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Research paper

Fast Load Voltage Stability Index constrained PMU Placement for Complete Observability

P.Lakshmi Narayana[1]*, M.Venkatesan[2], S.Ravindra[3]

[1] *Research Scholar (EEE),Vignan's Foundation for Science, Technology & Research, Vadlamudi, Guntur, A.P. India*
[2] *Associate Professor, Department of EEE, VLITS,Vadlamudi, Guntur, A.P. India.*
[3] *Professor, Department of EEE, VVIT, Nambur, Guntur, A.P. India.*
**Corresponding author E-mail:p.lakshminarayanaeee@gmail.com*

Abstract

This paper proposes a Fast Load Voltage Stability Index (FLVSI) constrained Binary Integer Programming (BIP) method for Phasor Measurement Unit placement at optimal locations in network to obtain complete observability. Every load bus of network is considered to sort out weak load bus from proposed FLVSI approach. PMUs are constrained to place at weak load buses using BIP approach for observability of network. Zero Injection (ZI) modeling is suggested to reduce PMU placement locations in network. Single line outage or PMU loss constraints are formulated for placement of PMUs. Bus Redundancy Index (BRI) is formulated and considered for every bus of network. With and without ZI modeling under normal and line outage cases is compared to present effectiveness of approach. IEEE – 14- 30-and 57- bus networks are tested with MATLAB Programming and compared with other methods to show its effectiveness.

Keywords: *BIP; Fast Voltage Stability Index; Observability; PMUs; Redundancy; ZI buses*.

1. Introduction

In power grid, there are blackouts occurred in USA and in India for example a blackout occurred in Lucknow in 2012. There are many reasons for the cause of occurrence like generation failure or due to heavy load changes or due to contingencies etc., The present existing system uses SCADA monitoring system for state estimation of the power system with data available of single snapshots. This is considered as steady state data and state estimated with this data is static state estimation.

Due to the development of synchrophasor technology and introduction of PMU devices into power system, this can change the static power system in to dynamic power system. PMU devices located at different places are synchronized with synchrophasor technology that uses time as common reference which are all connected to Global Positioning Systems (GPS) [1],[2]. In power system network placement of PMUs at all nodes of the system leads to heavy cost as each PMU is in between 50,000 $ to 180,000$ cost. This lead to development of different optimization techniques for PMU placement with complete observability but when we perform state estimation with PMU measurements obtained, this will give more measurement errors which are not accurate to provide exact states of system. In the network every load-bus does not show same result with load change, there are sensitive buses, critical buses and also weak load buses which change with reactive load change. The author in [3] introduced systematic equivalent circuit for real-time voltage stability valuation and proposed an index to calculate factors effecting voltage stability. In [4] voltage variation is considered to detect weak voltage buses with same load rate of deviation and proposed PMU configurations to monitor feeble voltage area to deliver real-time data. In order to monitor fragile areas of power system PMUs and their channels are located to find voltage stability status of buses to avoid voltage collapse [5]. In [6] PMU are used to evaluate FVSI to monitor system network. A new Voltage Stability Load Index [VSLI] is proposed based on the measurement obtained through PMU data [7]. Many authors proposed PMU placement to analyze voltage stability at buses, but did not place PMUs based on voltage stability index. A novel Technique is proposed to identify the weakest bus in large scale networks with reactive power load in [8]. Several Line Voltage Stability Indices [LVSI] are differentiated to determine weakest lines of network [9]. A new strategy considering analytical approach and graph theory process is proposed for PMU placement in network for complete observability [10]. A BPSO [11] method is suggested for optimal PMU placement (OPP) to acquire full observability of network. OPP utilizing TLBO [12] is presented to attain complete observability of network. The author in [13] proposed BCSO [14] to achieve complete observability of network. Sensitive analysis is done based on Newton Raphson load flow in [14] and PMU placement is considered based on sensitive analysis.

A New Fast Load Voltage Stability Index[FLVSI] is defined in this paper to identify weak load buses at which PMUs are placed using BIP method. ZI modeling is considered to reduce PMU placement location which reduces cost of PMUs. Contingency conditions such as single line outage are considered to show effectiveness of proposed method.

The remaining presentation of paper is as follows; Section II deals with problem formulation, Identification of weak buses, Zero injection bus modeling; section III describes FLVSI constrained PMU placement; Section IV presents results and analysis and section V concludes problem.

2. Problem Formulation

2.1. PMU Placement formulation for Observability of Network

The proposed approach is formulated as optimization problem for placement of PMUs in network with highest preference at identified weak load buses for complete observability in view of cost criteria.
The general optimization problem for OPP is formulated as:

$$Min\sum_{p=1}^{N} W_p Y_p \tag{1}$$

Subjected to

$$Y(p)=\begin{cases} B-1=y_1+y_2+y_5\geq 1 \\ B-2=y_1+y_2+y_3+y_4+y_5\geq 1 \\ B-3=y_2+y_3+y_4\geq 1 \\ B-4=y_2+y_3+y_4+y_5+y_7+y_9\geq 1 \\ B-5=y_1+y_2+y_4+y_5+y_6\geq 1 \\ B-6=y_5+y_6+y_{11}+y_{12}+y_{13}\geq 1 \\ B-7=y_4+y_7+y_8+y_9\geq 1 \\ B-8=y_7+y_8\geq 1 \\ B-9=y_4+y_7+y_9+y_{10}+y_{14}\geq 1 \\ B-10=y_9+y_{10}+y_{11}\geq 1 \\ B-11=y_6+y_{10}+y_{11}\geq 1 \\ B-12=y_6+y_{13}+y_{12}\geq 1 \\ B-13=y_6+y_{12}+y_{13}+y_{14}\geq 1 \\ B-14=y_9+y_{13}+y_{14}\geq 1 \end{cases} \tag{2}$$

Where N is number of buses, W_p is presented as cost coefficient of PMU located at bus p, it is considered as diagonal matrix with each PMU cost to 1.p.u.

$$y_p=\begin{cases} 1 & \text{if PMU is allocated at bus } p \\ 0 & \text{otherwise} \end{cases}$$

PMUs are placed in network with highest preference at weak load buses identified through FLVSI as follows

2.2. Formulation of FLVSI to identify weak load buses

Consider a 2-bus network as in Fig. 1

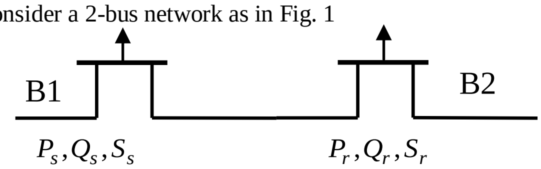

Fig. 1. Single-line diagram of 2-bus network

FLSVI is computed from voltage equation derived at receiving end. The voltage equation derived is presented as follows

$$V_r^2-\left[\frac{R}{X}\sin\delta+\cos\delta\right]V_sV_r+\left(X+\frac{R^2}{X}\right)Q_r=0 \tag{3}$$

The equation is rewritten as

$$\left(\left[\frac{R}{X}\sin\delta+\cos\delta\right]V_s\right)^2+4\left(X+\frac{R^2}{X}\right)Q_r\geq 0 \tag{4}$$

Further equation is simplified as

$$\frac{4Z^2Q_rX}{V_s^2(R\sin\delta+X\cos\delta)^2}<1 \tag{5}$$

Since δ is very small which is approximately equal to zero or negligible. i.e., $R\sin\delta=0$, and $X\cos\delta=X$.
FLVSI is formulated as

$$FVSI=\frac{4Z^2Q_r}{V_s^2(X)} \tag{6}$$

The value of bus voltage calculated should be less than 1 and values nearer to 1 are considered as weak load buses. The weak load buses identified through FLVSI are considered for PMU placement in prior. Algorithm to identify weak load buses in network is shown in Fig.2.

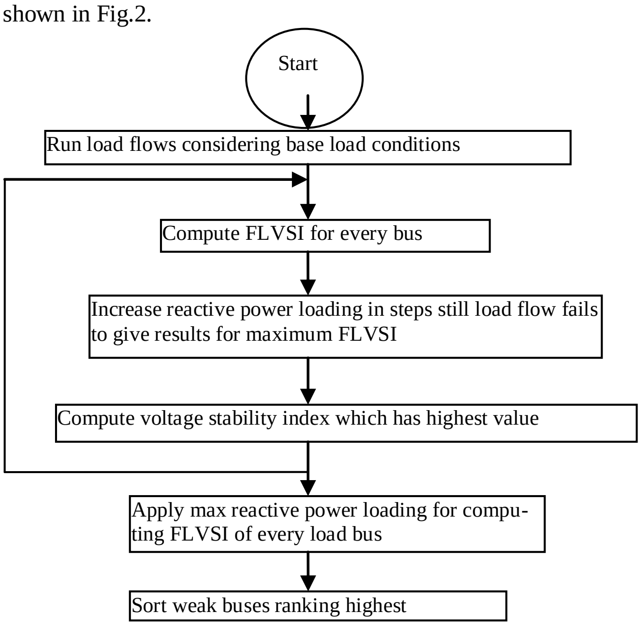

Fig.2. Algorithm to find weak load buses using FLVSI

For example consider 14-bus system shown in Fig.3. which reactive load is increased

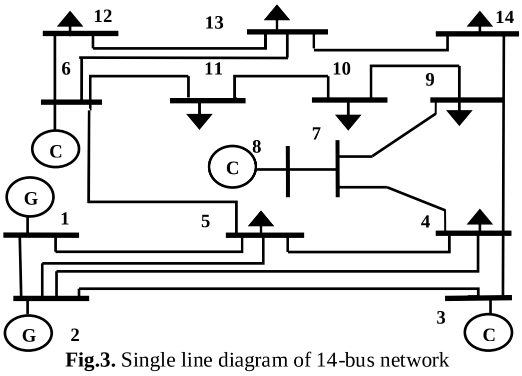

Fig.3. Single line diagram of 14-bus network

Table.1: flvsi for 14-bus system

Reactive Load applied(Q)	Load Buses	FLVSI	Ranking
0.745	14	0.998	1
0.910	12	0.992	2

1.006	11	0.995	3
1.390	10	0.988	4
1.473	13	0.997	5
1.572	9	0.989	6
3.002	5	0.998	7

From table I it is observed that most weak load buses obtained are 14 and 12 buses with less reactive load.

2.3. ZI Bus Modeling

The power neither injected nor flown in bus is considered as ZI bus. The current flown through these buses is approximately equal to zero. These buses are considered in optimization for OPP. Modeling of ZI constraints for linear networks is considered in this work

For example from the figure

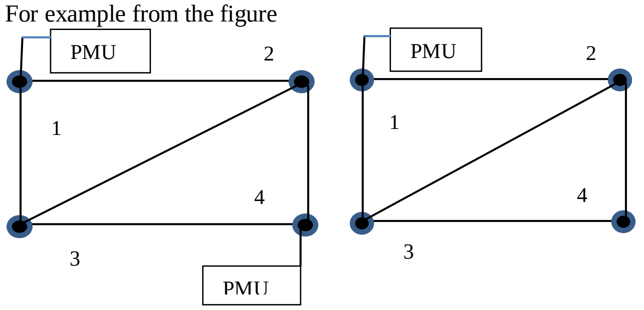

Fig. 2. 4-bus network

Consider ZI bus at node 4 as shown in Fig.2.

When voltage phasors from buses 1 to ($p-1$) are recognized then current $I_{l,1}$ is computed as

$$I_{l,1} = Y_{l,1}[e_l - e_1] \tag{8}$$

where $Y_{l,1}$ is line admittance between bus 1 and l

Bus p can be computed by calculating bus voltage as follows:

$$e_p = V_1 - Z_{1,p} \sum_{l=2}^{p-1} I_{l,1} \tag{9}$$

where $Z_{1,p}$ is impedance between buses 1 and l.

Every ZI node indicates one supplementary constraint. The minimum number of PMUs essential for observability is minimized by reducing ZI buses in network.

As Bus- 2 is ZI bus

$$I_{24} = I_{12} + I_{32}$$

As we know line currents, voltage at bus 4 is calculated as

$$V_4 = V_2 - (I_{12} + I_{32})Z_{24}$$

Therefore by applying Kirchhoff law, bus-4 voltage can be computed. Hence PMU at node 4 is not essential. Consider 14 -bus system for optimal allocation of PMUs in which bus-7 is ZIB of the system. The buses connected with bus-7 are 4, 8 and 9 buses are considered in optimization of constraints. Following sub set rule if $A \subset B$ then $A \cup B = B$, bus-1 and 3 are subsets of bus-2, bus-12 is subset of bus-13 and bus-8 is subset of bus-7.

$$Minimize \sum_{p=1}^{14} W_p Y_p \tag{10}$$

Subjected to observability constraints

$$Y(p) = \begin{cases} B-1 = y_1 + y_2 + y_5 \geq 1 \\ B-3 = y_2 + y_3 + y_4 \geq 1 \\ B-4 = y_2 + y_3 + y_4 + y_5 + y_7 + y_9 \geq 1 \\ B-5 = y_1 + y_2 + y_4 + y_5 + y_6 \geq 1 \\ B-6 = y_5 + y_6 + y_{11} + y_{12} + y_{13} \geq 1 \\ B-7 = y_4 + y_7 + y_8 + y_9 \geq 1 \\ B-9 = y_4 + y_7 + y_9 + y_{10} + y_{14} \geq 1 \\ B-10 = y_9 + y_{10} + y_{11} \geq 1 \\ B-11 = y_6 + y_{10} + y_{11} \geq 1 \\ B-13 = y_6 + y_{12} + y_{13} + y_{14} \geq 1 \\ B-14 = y_9 + y_{13} + y_{14} \geq 1 \end{cases} \tag{11}$$

With BIP approach, objective function with subjected constraint provides solution of PMU placement locations at 2, 6, and 9 in 14-bus network.

3. FLVSI Constrained Optimal PMU Placement

3.1. Optimal PMU Placement Considering FLVSI for observability of Power system

The most weak load buses obtained through FLVSI is 14 and 12 buses, PMU is considered as must to be located at 14-bus and 12-bus and constraints related to it to place PMU is formulated through BIP approach with other constraints used to make system complete observable. The problem is formulated as follows

$$Min \sum_{p=1}^{14} W_p Y_p \tag{12}$$

$$\text{Subject to } y_{12} = 1,\ y_{14} = 1 \tag{13}$$

$$Y(p) = \begin{cases} B-1 = y_1 + y_2 + y_5 \geq 1 \\ B-2 = y_1 + y_2 + y_3 + y_4 + y_5 \geq 1 \\ B-3 = y_2 + y_3 + y_4 \geq 1 \\ B-4 = y_2 + y_3 + y_4 + y_5 + y_7 + y_9 \geq 1 \\ B-5 = y_1 + y_2 + y_4 + y_5 + y_6 \geq 1 \\ B-7 = y_4 + y_7 + y_8 + y_9 \geq 1 \\ B-8 = y_7 + y_8 \geq 1 \\ B-10 = y_9 + y_{10} + y_{11} \geq 1 \\ B-11 = y_6 + y_{10} + y_{11} \geq 1 \end{cases} \tag{14}$$

3.2. Optimal PMU Placement Considering FLVSI Constraints with ZI Modeling

To further reduce PMU locations we use ZI modeling for weak load bus constrained locations. The problem is formulated as follows

$$Minimize \sum_{p=1}^{14} W_p Y_p \tag{15}$$

Subject to $y_{12}=1,\ y_{14}=1$ (16)

$$Y(p)=\begin{cases} B-1=y_1+y_2+y_5\geq 1 \\ B-3=y_2+y_3+y_4\geq 1 \\ B-4=y_2+y_3+y_4+y_5+y_7+y_9\geq 1 \\ B-5=y_1+y_2+y_4+y_5+y_6\geq 1 \\ B-7=y_4+y_7+y_8+y_9\geq 1 \\ B-10=y_9+y_{10}+y_{11}\geq 1 \\ B-11=y_6+y_{10}+y_{11}\geq 1 \end{cases} \quad (17)$$

Substituting equation (16) in (11) results equation (17). With FLVSI constrained BIP approach the locations obtained are 2, 10, 12, and 14.

3.3. PMU Placement Considering FLVSI and ZI Constraints with Single Line Contingency Modeling

In case of single line outage or PMU loss case each line should be observed by at least two PMUs. The optimization problem with single line outage can be formulated as follows:

$$Minimize \sum_{p=1}^{14} W_p Y_p \quad (18)$$

Subject to $y_{12}=2,\ y_{14}=2$ (19)

$$Y(p)=\begin{cases} B-1=y_1+y_2+y_5\geq 2 \\ B-3=y_2+y_3+y_4\geq 2 \\ B-4=y_2+y_3+y_4+y_5+y_7+y_9\geq 2 \\ B-5=y_1+y_2+y_4+y_5+y_6\geq 2 \\ B-7=y_4+y_7+y_8+y_9\geq 2 \\ B-10=y_9+y_{10}+y_{11}\geq 2 \\ B-11=y_6+y_{10}+y_{11}\geq 2 \end{cases} \quad (20)$$

Substituting equation (19) in (17) results equation (20) by changing constraints ≥ 2 With FLVSI constrained BIP approach the locations obtained are 2, 4, 5, 6, 9, 10, 12, and 14.

3.4. Performance of Observability through CSBOI

Observability of the complete system is computed with Bus Observability Index (BOI) at every bus and it is presented as

$$\beta_p \leq \Re_p + 1 \quad (21)$$

BOI is limited to number of incident branches ($\Re_p$) plus one. *BOI* (β) for a bus-*p* gives number of PMUs located to measure bus. Complete System Bus Observability Index (CSBOI) can be derived as sum of the incident branches at every bus of network.

$$CSBOI=\sum_{p=1}^{N}\beta_p \quad (22)$$

BRI is computed at every bus to estimate number of times bus is observed by PMU to achieve complete observability of bus network. BRI of network can be formulated as

$$BRI = AY \quad (23)$$

where $Y=[y_1 \quad y_2 \quad y_3 \quad \ldots \quad y_n]^T$ is a binary variable matrix in which y_j is binary decision variable and *A* is incidence matrix.

4. Results and Discussions

The optimal PMU placement considering FLVSI for different test case systems is formulated using BIP approach and simulated using MATLAB Programming. Test cases 14, 30- and 57-buses are run on Intel(R) core(TM), i3 processor at 2.20 GHz, 4 GB of RAM.The data of weak load buses and ZI buses is shown in Table II.

Table.2: Weak Load Buses and ZI Buses

IEEE test systems	Weak Load buses	ZI buses
14 bus	14,12	7
30 bus	26, 29,30	6,9,22,25,27,28
57 bus	37, 15,31,52	4,7,11,21,22,24,26,34, 36,37,39.40,45,46,48

Table. 3: Optimal Placement of PMU considering FLVSI

IEEE test systems	No of PMUs	PMU Locations
14 bus	5	2.7,10,12,14
30 bus	11	1,7,9,10,12,18,23,26,28,29,30
57 bus	19	1,4,9,15,20,24,28,29,31,32,36,37,38,43,51,52,53,56

Table. 4: Optimal Placement of PMUs considering FLVSI with ZI Modeling

IEEE test systems	No of PMUs	PMU Placement locations
14 bus	4	2,10,12,14
30 bus	9	1,5,10,12,18,24,26,29,30
57 bus	16	1,4,9,15,20,24,28,31,32,34,37,38,50,52,53,56

Table. 5: Without ZI Modeling for single line outage

IEEE test systems	No of PMUs	PMU Placement locations
14 bus	10	2, 4, 5, 6,7, 8, 9, 10, 12, 14
30 bus	25	1,3,5,6,7,9,10,11,12,13,15,17,19,20
57 bus	34	1,3,4,6,9,11,12,15,19,20,22,24,26,28,29,30,31,32, 33,35,36,37,38,41,44,46,47,50,51,52,54,55,56,57

FLVSI constrained PMU placement with and without ZI modeling are shown in Table III and IV. Comparing these two tables it is observed that, PMU with ZI modeling minimizes PMU number thereby reducing cost of installation in network. Single line outage problem is considered in this paper to show effectiveness of placement of PMUs at weak load buses i.e., considering FLVSI and advantage of ZI modeling to this condition. PMU placement considering weak load buses for single line outage problem with and without considering ZI modeling is shown in Table V and VI. Comparing single line outage problem with and without ZI modeling it is observed that, ZI modeling reduces PMUs locations without losing observability.

Table. 6: With ZI Modeling for single line outage

IEEE test systems	No of PMUs	PMU Placement locations
14 bus	8	2,4,5,6,9, 10,12,14
30 bus	17	2,3,4,7,10,12,13,15,17,19,20,22,24,26,27,29,30
57 bus	30	1,3,4,7,9,12,15,18,20,21,24,25,27, 28,31,32,33,34,36,37,38,41,42,44,48,50,51,52,53,55.

Table. 7: BRI for 14-bus considering FLVSI

Bus no	B1	B2	B3	B4	B5	B6	B7
BRI	1	1	1	2	1	1	1
Bus no	**B8**	**B9**	**B10**	**B11**	**B12**	**B13**	**B14**
BRI	1	3	1	1	1	2	1

Table. 8: BRI for 14-bus in case of Single line outage considering FLVSI

Bus no	B1	B2	B3	B4	B5	B6	B7
BRI	2	3	2	5	3	4	4
Bus no	**B8**	**B9**	**B10**	**B11**	**B12**	**B13**	**B14**
BRI	2	4	3	2	2	2	2

Table. 9: BRI for 30 bus system considering FLVSI

Bus No	B1	B2	B 3	B 4	B 5	B 6	B 7	B 8	B 9	B 10	B 11	B 12	B 13	B 14	B 15
BRI	**1**	**1**	**1**	**1**	**1**	**4**	**1**	**1**	**2**	**2**	**1**	**1**	**1**	**1**	**3**
Bus. No	B 16	B 17	B 18	B 19	B 20	B 21	B 22	B 23	B 24	B 25	B 26	B 27	B 28	B 29	B 30
BRI	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**3**	**1**	**2**	**2**

Table. 10: BRI for 57-bus system considering FLVSI

Bus No	B1	B2	B3	B4	B5	B6	B7	B8	B9	B10	B11	B12	B13	B14	B15	B16	B17	B18	B19	B20	B21	B22	B23	B24	B25	B26	B27	B28	B29	B30
BRI	**2**	**1**	**2**	**1**	**1**	**1**	**1**	**1**	**1**	**2**	**2**	**1**	**3**	**1**	**2**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**2**	**3**	**1**

Bus. No	B31	B32	B33	B34	B35	B36	B37	B38	B39	B40	B41	B42	B43	B44	B45	B46	B47	B48	B49	B50	B51	B52	B53	B54	B55	B56	B57
BRI	**2**	**2**	**1**	**1**	**1**	**2**	**3**	**2**	**1**	**2**	**2**	**1**	**1**	**1**	**1**	**1**	**1**	**2**	**1**	**1**	**1**	**3**	**2**	**1**	**1**	**1**	**1**

Bus Redundancy Index (BRI) for 14- bus system considering FLVSI under normal and single line outage conditions is show in Table VII and VIII. BRI considering VSI for 30 bus system is shown in Table IX, and for 57-bus system it is shown in Table X. Complete System Bus Observabilty Index (CSBOI) under normal conditions and single line outage condition is shown in Table XI. From the Table, it is observed that PMU placement increases redundancy without ZI modeling. Theoptimal redundancy is obtained with ZI modeling with complete observability.

Table. 11: CSBOI under normal and single line outage

IEEE Test systems	Without Single Line Outage		With Single Line Outage	
	with ZI modeling	without ZI modeling	with ZI modeling	without ZI modeling
14 bus	14	18	34	40
30 bus	34	49	66	79
57 bus	67	78	115	133

Table. 12: Comparison of proposed optimization method with other methods of complete observability

Methods	14-bus system	30-bus system	57-bus system
Graph Theory &MCDM Approach[10]	4	10	17
BPSO[11]	4	10	-
TLBO[12]	4	10	17
BCSO[13]	4	10	-
BABC[15]	4	10	-
Multi scheduling-ILP[16]	4	10	17
Proposed FLVSI constrained BILP Method	4	9	16

Table XII shows comparison of proposed method with other methods available in literature. The method followed for optimal PMU Placements [10]-[16] shows complete observability. The proposed FLVSI constrained PMU placement shows complete observability in case of normal and contingency conditions with optimal redundancy. The ZI modeling proposed in this work is effective in reducing PMU placement locations which reduces cost of installation when compared to other methods proposed.

5. Conclusion

This paper presented a novel Fast Load Voltage Stability Index to identify weak load buses in network. A new BIP method is formulated and presented PMU placement at weak load buses as main priority for complete observability of network. The proposed ZI modeling is able to minimize PMU locations in case of normal outage conditions. With and without ZI modeling in case of normal and single line outage conditions is compared to show effectiveness of proposed constrained optimization method. Bus redundancy at every bus of networks is computed to show that how many times the bus is observed by PMUs. The performance of optimization is checked using proposed Complete System Bus Observability Index (CSBOI). The proposed constrained optimization method is compared with other methods available in literature to show its effectiveness.

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