



# Mathematical Modelling of Linear Induction Motor

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## Abstract

The Linear Induction Motor is a special purpose electrical machines it produces rectilinear motion in place of rotational motion. By using D-Q axes equivalent circuit the mathematical modelling is done because to distinguish dynamic behavior of LIM, because of the time varying parameters like end effect, saturation of core, and half filled slot the dynamic modelling of LIM is difficult. For simplification here we are using the two axes modelling because to evade inductances time varying nature it becomes complex in modelling, this also reduces number of variables in the dynamic equation. Modelling is done using MATLAB/SIMULINK. LIM can be controlled by using sliding model control, vector control, and position control.

**Keywords:** longitudinal end effect; transverse edge effect; equivalent circuit; applications; Dynamic performance.

## Nomenclature

### Symbols Description

V	Voltage
C	Current
$V_{dp}, V_{qp}$	Primary voltage in the d-q axes(V)
$V_{dl}, V_{ql}$	Linor voltage in the d-q axes (V)
$i_{dp}, i_{qp}$	d-q axis primary current (A)
$i_{dl}, i_{ql}$	d-q axis linor current (A)
$\lambda_{dp}, \lambda_{qp}$	d-q axis primary flux linkages
$\lambda_{dl}, \lambda_{ql}$	d-q axis linor flux linkages
$R_p, R_l$	primary and linor resistance (A)
$L_{lp}, L_{ll}$	primary and linor leakage inductance(H)
$L_m$	magnetizing or mutual inductance (H)
$L_p, L_l$	primary and linor self inductances (H)
P	no. of poles
T	pole pitch (m)
D	length of the linor (m)
Q	factor associated with linor length
V	velocity (m/s)
$\omega$	primary angular velocity (rad/sec)
$\omega_l$	linor angular velocity (rad/sec)
$\omega_{sl}$	slip frequency (rad/sec)

## 1. Introduction

LINEAR INDUCTION MOTOR the name itself says linear so it produces linear motion. In industries these LIM are usually used for automated systems. Because of the occurrence of end effects the dynamic modelling of these kind of motors are complicated to model. For understanding motor behavior during disturbance of load and at normal condition motor modelling is done, by using rotor reference frame, arbitrary reference frame etc. It have many popular performance features, together with high-speed operation, during starting it has high thrust force, mechanical construction is

simple, silence operation, Simple structure, and easy maintenance, low cost, it do not require any gear mechanism, good reliability, reduction of mechanical losses.

By considering the end effects LIM is simulated in synchronously rotating reference frame, based on our requirement the reference frames are selected. For obtaining easy solution in hybrid computer the two-axis modeling is done to evade inductance time varying nature and reduce variables in equations. Compared to AC quantities DC quantities are chosen for controlling of LIM. The DC quantities will decide the operating point it is simple to model in small signal equation and difficult to model in non-linear equation. The park's transformation converts 3 $\Phi$  quantities to 2 $\Phi$  quantities for mmf equality. For maintaining voltage unbalance and inversion of park's transformation the Zero sequence component which is a new variable is introduced.

LIM's are used in many applications particularly in electromechanical conversion units such as Elevators, Baggage handling, Automatic sliding doors, Accelerators, Horizontal conveyance systems, Cranes, Material handling and storage, actuator, transportation, piston pumps, electric traction, automotive control and robotics etc.

## 2. End effect in LIM

When the primary moves, a new flux is always generated at the primary entry side, while at the exit side flux will be disappears. Ther will be a rapid generation and disappearance of the magnetic lines produce statically induced currents in the secondary sheet. The air gap flux is affected by the eddy currents. With the increase of speed the losses, and the flux-profile become sever this is called End-Effect in LIM. If velocity increases the primarie's length decreases this increases end effect which causes reduction of magnetization currents of LIM. For zero velocity the length of the primary is considered as infinite to reduce the end effect.

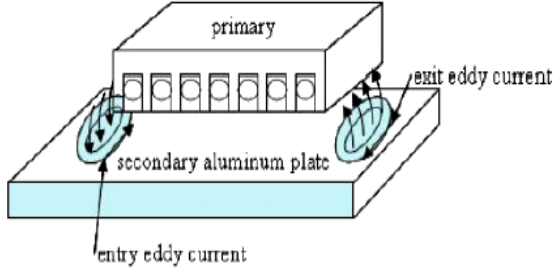


Fig. 1: Eddy-current generation at the entry and exit ends

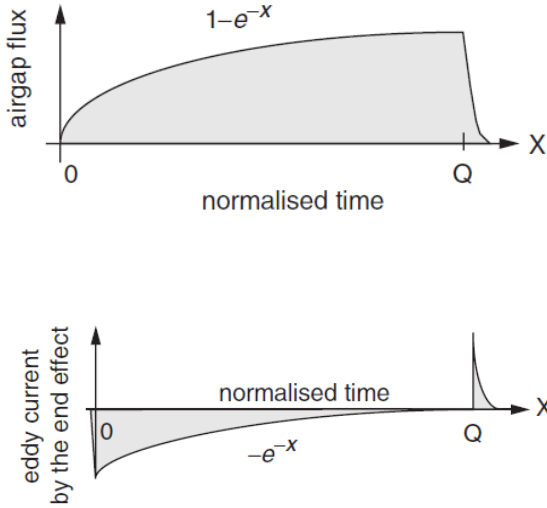


Fig. 2: Airgap flux profile and Eddy-current density profile along the length of LIM

### 3. Dynamic Model of LIM

More currents are taken by the motor while starting and load changing condition which causes dips in voltage, harmonics and oscillations in supply side. The Synchronously Rotating Reference Frame in d-q model reduces these problems precisely. Fig.3 and Fig.4 shows the LIM d-q axes equivalent circuit with end effects.

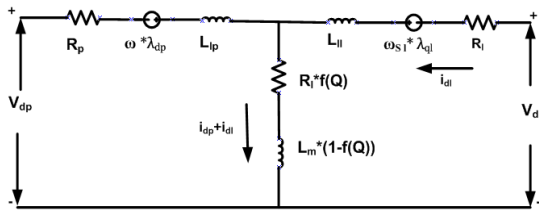


Fig. 3: Equivalent circuit for d-axis

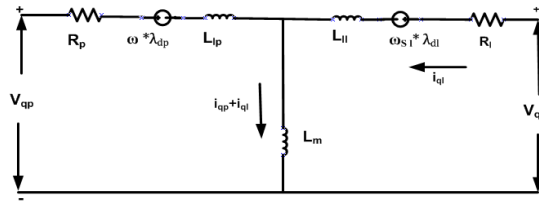


Fig. 4: Equivalent circuit for q-axis

When KVL is applied to the above circuits the below voltage equations are obtained.  $F(Q)$  is zero when end effect is neglected

$$V_{dp} = R_p i_{dp} + R_l F(Q)(i_{dp} + i_{dl}) + \dot{\lambda}_{dp} - \omega \dot{\lambda}_{qp} \quad (1)$$

$$V_{dl} = R_l i_{dl} + R_l F(Q)(i_{dp} + i_{dl}) + \dot{\lambda}_{dl} + (\omega - \omega_l) \dot{\lambda}_{ql} \quad (2)$$

$$V_{qp} = R_p i_{dp} + \dot{\lambda}_{qp} + \omega \dot{\lambda}_{dp} \quad (3)$$

$$V_{ql} = R_l i_{ql} + \dot{\lambda}_{ql} + (\omega - \omega_l) \dot{\lambda}_{dl} \quad (4)$$

$F(Q)$  is denoted as

$$F(Q) = \frac{1-e^{-Q}}{Q} \quad (5)$$

$$Q = \frac{DR_r}{(L_m + L_{lp})v} \quad (6)$$

$$Q = \frac{DR_r}{L_p v} \quad (7)$$

Primary and linor (Secondary) flux linkage equations  
The primary and linor flux linkages in d-q axis is given by

$$\lambda_{dp} = L_{lp} i_{dp} + \dot{L}_m (i_{dp} + i_{dl}) \quad (8)$$

$$\lambda_{qp} = L_{lp} i_{qp} + L_m (i_{qp} + i_{ql}) \quad (9)$$

$$\lambda_{dl} = L_{ll} i_{dl} + \dot{L}_m (i_{dp} + i_{dl}) \quad (10)$$

$$\lambda_{ql} = L_{ll} i_{ql} + L_m (i_{qp} + i_{ql}) \quad (11)$$

Self-Inductance = leakage inductance + mutual inductance

$$L_{lp} + L_m = L_p \quad (12)$$

$$L_{ll} + L_m = L_l \quad (13)$$

$$L_a = L_p L_l - L_m^2 \quad (14)$$

The obtained thrust force is given by

$$F = \frac{3\pi P}{4\tau} (\lambda_{dp} i_{qp} - \lambda_{qp} i_{dp}) \quad (15)$$

The primary and linor currents in d-q axes can be derived from equation (8) to (11)

$$i_{dp} = \frac{\lambda_{dp} - L_m i_{dl}}{L_{lp} + L_m} \quad (16)$$

$$i_{qp} = \frac{\lambda_{qp} - L_m i_{ql}}{L_{lp} + L_m} = \frac{\lambda_{qp} - L_m i_{ql}}{L_p} \quad (17)$$

$$i_{dl} = \frac{\lambda_{dl} - L_m i_{dp}}{L_{ll} + L_m} \quad (18)$$

$$i_{ql} = \frac{\lambda_{ql} - L_m i_{qp}}{L_{ll} + L_m} = \frac{\lambda_{ql} - L_m i_{qp}}{L_l} \quad (19)$$

By substituting equation (18) in (16) and (16) in (18) we get

$$i_{dp} = \frac{\lambda_{dp} L_l - L_m \lambda_{dl} + L_m F(Q)(\lambda_{dl} - \lambda_{dp})}{L_a - L_m F(Q)(L_l + L_p - 2L_m)} \quad (20)$$

$$i_{dl} = \frac{\lambda_{dl} L_p - L_m \lambda_{dp} + L_m F(Q)(\lambda_{dp} - \lambda_{dl})}{L_a - L_m F(Q)(L_l + L_p - 2L_m)} \quad (21)$$

By substituting equation (19) in (17) and (17) in (19) we get

$$i_{qp} = \frac{\lambda_{dp} L_l - \lambda_{ql} L_m}{L_a} \quad (22)$$

$$i_{q1} = \frac{\lambda_{q1}L_p - L_m\lambda_{qp}}{L_a} \tag{23}$$

For the modelling of LIM with end effect the total thrust (F) is given by

$$F = \frac{3\pi P}{2\tau^2} \frac{L_m(1-f(Q))}{L_{l1}+L_m(1-f(Q))} * (\lambda_{d1}i_{qp} - \frac{L_{l1}}{L_1} \frac{f(Q)}{1-f(Q)} i_{dp}i_{q1}) \tag{24}$$

### 4. Transformation Technique:

By Clark’s transformation  $V_a, V_b$  and  $V_c$  the 3-ph quantities are converted to  $V_\alpha, V_\beta$  in stationary reference frame.  $V_\alpha, V_\beta$  are converted into rotating reference frame as  $V_d$  and  $V_q$  using Park’s conversion. The 3Φ- 2Φ conversion matrix is described as

$$\begin{bmatrix} V_d \\ V_q \\ V_0 \end{bmatrix} = \begin{bmatrix} \cos \omega t & \cos(\omega t - \frac{2\pi}{3}) & \cos(\omega t - \frac{4\pi}{3}) \\ -\sin \omega t & -\sin(\omega t - \frac{2\pi}{3}) & -\sin(\omega t - \frac{4\pi}{3}) \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \tag{25}$$

### Q 5. MATLAB Simulation of LIM

The mathematical model of LIM is conventionally constructed in 3-ph parameters it is really complex to analyze, so we use a d-q axes equivalent circuit model. A convenient way of representing the linear induction motor is equivalent circuit. The equivalent circuit contains inductances and resistances. The 3-ph voltages are converted and phase shift by 120°,  $V_d$  and  $V_q$  are d-q axes input voltages.

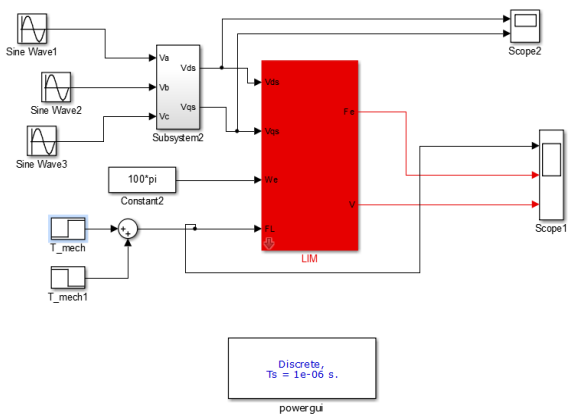


Fig. 5: MATLAB Simulation of Linear Induction Motor

Table 1: Parameters involved in LIM model

SL.NO	PARAMETERS	VALUES
1.	Length of the primary	1 m
2.	Length of the linor(D)	0.25 m
3.	No. of slots (S)	8
4.	No. of poles(P)	2
5.	Primary resistance( $R_p$ )	1.298 $\Omega$
6.	Linor resistance( $R_l$ )	0.976 $\Omega$
7.	Primary inductance( $L_p$ )	0.0684 H
8.	Linor inductance( $L_l$ )	0.0416 H
9.	Magnetizing inductance( $L_m$ )	0.0416 H
10.	Inertia movement (J)	0.00247 kg/m <sup>2</sup>
11.	Pole pitch ( $\tau$ )	0.027
12.	Mass (M)	4.775 kg
13.	Motor length	0.82 m

## 6. Simulation Results

The Linear Induction Motor was modelled, by considering end effect the behavior of the motor under transient condition was observed when a 30N load force is applied in the duration from 0.5 sec to 1.2 sec. The speed, force, thrust, currents, flux linkages waveforms of Linear Induction Motor are shown below. In LIM the thrust force oscillations are caused because of the presence of end effect.

### 6.1. Speed waveform:

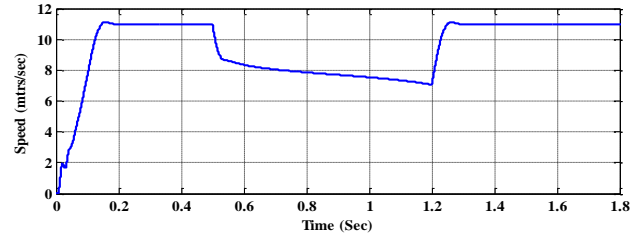


Fig. 6: Speed waveform of LIM

when the motor is at steady state it runs with a speed of 11m/s, suddenly when a 30N load force is applied at a time period between 0.5 sec to 1.2 sec the speed reduces from 11 m/s to 7 m/s. when the load force is removed again it comes to normal speed of 11m/s.

### 6.2. Thrust Force waveform:

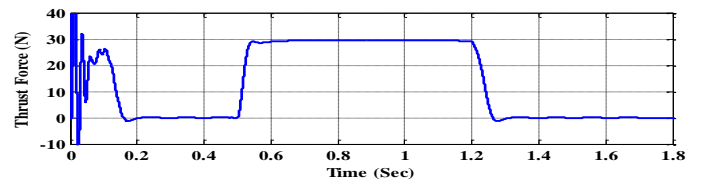


Fig. 7: Thrust force wave form of LIM

During starting of the motor the thrust force is zero, when a 30N load force is applied at a time period between 0.5 sec to 1.2 sec the thrust force increases from 0N to 30N. When a 30N load force is removed again it falls to zero.

### 6.3. Current waveforms

#### 6.3.1. idp Current:

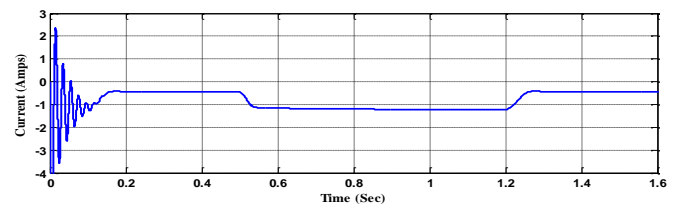


Fig. 8: Primary d-axis current wave form

The primary d-axes current at steady state is -0.5 amps when a 30N load force is suddenly applied at a time period of 0.5 sec to 1.2 sec the current falls from -0.5 amps to -1.1 amps when the load force is removed it comes to -0.5 amps.

6.3.2  $i_{qp}$  Current:

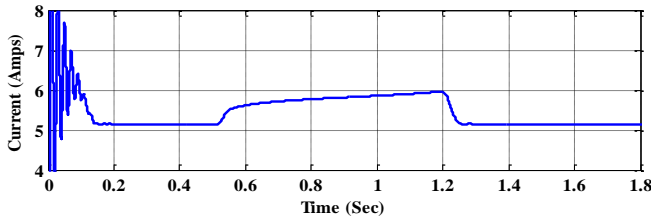


Fig. 9: Primary q-axis current waveform

The primary q-axis current at steady state is 5.1 Amps, when a 30N load force is applied at a time period from 0.5 sec to 1.2 sec the current increases from 5.1 Amps to 6 Amps. When the load force is removed again it comes to steady state of 5.1 Amps.

6.3.3.  $i_{dl}$  Current:

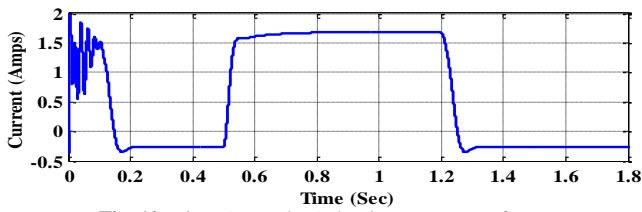


Fig. 10: Linor (secondary) d-axis current waveform

The Linor d-axis current at steady state is -0.3 Amps when a 30N load force is applied at a time period of 0.5 sec to 1.2 sec then the current varies from -0.3 Amps to 1.7 Amps. It reduces to -0.3 when the load force is removed.

6.3.4.  $i_{ql}$  Current:

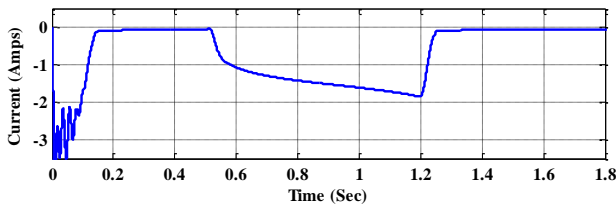


Fig. 11: Linor q-axis current waveform

The Linor q-axis current at steady state is zero when a 30N load force is applied at a time period of 0.5 sec to 1.2 sec the current varies from 0 Amps to -0.9 Amps. Again it comes to zero when the load force is removed.

6.3.5.  $\lambda_{dp}$ :

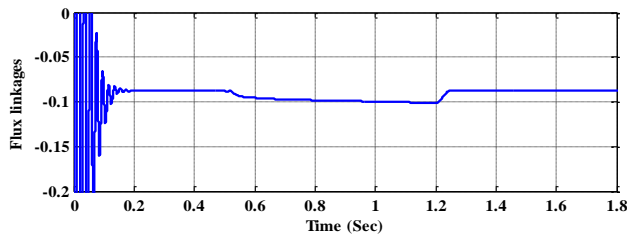


Fig. 12: Primary d-axis flux linkage waveform

The primary d-axis flux at steady state is -0.09 when a 30N load force is applied at a time period of 0.5 sec to 1.2 sec there is a slight variation of flux from -0.09 to -0.1.

6.3.6.  $\lambda_{qp}$ :

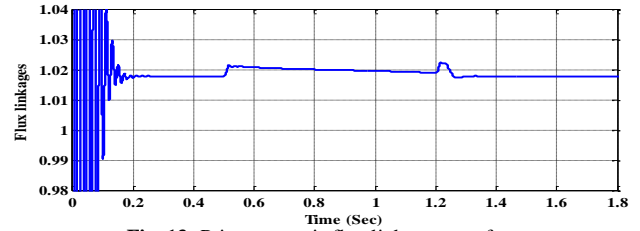


Fig. 13: Primary q-axis flux linkage waveform

The Primary q-axis flux at steady state is 1.009 when a 30N load force is applied at a time period of 0.5 sec to 1.2 sec there is a slight variation of flux from 1.009 to 1.02.

6.3.7.  $\lambda_{dl}$ :

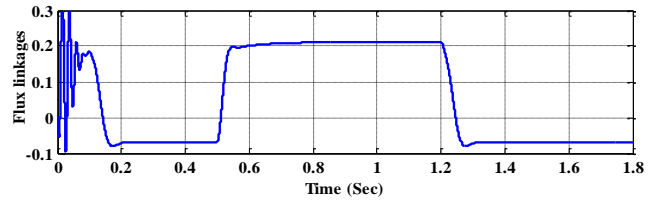


Fig. 14: Linor d-axis flux linkage waveform

Linor d-axis flux at steady state is -0.09 when a 30N load force is applied at a time period of 0.5 sec to 1.2 sec there is a variation of flux from -0.09 to 0.2. When the load force is removed it comes to normal condition.

6.3.8.  $\lambda_{ql}$ :

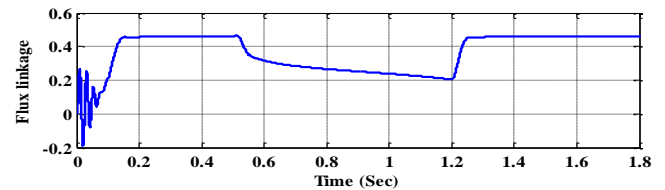


Fig. 15: Linor q-axis flux linkage waveform

The Linor q-axis flux at steady state is 0.45 when a 30N load force is applied at a time period of 0.5 sec to 1.2 sec there is a variation of flux from 0.45 to 0.2 again it reaches to 0.5 when a load force is removed.

7. Conclusion

The transient behavior of the LIM by considering end effects was modelled. For the reason that the occurrence of end affects the thrust force oscillations are generated in Linear Induction Motor. The thrust force behavior without involvement of the end effects does not have oscillations. This can be extending by splitting the currents and flux linkages method as is considered for reducing oscillations.

References

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