



Forecasting Electricity Consumption Using Fuzzy Time Series

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Abstract

Electricity consumption forecasting is important for effective operation, planning and facility expansion of power system. Accurate forecasts can save operating and maintenance costs, increased the reliability of power supply and delivery system, and correct decisions for future development. There is a great development of Universiti Tun Hussein Onn Malaysia (UTHM) infrastructure since its formation in 1993. The development will be accompanied with the increasing demand of electricity. Hence, there is a need to forecast the UTHM electricity consumption for future decisions on generating electric power, load switching, and infrastructure development. Therefore, in this study, the Fuzzy time series (FTS) with trapezoidal membership function was implemented on the UTHM monthly electricity consumption from January 2011 to December 2017 to forecast January to December 2018 monthly electricity consumption. The procedure of the FTS and trapezoidal membership function was described together with January data. FTS is able to forecast UTHM electricity consumption quite well.

Keywords: Fuzzy time series; FTS; MAE; MAPE; MSE; RMSE.

1. Introduction

Time series is a collection of equally spaced temporal data. A time series can consist some or all of the components such as trend (long term pattern), cyclical (repeated up and down movements), seasonal (regular fluctuations during the same month or quarter) and irregular (unexplained random component)[1].

Zadeh [1] was the first to propose the concept of fuzzy set theory. Song and Chissom [2] presented the definition, properties and procedure to develop a first-order time-invariant Fuzzy time series (FTS) model based on Zadeh's [3-6] works. Their definition of FTS is as follows:

Let $Y(t)$ ($t = \dots, 0, 1, 2, \dots$), be a time series, a subset of R and be the universe of discourse on which fuzzy sets $f_i(t)$ ($i = 1, 2, \dots$) are defined. Let $F(t)$ be a collection of $f_i(t)$. Then, $F(t)$ is called a fuzzy time series on $Y(t)$ ($t = \dots, 0, 1, 2, \dots$).

Forecasting is predicting future values based on past and current time series data. Forecasting for future load demand is essential for future power system planning and control. Load forecasting can be divided into short-term load forecasting (STLF), medium-term load forecasting (MTLF) and long-term load forecasting (LTLF). STLF up to one day or one week at most, MTF ranges from one day to several months while LTF forecasts more than a year ahead [7]. STLF is used for scheduling the generation and transmission of electricity, MTLF is used to plan the fuel purchases, whereas LTLF is aimed to develop the power supply and delivery system (generation units, transmission system, and distribution system) [8].

Song and Chissom [9] applied first-order time-invariant FTS on the enrolment of Alabama University from the years 1971-1990. Their proposed procedure using FTS to forecast are as follow:

1. Define the universe of discourse

$$U = [D_{min} - D_1, D_{max} + D_2].$$

2. Partition the universe of discourse into several even equal length intervals as u_1, u_2, \dots, u_m .
3. Define some fuzzy sets on the universe.
4. Fuzzify the historical data using the memberships of each year's enrollment in each fuzzy set A_i .
5. Derive fuzzy logical relationship (FLR).

$$R_i = A_j^T \times A_k, \text{ for all } n \text{ relations } A_j \rightarrow A_k, R = \bigcup_{i=1}^n R_i$$

where \times is the min operator, T is the transpose operator, and \cup is the union operator.

6. Forecasted output using $A_i = A_{i-1} \circ R$, where A_{i-1} and A_i are the fuzzified enrollments of year $i-1$ and i represented by a fuzzy set. The symbol \circ denotes the Max-Min composition operator.
7. Defuzzify forecasted results.

In [9], Song and Chissom's forecasting method requires a huge computation to derive the fuzzy relation R and the max-min composition operator when the fuzzy relation R is very big. Song and Chissom [10] applied time-variant FTS models with a three-layer back-propagation neural network to convert the output and compared three different defuzzification methods on the enrolment of Alabama University from the years 1971-1992. They also investigated the difference of time-invariant [9] and time-variant [10]. Results showed that the neural network model yielded the smallest average forecasting error. Their procedure is similar with Song and Chissom [9] from steps 1-5 except the fuzzy relation R was defined as

$$R_i = f^T(t-i) \times f^T(t-i+1) \text{ for all } n \text{ relations}$$

$$A_j \rightarrow A_k, R^W(t, t-1) = \bigcup_{i=2}^W R_i$$

where w is the window base, T is the transpose operator, \times is the Cartesian product, and \cup is the union

Later, in step 6, they picked a suitable parameter w , where $w > 1$, calculate $R^W(t, t-1)$ and forecast the enrollments as follows:

$$F(t) = F(t-1) \circ R^W(t, t-1),$$

where $F(t)$, $F(t-1)$ represent the forecasted fuzzy enrollment of year t , $t-1$ respectively and

Step 7: Defuzzify the forecasted fuzzy enrollment using a 3-layer backpropagation neural network.

Sullivan and Woodall [11] reviewed the first-order time-invariant and time-variant fuzzy time series models proposed by Song and Chissom [9-10] and compared with a time-invariant Markov model using linguistic labels with probability distributions.

Chen [12] proposed a simplified procedure to implement first-order FTS with triangular Fuzzy membership function on the enrolment of Alabama University from the years 1971-1992. The MAPE of Chen was 3.11%. Chen [12] procedure is similar to Song and Chissom [9] from steps 1-5 except in step 6, they established a fuzzy logical relationship group (FLRG) and Step 7 Defuzzify the forecasted outputs using several simplified rules.

Hwang, Chen and Lee [13] adopted the differences of the enrollments on Song and Chissom's [9] procedure to forecast the enrollments of University of Alabama based on FTS.

Chen and Hwang [14] proposed a two-factor time-variant fuzzy time series model to forecast temperature.

Huang [15] introduced the heuristic knowledge and establish the heuristic FLRG into Chen's [10] model to improve forecasting of university enrollment.

Huang [16] examined the effect of interval length on the forecasting results. He proposed two heuristic approaches, namely distribution and average-based, to determine the length of the interval

Chen [17] presented a high-order FTS model to forecast enrolment of Alabama University in their previous work [10]. and obtained MAPE of 1.52%.

Chen and Hsu [18] suggested first order and time-variant FTS model to forecast enrollments of the University of Alabama. Their procedure of Steps 1 and 2 are the same as Song and Chissom [9]. Nevertheless, in

Step 4: Sort the intervals based on the number of historical enrollment data in each interval from the highest to the lowest. Divide the interval having the largest, second largest, third largest, and fourth largest number of historical enrollment data into four, three, two and one intervals respectively. If there are no data distributed in an interval, then discard this interval. Lastly, a set of rules was used to determine whether the trend of the forecasting goes up or down and to forecast the enrollments.

Eleruja, Mu'azu and Dajab [19] implemented TFA and particle swarm optimization (PSO) on maximum temperature data of Zaria for the period 1990-2003 to forecast 2014 maximum temperature. They used the revised standard deviation of the distance between two consecutive data points to replace D_1 and D_2 in Song and Chissom [9].

Cheng, Chang and Yeh [20] implemented FTS with the trapezoidal Fuzzy approach (TFA) on the enrolment of Alabama University from the years 1971-1992. They obtained a MAPE of 2.66%. They used the standard deviation of the data to replace the arbitrary numbers of D_1 and D_2 in Song and Chissom [7- 8] and fuzzified data using TFA.

Poulsen [21] utilized a new high-order FTS model, TFA, aggregation and PSO on enrolment of the University Of Alabama.

Konica and Hanelli [22] adopted fuzzy interference system toolbox in Matlab for a short-term load forecasting electricity consumption for Albania. Time, historical and forecasting value

of the temperature and previous day load were used as the input to predict the next-day electricity consumption.

In this paper, we have only monthly UTHM electricity consumption from January 2011 to December 2017. Therefore, the FTS was applied to these data to forecast the UTHM monthly 2018 electricity consumption. The accuracy of FTS will be compared with historical data.

2. Fuzzy Time Series (FTS)

We adopted Poulsen's [21] algorithm of first-order FTS with trapezoidal fuzzification set in Microsoft Excel spreadsheet, but did not apply PSO. The input is the same month of electricity consumption from year 2011-2017, for example with the input of January electricity consumption from 2011-2017, the January 2018 electricity consumption will be forecasted. The process is repeated for February, March, ... till December. The following steps show the general procedure to be taken in order to forecast monthly 2018 electricity consumption.

1. Sort the values of the same month electricity consumption from 2011-2017 in ascending order.
2. Compute distance, D_i between any two consecutive electricity consumptions in the sorted dataset as

$$D_i = |x_{i+1} - x_i| \quad (1)$$

3. Find average distance, AD between any two consecutive electricity consumptions in the sorted dataset as

$$AD = \frac{1}{n-1} \sum_{i=1}^{n-1} D_i \quad (2)$$

4. Compute the corresponding standard deviation, σ_{AD} of the average distance as

$$\sigma_{AD} = \frac{1}{n-1} \sum_{i=1}^{n-1} (D_i - AD)^2 \quad (3)$$

5. D should be in the following interval

$$AD - \sigma_{AD} \leq D \leq AD + \sigma_{AD} \quad (4)$$

Eliminate the outlier D_i which is not in the range.

6. Recalculate the revised average distance AD_r from the remaining D_i .
7. Define the universe of discourse

$$U = [D_{MIN} - AD_r, D_{MAX} + AD_r] \quad (5)$$

where D_{MIN} is the minimum value of the electricity consumption while D_{MAX} is the maximum value of the electricity consumption.

8. Fuzzify the universe of discourse using the trapezoidal fuzzification approach. In the trapezoidal fuzzy set, there is four quartets of trapezoidal fuzzy numbers (a , b , c , d). The leftmost value of the trapezoidal set is $D_{MIN} - AD_r$, whereas the rightmost value of the trapezoidal set is $D_{MAX} + AD_r$ and the distance for each number of a , b , c and d are the revised average distance, AD_r .
9. Sometime a data may belong to two fuzzy sets, so to determine the data belong to which fuzzy set, the trapezoidal Fuzzy number will be used to find the membership degree:

$$\mu_A = \begin{cases} 0 & x_i < a \\ \frac{x_i - a}{b - a}, & a \leq x_i \leq b \\ 1 & b \leq x_i \leq c \\ \frac{d - x_i}{d - c}, & b \leq x_i \leq c \\ 0, & x_i > d \end{cases} \quad (6)$$

10. The highest membership degree will be used to determine the membership of a fuzzy set.
11. Next, a Fuzzy set relationship will be determined. If the time series variable $F(t-1)$ as fuzzified as A_i and $F(t)$ as A_j , then A_i is related to A_j and denoted as $A_i \rightarrow A_j$.
12. Then, fuzzy linear relation group (FLRG) will be determined by grouping the same fuzzy set which is related to more than one set.
13. Later, the midpoint of each of the FLRG will be computed.
14. The forecasted value will be the average value of the midpoint of the FLRG values.

3. Error Analysis

The performance of the above time series methods can be measured by absolute error (AE), mean absolute error (MAE), mean absolute percentage error, sum of square of error (SE), mean square error (MSE), root mean square error (RMSE) as below:

$$AE = |error| = |y_i - \hat{y}_i|,$$

$$MAE = \frac{\sum_{i=1}^n |y_i - \hat{y}_i|}{n},$$

$$MAPE = \frac{\sum_{i=1}^n \frac{|y_i - \hat{y}_i|}{y_i}}{n} \times 100\%,$$

$$SE = |error|^2 = |y_i - \hat{y}_i|^2,$$

$$MSE = \frac{\sum_{i=1}^n |y_i - \hat{y}_i|^2}{n},$$

$$RMSE = \sqrt{\frac{\sum_{i=1}^n |y_i - \hat{y}_i|^2}{n}},$$

where y_i, \hat{y}_i are real and forecasted data respectively, n is the number of real data. Here, MAE measures the average value of the absolute error or the average of the spread of error. All errors are assigned equal weights in MAE [23]. MAPE is a relative percentage error corresponds to MAE. Lewis [24] stressed that the MAPE is the most useful measure to compare the accuracy of forecasting methods as it measures relative performance. A MAPE which is less than 10 percent is considered as highly accurate forecasting, between 10 - 20 percent is good forecasting, between 20 -50 percent is interpreted as reasonable forecasting and over 50 percent is inaccurate forecasting [24]. On the other hand, MSE measures the average of the squares of the errors, hence large errors are given additional weight [23]. Whereas RMSE is the square root of the MSE.

4. Methodology

The forecasting process of January 2018 electricity consumption will be started on the January data from the year 2011 to 2017 which is shown in Table 1 as the following steps:

Table 1: January data set

Year	Jan	Sort	D	D-AD ^2
2011	1757.133	1757.133		
2012	2646.807	2256.096	498.963	97822.05
2013	2855.407	2379.815	123.719	3903.605
2014	2379.815	2646.807	266.992	6527.697
2015	2774.32	2774.32	127.513	3443.91
2016	2874.32	2855.407	81.087	11048.29
2017	2256.096	2874.32	18.913	27984.22
		AD	186.1978	
		Sigma	158.498	

- 1) The sorted values of the January electricity consumption from the year 2011-2017 in the current dataset in ascending order are shown in column 3 of Table 1.
- 2) Compute distance, D using Eq. (1) and the results are shown in the fourth column of Table 1.
- 3) The average distance, AD between any two consecutive electricity consumption in the sorted dataset was found as 186.1978.
- 4) The corresponding standard deviation, σ_{AD} of the average distance was obtained as 158.498.
- 5) D should be in the interval $27.6998 \leq D \leq 344.6959$. Eliminate the outlier D (498.963 and 18.913) which are not in the range.
- 6) The revised average distance AD_r from the remaining D was calculated as $AD_r = 149.8278$
- 7) Define the universe of discourse $U = [1607.305, 3024.148]$
- 8) The trapezoidal fuzzy sets were obtained as given in Table 2:

Table 2: Trapezoidal Fuzzy numbers (a, b, c, d)

Fuzzy set	a	Fuzzy b	Numbers c	d
A ₁	1607.305	1757.133	1906.961	2056.789
A ₂	1906.961	2056.789	2206.616	2356.444
A ₃	2206.616	2356.444	2506.272	2656.1
A ₄	2506.272	2656.1	2805.927	2955.755
A ₅	2805.927	2955.755	3105.583	3255.411

Where the left most is

$D_{MIN} - AD_r = 1757.133 - 149.8278 = 1607.305$. Then the following values will be added with AD_r . Next the values of a and b for the following fuzzy set are equal to the preceding values of c and d from preceding Fuzzy set. The right most value of the fuzzy set will not necessary equals to $D_{MAX} + AD_r$ but will be greater than it.

9. The fuzzified membership set of January electricity consumption is given in Table 3. It is clearly shown that January 2011, 2014, 2015 belong to fuzzy set A1, A3 and A4. January 2012, 2013, 2016 and 2017 belong to both fuzzy sets, hence the formula (6) was used to determine their higher membership degree as given in columns 3 and 4 in Table 3. The fuzzified membership set was determined based on the highest membership degree. For example, electricity consumption in January 2012 (2646.807) belongs to fuzzy sets A3 and A4. The membership degrees of A3 and A4 are shown in columns 3 and 4 in Table 3. The highest membership de-

gree is A_4 and thus the January 2012 data is fuzzified in Fuzzy set A_4 .

Table 3: Fuzzified membership set

Year	Jan	Membership degree		Fuzzified Data
2011	1757.133	-	-	A_1
2012	2646.807	0.062021	0.937979	A_4
2013	2855.407	0.669756	0.330244	A_4
2014	2379.815	-	-	A_3
2015	2774.32	-	-	A_4
2016	2874.32	0.543524	0.456476	A_4
2017	2256.096	0.669756	0.330244	A_2

- The fuzzy logical relationships (FLR) were obtained as
 $A_1 \rightarrow A_4, A_4 \rightarrow A_4, A_4 \rightarrow A_3,$
 $A_3 \rightarrow A_4, A_4 \rightarrow A_4, A_4 \rightarrow A_2.$
- Next, a fuzzy logical relation group (FLRG) were determined by grouping the same group of fuzzy set relationships as shown in Table 4.

Table 4: FLRG

Year	Jan	Fuzzified membership set	FLRG
2011	1757.133	A_1	$A_1 \rightarrow A_4$
2012	2646.807	A_4	$A_4 \rightarrow A_2, A_3, A_4$
2013	2855.407	A_4	$A_4 \rightarrow A_2, A_3, A_4$
2014	2379.815	A_3	$A_3 \rightarrow A_4,$
2015	2774.32	A_4	$A_4 \rightarrow A_2, A_3, A_4$
2016	2874.32	A_4	$A_4 \rightarrow A_2, A_3, A_4$
2017	2256.096	A_2	$A_2 \rightarrow A_2$

- The midpoint of each of the FLRG is given in Table 5.

Table 5: Midpoint of FLRG

FLRG	Midpoint		
$A_1 \rightarrow A_4$	2731.013		
$A_4 \rightarrow A_2, A_3, A_4$	2131.702	2431.358	2731.013
$A_4 \rightarrow A_2, A_3, A_4$	2131.702	2431.358	2731.013
$A_3 \rightarrow A_4$	2731.013		
$A_4 \rightarrow A_2, A_3, A_4$	2131.702	2431.358	2731.013
$A_4 \rightarrow A_2, A_3, A_4$	2131.702	2431.358	2731.013
$A_2 \rightarrow A_2$	2131.702		

- Finally, the forecasted January electricity consumption is the average of the previous year electricity consumption as given in Table 6.

Table 6: Forecast results

Year	Jan	Forecasted
2011	1757.133	
2012	2646.807	2731.013
2013	2855.407	2431.358
2014	2379.815	2431.358
2015	2774.32	2731.013
2016	2874.32	2431.358
2017	2256.096	2431.358
		2131.702

5. Results and Discussions

UTHM electricity consumption patterns versus month for years 2011-2018 is shown in Figure 1. It is noticed that the electricity consumption has increased since the year 2011. The electricity consumption fluctuates for each month. The electricity consumption

in December 2017 is the minimum if compared to the same December month as 3 faculties of UTHM moved to Pagoh since Aug 2017. The year 2015 has the highest electricity consumption of 3869.05MWh, while the year 2011 has the minimum electricity consumption if compared to other years. The electricity consumption for certain months are low than usual may because the month is mostly semester break of UTHM. There are fewer students in the campus and therefore, the electricity consumption will be less if compared to the months that are not semester break.

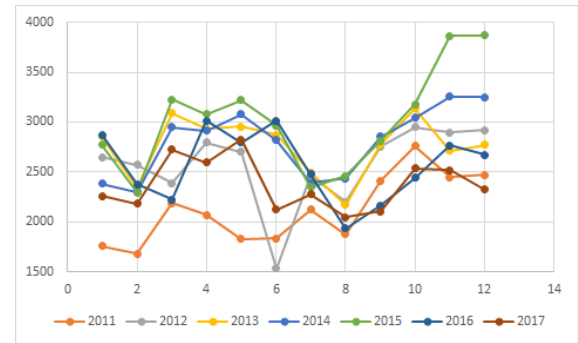


Fig. 1: Actual Load by years

Figure 2 shows the time series of UTHM electricity consumption continuously from Jan 2011-Dec 2017. The electricity consumption is range from 1500 MWh to 4000 MWh. The time series seems is not stationary and is increasing. The electricity consumption in the year 2018 will be forecasted by using six different time series analysis methods.

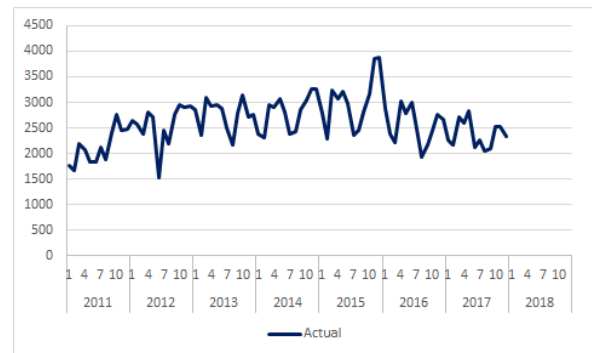


Fig. 2: Actual Electricity Consumption for all years

The actual UTHM electricity consumption (blue colour) from Jan 2011 to December 2017 and forecasted electricity consumption (red colour) using FTS from January 2012 to December 2018 were depicted in Figure 3. FTS uses the data in 2011 to forecast the electricity consumption for 2012 and so on. Hence, the forecasted results start from January 2012. FTS can forecast the pattern of the actual data quite close especially in May 2012 as the absolute percentage error (APE) in that month is the minimum of 0.02%. The highest APE is on June 2017 which is 20.01%.

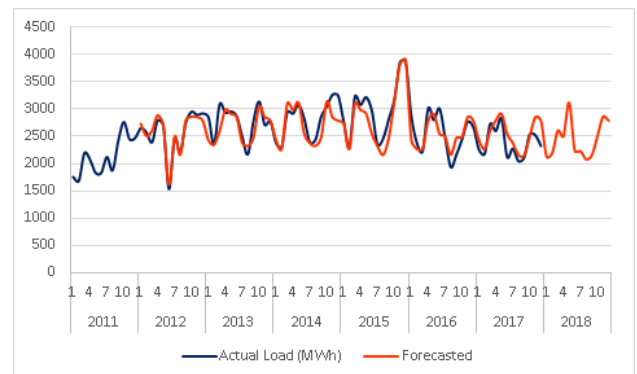


Fig. 3: Actual Electricity Consumption and FTS

Table 7 gives the MAE, MAPE, MSE and RMSE values for FTS method.

Table 7: Errors

	MAE	MAPE(%)	MSE	RMSE
FTS	153.1087	5.74	43543.899	208.671

6. Conclusion

FTS was applied on monthly UTHM electricity consumption from Jan 2011-December 2017 to forecast monthly 2018 UTHM electricity consumption. FTS gives 5.74% of MAPE which is quite low and can consider as high accuracy forecasting tool.

Acknowledgment

We would like to thank the UTHM Research Fund and UTHM Tier 1 2018 research grant vote number H258for financial support of this project.

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