST particle studio’’ is used for the cold simulation to carried out its EM behavior between input into the RF interaction region and their both input and output ports. At boundary condition into the interaction chamber, the tangential compound should become null (i.e.,  $\vec{E}_t = 0$ ). Fig. 3 (a, b, c, d & e) shows the contour plot and vector plot of the magnetic fields and electric fields inside the input port and out port into the interaction chamber of the FEL amplifier, which clarify that there is zero variation in azimuthal and radial direction that is defined the operation of  $TM_{01}$  confirmation into the interaction chamber and same operating mode (i.e.,

$TM_{01}$ ) is coming out at the output port for linear amplification in the FEL amplifier. In the Fig. 4 (a, b & c), the dielectric loss, impedances, power accepted and s-parameters (the standard values of the s-parameters magnitude is varies between  $-30dB$  to  $-180dB$ ) calculation is readily available through the 3-D/2-D -

field processing and as a results shown in the given figure for the specific parameters and dominant mode  $S_{21} \rightarrow S2(3),1(3)$ , which is shows the availability of the device. And also sustain the accepted power in Watts.

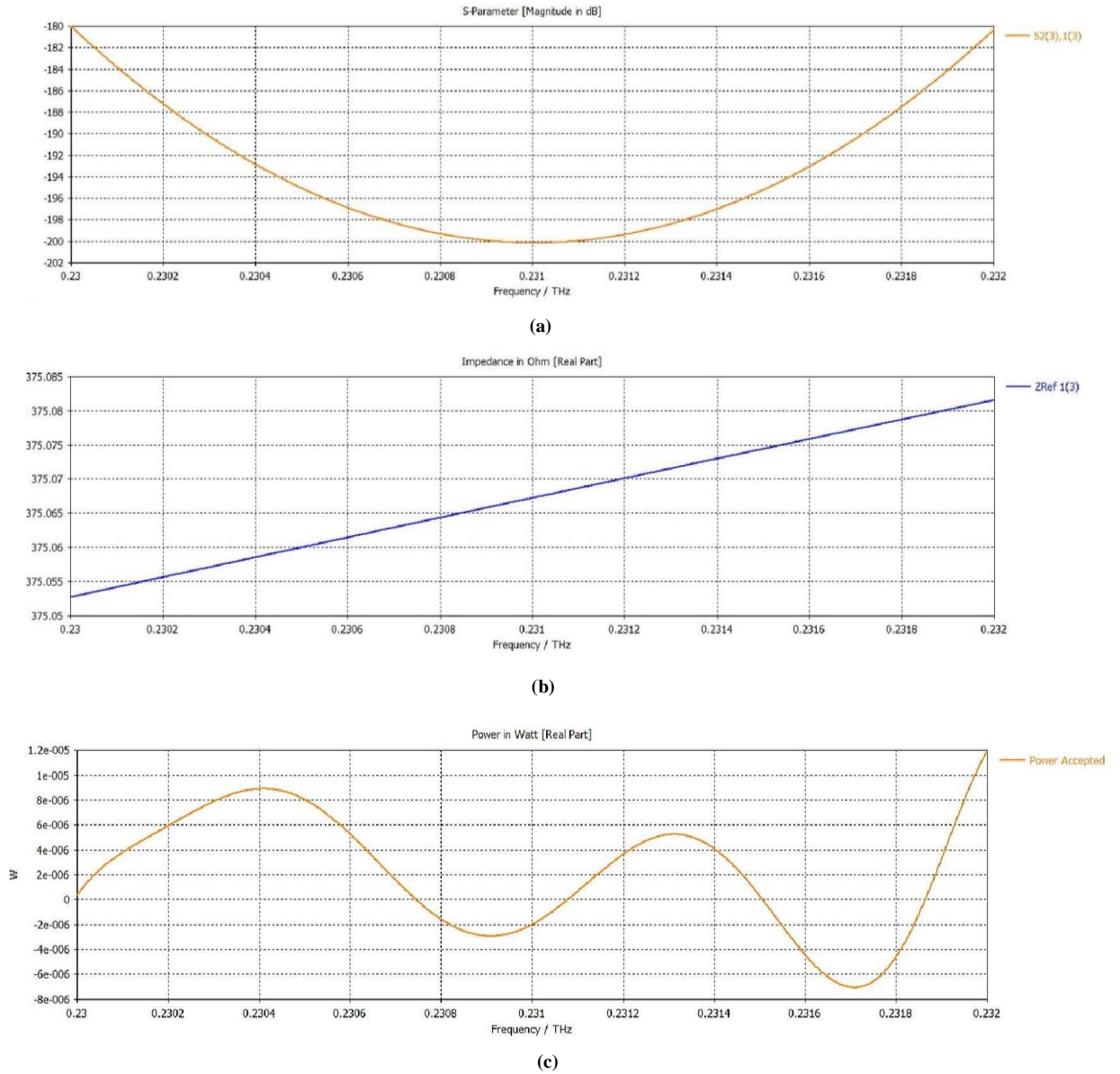


Fig. 4: FEL amplifier simulated and desired observation for (a) S-parameters (b) Impedance in ohm (c) Power accepted

C. Parametric Analysis

The parametric analysis of the free electron laser amplifier is estimated by using the particle-in-cell (PIC) simulation in “CST Particle studio” to study the device sensitivity for the various parameters of the beams. The efficiency variation and RF output power has been estimated to the different frequencies radiation. Fig. 5 shown the RF power output of the FEL amplifier is increases after increasing the taper axial magnetic fields ( $B_s$ ). The overall the phenomenon of the extraction efficiency is increases of the device.

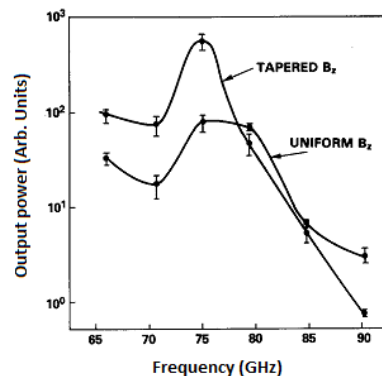


Fig. 5: FEL amplifier output power vs radiated frequencies

## 5. Conclusion

The detailed description of the design methodology and simulation study of the highly radiated power microwave (HPM) device-Free Electron Laser Amplifier (FELA) has been presented. The FEL amplifier shows the simulated results as 50MW RF power output and 20% increases with extraction efficiency with linear axial taper strong guided magnetic fields. Therefore the device has been found a good agreement presented with  $\approx 5\%$  experimental values as reported as previous paper and the overall performance of the FEL amplifier has been spread, hence, the parametric analysis behavior through the particle-in-cell (PIC) simulation in "CST Particle studio" has been performed and validate.

## Acknowledgement

One of the authors, Ram Gopal (RG), is grateful to the Indian Institute of Technology (IIT-BHU), Varanasi, India and Rajiv Gandhi National Fellowship (RGNF-UGC), New Delhi, India for financial support. The author is also thankful to the reviewer of the manuscript for improving the quality.

## References

- [1] Gold SH, Hardesty DL, Kinkead AK, Barnett LR & Granatstein VL, "High gain 35-GHz free electron laser amplifier experiment", *Physical review letters*, Vol. 52, No. 14, (1984), p. 1218-1221. <https://doi.org/10.1103/PhysRevLett.52.1218>.
- [2] Gold SH, Black WM, Freund HP, Granatstein VL & Kinkead AK, "Radiation growth in a millimetre-wave free electron laser operating in the collective Regime", *Physics of Fluids*, Vol. 27, No. 3, (1984), p. 746-754. <https://doi.org/10.1063/1.864650>.
- [3] Gold SH, Freund HP & Bowie, "Free electron laser with tapered axial magnetic field", The United States of America as represented by the Secretary of the Navy, Washington, DC, (1987), Patent Number: 4,644,548.
- [4] Pant KK & Tripathi VK, "Free Electron Laser Operation in the Whistler Mode", *IEEE Transactions on plasma science*, Vol. 22, No. 3, (1994). <https://doi.org/10.1109/27.297869>.
- [5] Gopal R & Jain PK, "Tapering effect of an axial magnetic field on whistler-pumped FEL Amplifier", *IRACTS-Engineering Science and Technology: An International journal (ESTIJ)*, Vol. 8, No. 2, (2018).
- [6] Gopal R & Jain PK, "Effect of axial magnetic field tapering on whistler-pumped FEL amplifier in collective Raman regime operation", *International Journal of Engineering & Technology*, Vol. 7, No. 4, (2018), pp.2044-2050.
- [7] Motz H, "Applications of the Radiation from Fast Electron Beams," *Journal of Applied Physics*, 1951, vol. 22, no. 5.
- [8] Sharma A & Tripathi VK, "Kinetic theory of a whistler-pumped free electron laser," *Phys. of Fluids*, Vol. B 5, No.1, (1993).
- [9] Sharma A & Tripathi VK, "A plasma filled gyrotron pumped free electron laser", *Phys. Plasmas*, Vol. 3, (1996), pp. 3116. <https://doi.org/10.1063/1.871658>.
- [10] Marshall TC, *Free Electron Lasers*, MacMillan, New York, (1985).
- [11] Liu CS & Tripathi VK, *Interaction of electromagnetic waves with electron beams and plasmas*, World Scientific, (1994). <https://doi.org/10.1142/2189>.
- [12] Roberson CW and Sprangle P, *Physics Fluids*, Vol. B1, No. 3, (1989).
- [13] Dattoli G and Renieri AT, "Lectures on the free electron laser theory and related topics," World scientific publishing Co. Pte. Ltd, (1993).
- [14] Workie DW, "Basic Physical Processes and Principles of Free Electron Lasers," Lecture notes- Department of Physics, University of Cincinnati, Cincinnati, Ohio 45221, 2001.
- [15] Nguyen KT, et al, "Modeling of gyrokylystrons with MAGY," *IEEE Trans. Plasma Sci.*, Vol. 28, No. 3, (200), pp. 867-886.
- [16] Petillo J, Mankofsky A, Krueger W, Kostas C, Mondelli A, and Drobot A, "Applications of the ARGUS code in accelerator physics," in *Proc. AIP Conf. Comput. Accel. Phys.*, (1993), pp. 303-312.
- [17] Di J, Zhu DJ, and Liu SG, "Electromagnetic field algorithms of CHIPIC code," *J. Univ. Electron. Sci. Technol. China*, vol. 4, no. 4, (2005), pp. 485-488.
- [18] Warren G, Ludeking L, Nguyen KT, Smithe D, and Goplen B, "Advances/applications of MAGIC and SOS," in *Proc. AIP. Conf. Comput. Accel. Phys.*, (1993), pp. 313-322.
- [19] MAGIC User's Manual 2007, ATK Mission Systems, Newington, Virginia, (2007).
- [20] *User's Manual*, CST-Particle Studio, Darmstadt, Germany, (2013).
- [21] *User's Manual*, CST-Particle Studio, Darmstadt, Germany, (2014).
- [22] Kartikeyan MV, Borie E, and Thumn M, *Gyrotrons: High-Power Microwave and Millimeter Wave Technology*. Berlin, Germany: Springer, (2004).
- [23] Nusinovich GS, *Introduction to the Physics of Gyrotrons*. Baltimore, MD, USA: The Johns Hopkins Univ. Press, (2004).
- [24] Swati MV, Chauhan MS, and Jain PK, "Design Methodology and Beam-Wave Interaction Study of a Second-Harmonic D-Band Gyrokylystron Amplifier", *IEEE Transactions on plasma science*, Vol. 44, No. 11, (2016), p. 2845.
- [25] Mahto M, and Jain PK, "Design and Simulation Study of the HPM Oscillator-Reltron", *IEEE transactions on plasma science*, Vo. 44, No. 5, (2016).