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Research paper

Analysis of magnetic fields on a levitating coaxial and non-coaxial aluminum disc

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Abstract

The electromagnetic field calculations for a levitating aluminum disc involve integro-differential equations and the solution of these equations is difficult to obtain using conventional techniques especially when the disc is coaxially away from the axis of the coils. Over the years many analytical, semi-analytical and numerical techniques have been proposed to calculate the magnetic fields on the disc when it is coaxial with the coils. In this paper, a mathematical formulation has been developed to obtain the magnetic fields on a conducting disc using a numerical technique at different positions of the levitation and for different disc discretization's. The numerical technique developed here is based on Finite Difference Method. Since the magnetic fields on the disc are due to the coil currents and eddy currents in the disc, first a mathematical formulation is done to calculate fields due to exciting coil currents and then a numerical technique is used to calculate fields due to eddy currents on the disc. Also, the magnetic fields on the disc are calculated when the disc moves away from the axis of the coil. A MATLAB program is developed to calculate these fields.

Keywords: *Electromagnetic field analysis; Finite Difference Method; eddy currents, COMSOL*

1. Introduction

Nowadays the electromagnetic levitation principle is used in several electrodynamic applications like precision position systems, highs speed transportation, etc.Among the two types of levitation principle, attraction and repulsion, the repulsive magnetic levitation [1 - 3] is more widely used. The levitation by magnetic repulsive force is shown in Fig.1. It consists of an aluminum disc of placed on two concentric current carrying coils.The two concentric coils are exciting coil and stabilizing coil.The exciting coil develops the repulsive force between the disc and the coil due the interaction of fields on the coil and the disc. Due to this the disc lifts off from the coil when the repulsive force is greater than the disc weight. The levitated disc will move away from the axis of the coils as it always tries to follow low inductance path.
To stabilize the disc laterally, another coil known as stabilizing coil is placed concentrically outside the primary coil and is connected in series opposition with the primary coil. Since the current in stabilizing coil is in phase with that of the disc, it attracts the disc thus pulling it down and at the same time stabilizing it. Another requirement to achieve the above objective is to keep the disc diameter a little more than the outer diameter of the primary coil and less than the outer diameter of the stabilizing coil. The final lift of the disc depends on the ampere-turns of the lift and the stabilizing coil, the frequency of the current, conductivity, diameter and the thickness of the disc. Since the last two factors increase the weight of the disc, aluminum is the obvious choice for the disc as it is being lightin weight and possessing good conductivity. The magnetic fields and current density on the disc have been calculated by mathematical formulation

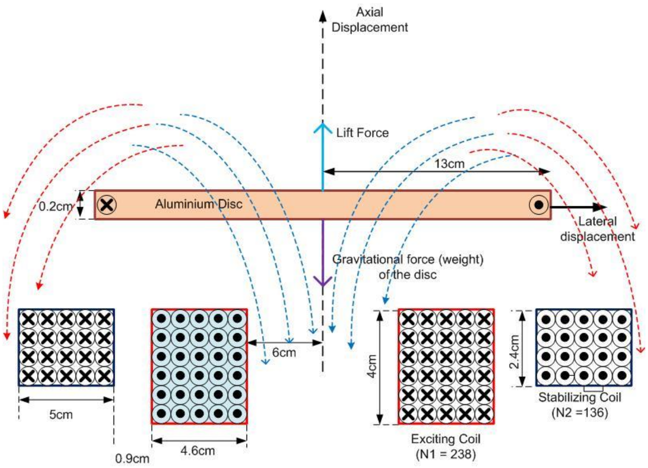

Fig. 1: Principle of MagneticLevitation due to repulsive force.

2. Magnetic fields on non-coaxial disc

The magnetic fields produced bythe exciting and stabilizing coil currents on the non-coaxial disc are calculated here.
A point P(r, θ) on the disc (with the center of the disc as origin) will have a radial distance r″ from the axis of the coils, given by

$$r'' = \sqrt{r^2 + \varepsilon^2 + 2r\varepsilon\cos(\theta)} \quad (1)$$

where, 'ε' is the displacement of the disc from the axis of the coils and r″ is the distance from the axis of the coils to the point P.

Let, the radius of the inner coil is 'a' and that of the outer coil is 'b'. The magnetic fields on the disc are due to currents in the coil and due to induced currents in the disc. If B_{Z_1} is the field due to coil currents, it has two components, axial component, B_Z and the radial component, B_r.These components are calculated at a point (r″, z), where 'z' is the height from the plane of the coils. As mentioned in classical field theory [4] and also in [5, 6]the components of B_{Z1}can be obtained as
The axial component of B_{Z1} is,

$$B_z = \frac{\mu_0 I}{2\pi}\left[\frac{1}{\sqrt{(r''+a)^2+z^2}}\right]\left\{\frac{a^2-(r'')^2-z^2}{(a-r'')^2+z^2}\,E(k)+K(k)\right\} \tag{2}$$

and, the radial component of B_{Z1} is,

$$B_r = \frac{\mu_0 I}{2\pi}\left[\frac{z}{r''}\right]\left[\frac{1}{\sqrt{(r''+a)^2+z^2}}\right]\left\{\frac{(r'')^2+a^2+z^2}{(a-r'')^2+z^2}E(k)-K(k)\right\} \tag{3}$$

where, E(k) and K(k) are the Elliptic integrals of the first kind and second kind respectively and is given by,

$$K(k) = \int_0^{\frac{\pi}{2}} \frac{d\alpha}{\sqrt{1-k^2\sin^2\alpha}} \tag{4}$$

$$E(k) = \int_0^{\frac{\pi}{2}} \left(\sqrt{1-k^2\sin^2\alpha}\right)d\alpha \tag{5}$$

$$\text{and, } k = \sqrt{\frac{4ar''}{(r''+a)^2+z^2}} \tag{6}$$

$$\vec{B}_{z_1} = \vec{B}_z + \vec{B}_r \tag{7}$$

If $B_{Z_2}$is the field to the induced currents on the disc, it can be obtained as discussed in [3]and rewritten here for convenience.

$$\vec{B}_{z_2} = \left(\frac{\mu_0\tau}{4\pi}\right)\int_0^a\int_0^{2\pi}\left\{\frac{[J_{r'}\, r\sin(\theta-\theta')]-[J_{\theta'}(r\cos(\theta-\theta')-r')]}{|\vec{r}-\vec{r}'|^3}\right\}r'dr'd\theta' \tag{8}$$

Combining equations (7) and (8) and using Maxwell's equations we get,

$$\nabla^2 U_Z = j\omega\sigma(B_{Z1}+B_{Z2}) \tag{9}$$

where, U_Z is a streaming function.
The equation (9) is a complex equation and the solution is difficult to obtain. By expanding the equation (9), we get

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial U_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 U_z}{\partial\theta^2}+j\omega\sigma\left(\frac{\mu_0\tau}{4\pi}\right)\int_0^a\int_0^{2\pi}\left\{\frac{[J_{r'}\, r\sin(\theta-\theta')]}{|\vec{r}-\vec{r}'|^3}\right\}-\left\{\frac{[J_{\theta'}(r\cos(\theta-\theta')-r')]}{|\vec{r}-\vec{r}'|^3}\right\}r'dr'd\theta' = j\omega\sigma B_{z_1} \tag{10}$$

3. Numerical technique using disc discretization

A numerical technique based on FDM is used to solve the equation (10) by discretizing the disc into several elements (m) as shown in Fig.2.

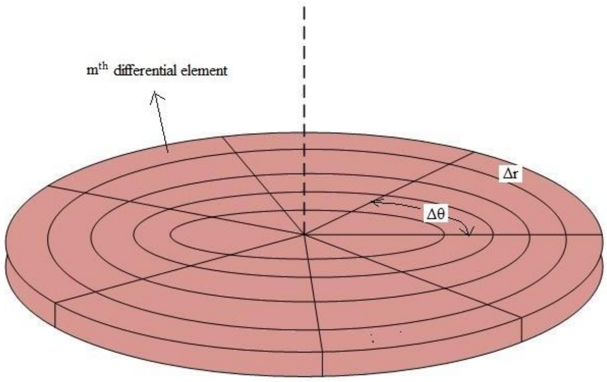

Fig. 2: Disc discretization into 'm' differential elements

Let, 'Δr' be the differential distance between any two consecutive circles and 'Δθ'be the differential length between any two consecutive lines.
Suppose P(i, j) is the point of interest where the magnetic field is to be calculated on the disc and P'(i', j') is the source point. Now, discretize the above integro-differential equation around the point P(i, j) by using central difference method and B_{Z2}can be represented in discretized form as

$$B_{z_2} = \left(\frac{\mu_0\tau}{4\pi}\right)\sum_{i'}\sum_{j'}\frac{r_{ij}r_{i'j'}J1}{R} - \left(\frac{\mu_0\tau}{4\pi}\right)\sum_{i'}\sum_{j'}\frac{r_{i'j'}J2}{R} \tag{11}$$

except for (i', j') = (i, j) where,

$$J_1 = J_r(i',j')\sin(\theta_{ij}-\theta_{i'j'})\,\Delta\theta\,\Delta r \tag{12}$$

$$J_2 = J_\theta(i',j')[r_{ij}\cos(\theta_{ij}-\theta_{i'j'})-r_{i'j'}]\,\Delta\theta\,\Delta r \tag{13}$$

$$R = [(r_{ij})^2+(r_{i'j'})^2-2r_{ij}r_{i'j'}\cos(\theta_{ij}-\theta_{i'j'})]^{\frac{3}{2}} \tag{14}$$

The equation (10) can be written in compact form as

$$B_{z_2} = C_1\sum_{i'}\sum_{j'}[K_r J_r + K_\theta J_\theta] \tag{15}$$

$$\text{where, } C_1 = \frac{\mu_0\tau}{4\pi}, \tag{16}$$

$$K_r = \text{kernel-r},\ K_r(ij;i'j') = \frac{r_{ij}r_{i'j'}J_1}{R} \tag{17}$$

$$K_\theta = \text{kernel-}\theta,\text{ and }\ K_\theta(ij;i'j') = \frac{r_{i'j'}J_\theta(i',j')J_2}{R} \tag{18}$$

$$\therefore B_{z_2} = C_1\sum_{i'}\sum_{j'}[(\text{kernel}-r)(J_r)+(\text{kernel}-\theta)(J_\theta)] \tag{19}$$

$$J_r = \frac{1}{r}\frac{\partial U_z}{\partial\theta} = \frac{1}{r_i}\frac{[U_i^{j+1}-U_i^j]}{\Delta\theta} \tag{20}$$

$$J_\theta = -\frac{\partial U_z}{\partial r} = -\frac{[U_{i+1}^j-U_i^j]}{\Delta r} \tag{21}$$

In matrix form,B_{Z2} can be represented as,

$$[B_{z_2}] = C_1([K_r][J_r]+[K_\theta][J_\theta]) \tag{22}$$

whereK_r, $K_\theta$is the unknown values of U_Z (including the center),and their order is (mn+1).
All the elements of K_r, K_θ are full except for (i', j') = (i, j) which is zero. J_r, J_θ are the column matrices of the same order,and their elements can be obtained from the difference values of U_Z at different nodes.

$$\therefore \{\nabla^2 - j\omega\sigma C_1([K_r][D_r]+[K_\theta][D_\theta])\}[U_z] = j\omega\sigma[B_{z_1}] \tag{23}$$

By solving the equation (23), (mn+1) values of $U_Z$are obtained. With these values of U_Z, B_{Z2} can be calculated by using the equation,

$$[\mathbf{B}_{\mathbf{z_2}}] = \mathbf{C_1}\{[\mathbf{K_r}][\mathbf{D_r}][\mathbf{U_z}] + [\mathbf{K_\theta}][\mathbf{D_\theta}][\mathbf{U_z}]\}$$

4. Results and Discussion

4.1 Magnetic fields on the coaxial disc

The net magnetic field on the disc for 101, 401, 901, 1601, and 2501 disc discretization elements have been calculated at centre of the disc and also at 0.026m, 0.052m, 0.078m, 0.104m, and 0.13m of the disc radius. The calculated values are given in tables (1 – 6).

Table 1: Magnetic flux Density on the disc at r = 0m

Levitated Disc position (m)	Magnetic flux Density for different disc discretization's (A/m^2)				
	m=101	m=401	m=901	m=1601	m=2501
0.040	0.0087	0.0097	0.0099	0.0100	0.0100
0.044	0.0083	0.0092	0.0094	0.0094	0.0095
0.048	0.0079	0.0086	0.0088	0.0088	0.0089
0.052	0.0073	0.0080	0.0082	0.0082	0.0082
0.056	0.0068	0.0074	0.0075	0.0076	0.0076
0.060	0.0063	0.0068	0.0069	0.0070	0.0070
0.064	0.0057	0.0062	0.0063	0.0063	0.0064
0.068	0.0052	0.0056	0.0057	0.0058	0.0058
0.072	0.0047	0.0051	0.0051	0.0052	0.0052
0.076	0.0042	0.0045	0.0046	0.0046	0.0047
0.080	0.0038	0.0040	0.0041	0.0041	0.0042
0.084	0.0033	0.0036	0.0037	0.0037	0.0037
0.088	0.0029	0.0032	0.0032	0.0033	0.0033
0.090	0.0027	0.0030	0.0030	0.0031	0.0031

The magnetic field values calculated at the centre of the disc and at 0.026m, 0.052, 0.078, 0.104, and 0.13m of disc radius for different disc discretization's have been plotted in Figs. (3 – 8) respectively.

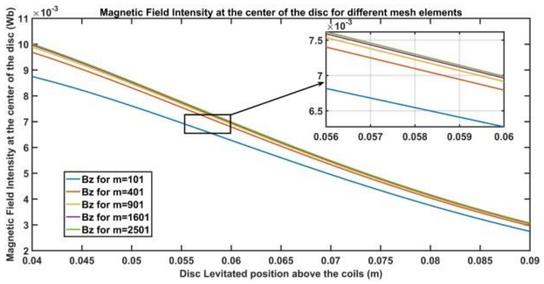

Fig. 3: Magnetic Field at the centre of the disc

Table 2: Magnetic flux Density on the disc at r = 0.026m

Levitated Disc position (m)	Magnetic flux Density for different disc discretization's (A/m^2)				
	m=101	m=401	m=901	m=1601	m=2501
0.040	0.0109	0.0112	0.0112	0.0112	0.0112
0.044	0.0102	0.0104	0.0105	0.0105	0.0105
0.048	0.0095	0.0097	0.0097	0.0097	0.0097
0.052	0.0087	0.0089	0.0089	0.0089	0.0089
0.056	0.0080	0.0082	0.0082	0.0082	0.0082
0.060	0.0073	0.0074	0.0074	0.0074	0.0074
0.064	0.0066	0.0067	0.0067	0.0067	0.0067
0.068	0.0060	0.0061	0.0061	0.0061	0.0061
0.072	0.0054	0.0055	0.0055	0.0055	0.0055
0.076	0.0049	0.0050	0.0050	0.0050	0.0050
0.080	0.0044	0.0045	0.0045	0.0045	0.0045
0.084	0.0040	0.0041	0.0041	0.0041	0.0041
0.088	0.0037	0.0037	0.0037	0.0037	0.0037
0.090	0.0035	0.0036	0.0036	0.0036	0.0036

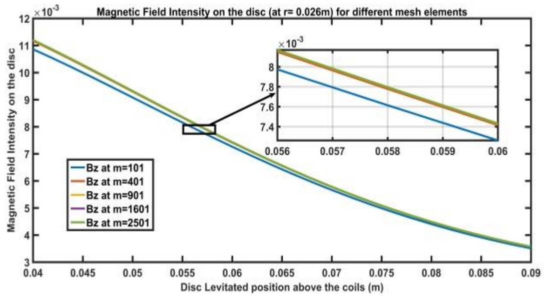

Fig. 4: Magnetic Field at r=0.026m radius of the disc

Table 3: Magnetic flux Density on the disc at r = 0.052m

Levitated Disc position (m)	Magnetic flux Density for different disc discretization's (A/m^2)				
	m=101	m=401	m=901	m=1601	m=2501
0.040	0.0142	0.0144	0.0144	0.0144	0.0144
0.044	0.0126	0.0127	0.0127	0.0127	0.0127
0.048	0.0110	0.0111	0.0111	0.0111	0.0111
0.052	0.0096	0.0097	0.0097	0.0097	0.0097
0.056	0.0084	0.0084	0.0084	0.0084	0.0084
0.060	0.0072	0.0073	0.0073	0.0073	0.0073
0.064	0.0063	0.0063	0.0063	0.0063	0.0063
0.068	0.0054	0.0054	0.0054	0.0054	0.0054
0.072	0.0046	0.0047	0.0047	0.0047	0.0047
0.076	0.0040	0.0040	0.0040	0.0040	0.0040
0.080	0.0034	0.0034	0.0034	0.0034	0.0034
0.084	0.0029	0.0029	0.0029	0.0029	0.0029
0.088	0.0025	0.0025	0.0025	0.0025	0.0025
0.090	0.0023	0.0023	0.0023	0.0023	0.0023

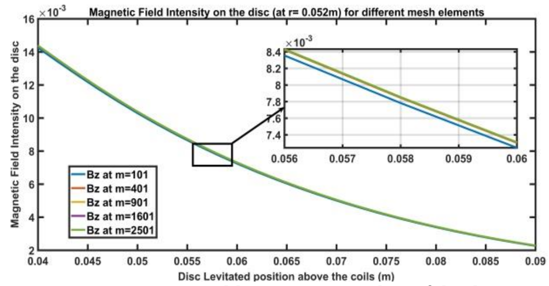

Fig. 5: Magnetic Field at r=0.052m radius of the disc

Table 4: Magnetic flux Density on the disc at r = 0.078m

Levitated Disc position (m)	Magnetic flux Density for different disc discretization's (A/m^2)				
	m=101	m=401	m=901	m=1601	m=2501
0.040	0.0151	0.0152	0.0152	0.0152	0.0152
0.044	0.0127	0.0128	0.0128	0.0128	0.0128
0.048	0.0106	0.0107	0.0107	0.0107	0.0107
0.052	0.0089	0.0090	0.0090	0.0090	0.0090
0.056	0.0075	0.0075	0.0075	0.0075	0.0075
0.060	0.0062	0.0063	0.0063	0.0063	0.0063
0.064	0.0052	0.0052	0.0052	0.0052	0.0052
0.068	0.0043	0.0044	0.0044	0.0044	0.0044
0.072	0.0036	0.0036	0.0036	0.0036	0.0036
0.076	0.0030	0.0030	0.0030	0.0030	0.0030
0.080	0.0024	0.0025	0.0025	0.0025	0.0025
0.084	0.0020	0.0020	0.0020	0.0020	0.0020
0.088	0.0016	0.0016	0.0016	0.0016	0.0016
0.090	0.0014	0.0014	0.0014	0.0014	0.0014

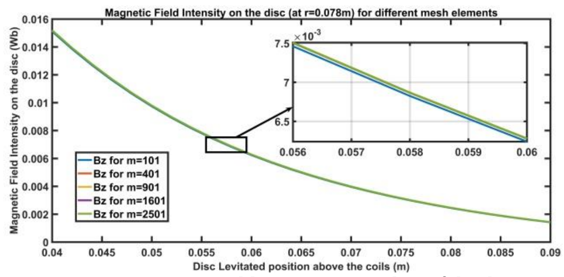

Fig. 6: Magnetic Field at r=0.078m radius of the disc

Table 5: Magnetic flux Density on the disc at r = 0.104m

Levitated Disc position (m)	Magnetic flux Density for different disc discretization's (A/m^2)				
	m=101	m=401	m=901	m=1601	m=2501
0.040	0.0154	0.0154	0.0155	0.0155	0.0155
0.044	0.0124	0.0125	0.0125	0.0125	0.0125
0.048	0.0102	0.0102	0.0102	0.0102	0.0102
0.052	0.0084	0.0084	0.0084	0.0084	0.0084
0.056	0.0070	0.0070	0.0070	0.0070	0.0070
0.060	0.0058	0.0058	0.0059	0.0059	0.0059
0.064	0.0049	0.0049	0.0049	0.0049	0.0049
0.068	0.0041	0.0041	0.0041	0.0041	0.0041
0.072	0.0034	0.0035	0.0035	0.0035	0.0035
0.076	0.0029	0.0029	0.0029	0.0029	0.0029
0.080	0.0025	0.0025	0.0025	0.0025	0.0025
0.084	0.0021	0.0021	0.0021	0.0021	0.0021
0.088	0.0018	0.0018	0.0018	0.0018	0.0018
0.090	0.0016	0.0016	0.0016	0.0016	0.0016

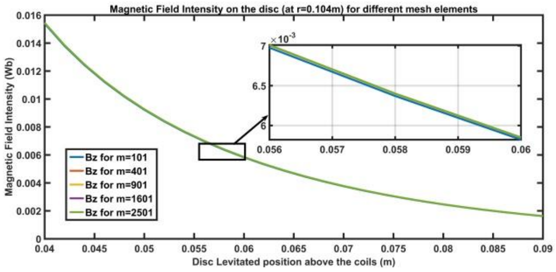

Fig. 7: Magnetic Field at r=0.104m radius of the disc

Table 6: Magnetic flux Density on the disc at r = 0.13m

Levitated Disc position (m)	Magnetic flux Density for different disc discretization's (Wb/m^2)				
	m=101	m=401	m=901	m=1601	m=2501
0.040	0.0128	0.0128	0.0128	0.0128	0.0128
0.044	0.0108	0.0109	0.0109	0.0109	0.0109
0.048	0.0092	0.0092	0.0092	0.0092	0.0092
0.052	0.0078	0.0078	0.0078	0.0078	0.0078
0.056	0.0066	0.0067	0.0067	0.0067	0.0067
0.060	0.0057	0.0057	0.0057	0.0057	0.0057
0.064	0.0049	0.0049	0.0049	0.0049	0.0049
0.068	0.0042	0.0042	0.0042	0.0042	0.0042
0.072	0.0036	0.0037	0.0037	0.0037	0.0037
0.076	0.0032	0.0032	0.0032	0.0032	0.0032
0.080	0.0028	0.0028	0.0028	0.0028	0.0028
0.084	0.0024	0.0024	0.0024	0.0024	0.0024
0.088	0.0021	0.0022	0.0022	0.0022	0.0022
0.090	0.0020	0.0020	0.0020	0.0020	0.0020

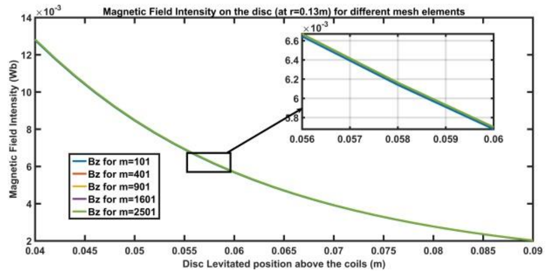

Fig. 8: Magnetic Field at r=0.13m radius of the disc

4.2Magnetic fields on the non-coaxial disc

The net magnetic field on the disc for 101, 401, 901, 1601, and 2501 disc discretization elements have been calculated at center of the disc and also at 0.026m, 0.052m, 0.078m, 0.104m, 0.13m of the disc radius when the disc is off from the axis of the coils. The calculated values are given in tables (7 – 12).

The magnetic field values calculated at the centre of the disc and at 0.026m, 0.052, 0.078, 0.104, and 0.13m of disc radius for different disc discretization's when the disc is off from the axis of the coils have been plotted in Figs. (9 – 14) respectively.

Table 7: Magnetic flux Density on the on the non-coaxial disc at r=0m

Off axis Disc position 'ε' (m)	Magnetic Flux Density for different disc discretization's (Wb/m2)				
	m=101	m=401	m=901	m=1601	m=2501
0	0.0069	0.0072	0.0072	0.0072	0.0072
0.1	0.0065	0.0063	0.0062	0.0062	0.0061
0.2	0.0015	0.0015	0.0015	0.0015	0.0015
0.3	0.0004	0.0004	0.0004	0.0004	0.0004
0.4	0.0002	0.0002	0.0002	0.0002	0.0002
0.5	0.0001	0.0002	0.0002	0.0002	0.0002

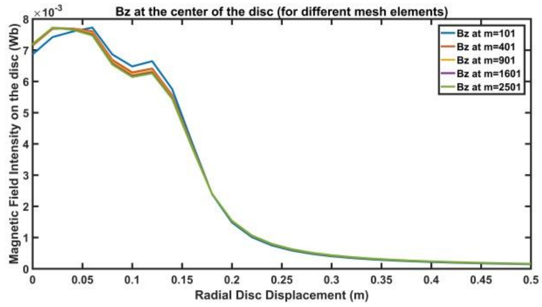

Fig. 9: Magnetic Field at r=0m radius of the non-coaxial disc

Table 8: Magnetic flux Density on the on the non-coaxial disc at r=0.026m

Off axis Disc position 'ε' (m)	Magnetic Flux Density for different disc discretization's (Wb/m2)				
	m=101	m=401	m=901	m=1601	m=2501
0	0.00804	0.00784	0.00776	0.007707	0.007679
0.1	0.00685	0.0064	0.00627	0.006197	0.006158
0.2	0.00097	0.00098	0.00098	0.000976	0.000976
0.3	0.00035	0.00036	0.00036	0.00036	0.00036
0.4	0.00021	0.00021	0.00021	0.000212	0.000212
0.5	0.00014	0.00015	0.00015	0.000147	0.000147

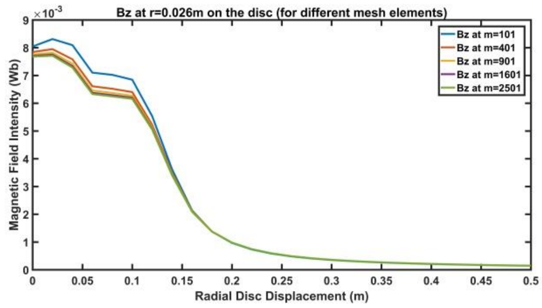

Fig. 10: Magnetic Field at r=0.026m radius of the non-coaxial disc

Table 9: Magnetic flux Density on the on the non-coaxial disc at r=0.052m

Off axis Disc position 'ε' (m)	Magnetic Flux Density for different disc discretization's (Wb/m2)				
	m=101	m=401	m=901	m=1601	m=2501
0	0.00847	0.00795	0.0078	0.00772	0.007676
0.1	0.00502	0.00473	0.00464	0.004601	0.004577
0.2	0.00069	0.00069	0.00069	0.00069	0.00069
0.3	0.0003	0.00031	0.00031	0.000306	0.000306
0.4	0.00019	0.00019	0.00019	0.00019	0.00019
0.5	0.00013	0.00014	0.00014	0.000136	0.000136

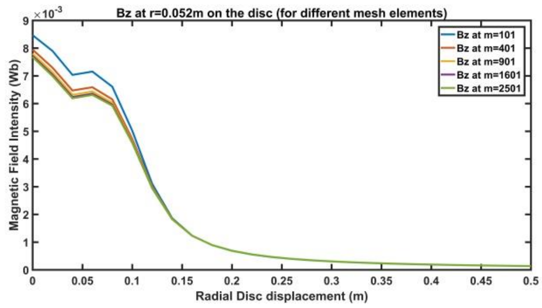

Fig. 11: Magnetic Field at r=0.052m radius of the non-coaxial disc

Table 10: Magnetic flux Density on the on the non-coaxial disc at r=0.078m

Off axis Disc position 'ε' (m)	Magnetic Flux Density for different disc discretization's (Wb/m2)				
	m=101	m=401	m=901	m=1601	m=2501
0	0.00759	0.00697	0.00681	0.006723	0.006675
0.1	0.00265	0.00258	0.00255	0.00254	0.002533
0.2	0.00052	0.00053	0.00053	0.000526	0.000526
0.3	0.00026	0.00026	0.00026	0.000265	0.000265
0.4	0.00017	0.00017	0.00017	0.000173	0.000173
0.5	0.00013	0.00013	0.00013	0.000126	0.000126

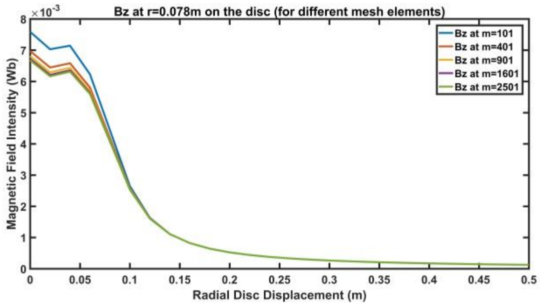

Fig. 12: Magnetic Field at r=0.078m radius of the non-coaxial disc

Table 11: Magnetic flux Density on the on the non-coaxial disc at r=0.104m

Off axis Disc position 'ε' (m)	Magnetic Flux Density for different disc discretization's (Wb/m2)				
	m=101	m=401	m=901	m=1601	m=2501
0	0.0071	0.0065	0.00635	0.006265	0.00622
0.1	0.00144	0.00143	0.00143	0.001426	0.001424
0.2	0.00042	0.00042	0.00042	0.000422	0.000422
0.3	0.00023	0.00023	0.00023	0.000233	0.000233
0.4	0.00016	0.00016	0.00016	0.000158	0.000158
0.5	0.00012	0.00012	0.00012	0.000117	0.000117

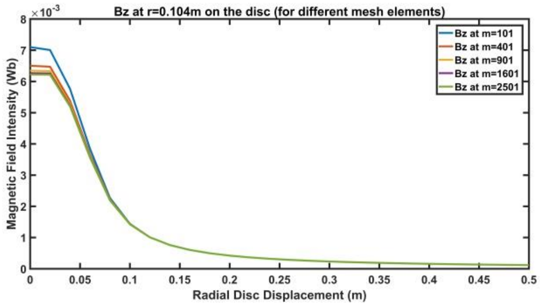

Fig. 13: Magnetic Field at r=0.104m radius of the non-coaxial disc

Table 12: Magnetic flux Density on the on the non-coaxial disc at r=0.13m

Off axis Disc position 'ε' (m)	Magnetic Flux Density for different disc discretization's (Wb/m2)				
	m=101	m=401	m=901	m=1601	m=2501
0	0.00675	0.00625	0.00612	0.006049	0.006012
0.1	0.00092	0.00092	0.00092	0.00092	0.000919
0.2	0.00035	0.00035	0.00035	0.00035	0.00035
0.3	0.00021	0.00021	0.00021	0.000208	0.000208
0.4	0.00014	0.00015	0.00015	0.000145	0.000145
0.5	0.00011	0.00011	0.00011	0.00011	0.00011

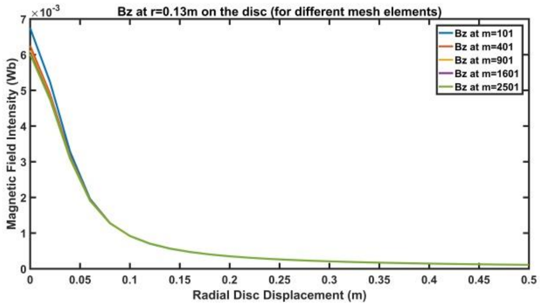

Fig. 14: Magnetic Field at r=0.13m radius of the non-coaxial disc

5. Conclusion

The magnetic flux density on the disc is calculated when the disc is both coaxial and non-axial with the axis of the coils. The stabilization of the disc can be obtained by the magnetic fields calculated when the disc is coaxial with the coils. Also the magnetic fields are analyzed on the disc when the disc moved away from its floating coaxial position. The magnetic fields are computed by mathematical programming of the numerical technique using FDM.

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