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Research paper

X-Type Tilted Quadrotor Flight Dynamic Modeling

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Abstract

Multirotor UAV such as quadrotor is ideal for photogrammetry application due to its hovering capability and flying at low altitude. It can be used for surveillance, photography, mapping, including Search and Rescue (SAR) operations. The flight of quadrotor requires that each motor produces the propulsion force against the gravitational force. Thrust differential approach is commonly used to control the movement of quadcopter (and also other type of multirotors). Control technique using thrust differential is not ideal to maintain platform attitude. The flight mission almost rely on additional stabilizer to carry a camera for capturing accurate images. Tilted rotor is an alternative control approach suitable for better platform flight attitude, which is horizontal flight. Flight dynamic behaviour of tilted rotor platform shall be understood. This paper shows an effort of development of the quadrotor flight dynamic mathematical model based on thrust differential and rotor tilted angle. X-type tilted configuration is used. Based on three dimensional kinetic analysis, the dynamic mathematical model were derived, and presented.

Keywords: *Flight Dynamics; Photogrammetry; Quadcopter; Tilted Quadrotor*

1. Introduction

At present, Unmanned Aerial Vehicles (UAV) have received a lot of attentions and have become a common selection for commercial, military and scientifically uses. It also can be used for surveillance, photography, mapping, including Search and Rescue (SAR) operations. Multirotor UAV is ideal for photogrammetry application due to its hovering capability and flying at low altitude. Compared to fixed wing UAV, multirotor is able to do Vertical Take-Off and Landing (VTOL) and good in maneuverability. Table 1 shows comparison of advantages and limitations of four of the main UAV types.

A multirotor UAV can perform autonomous flight by obtaining accelerometer and gyrometer information merging with barometer and Global Positional System (GPS) data. Hence, the flight controller recognises the correct orientation and position [1].

Nowadays, multirotors UAV are popular for Radio Control (RC) hobbyists. It can be produced at a moderately low cost due to technology improvement in power supplies (batteries), sensors, and other electronic devices. A common type of multirotor is a quadrotor which use four rotors. Adding extra rotors e.g. hexa or octorotor, will lift heavier payloads and as well gaining a higher degree of redundancy [2].

Quadrotor or quadcopter is commonly used for surveillance, inspection, monitoring, surveying, photography, and photogrammetry which need a stable platform. For example, most aerial photogrammetry is acquired to produce topographic maps [3]. The flight path to take aerial photos comprises of a progression of parallel lines layout to obtain overlapping photographs. Each flight line preferably takes a straight course, however unavoidable cross-winds can deviate the flight path.

For example, Figure 1 shows the flight paths of an aerial mission in cross wind situations which can affect crab and drift. Crab happens when the platform is slightly diverge into a cross-wind to retain a correct heading. In the event that the platform is not turned to recompense, the plane will drift away, the edges of the photos will not be parallel to the flight way.

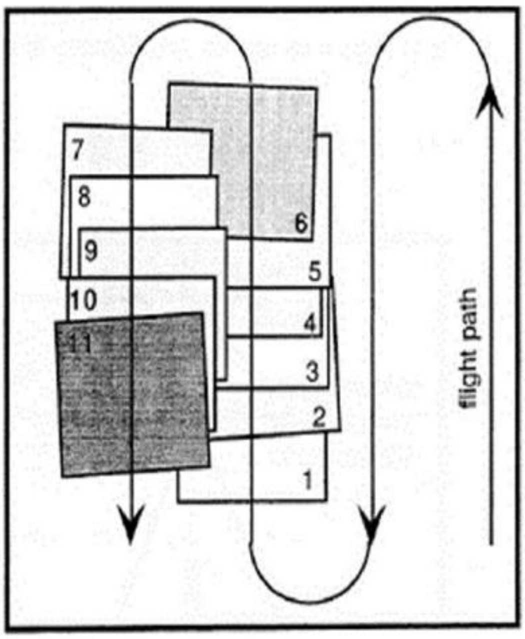

Fig. 1: Flight lines and aerial photogrammetry coverage.

Table 1: Comparison of advantages and limitations of existing flight systems [4]

Aircraft Type	Efficiency and range*	Flexibility and maneuverability	Weather dependency	Payload	Safety	Complexity and simplicity	Costs	Setup time
Airships	Very good	Average	Poor	Very good	Good	Good	Poor	Poor
Fixed wing aircraft	Very good	Poor	Good	Good	Average	Average	Average	Average
Helicopters	Average	Very good	Good	Very good	Poor	Poor	Average	Good
Multicopters	Poor	Very good	Good	Average	Average	Good	Very good	Very good

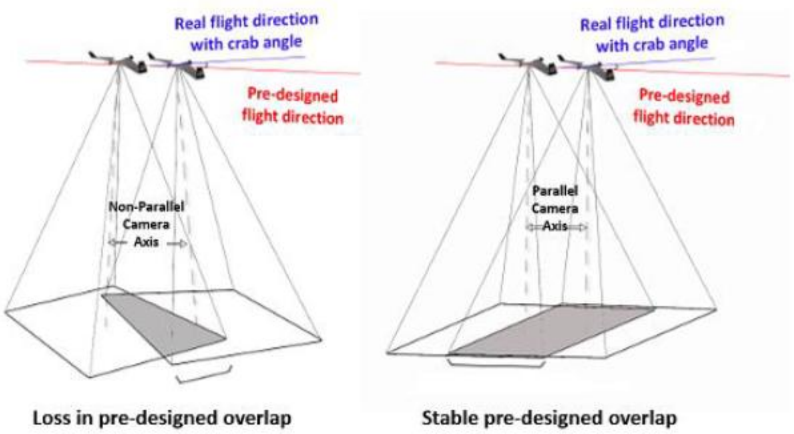

Fig. 2: Comparison between loss in pre-designed overlap and stable pre-designed overlap during capturing images

In the case of producing land surface profile aerial photogrammetry, a UAV is used to capture images at particular locations on its waypoints. These images are then stitched together to form a complete profile. To generate a good images using image stitching, the images are desired to have a certain percentage of overlap between them. For normal photographic flight, 60 percent images overlap is recommended along the flight line and 25 to 30 percent side lap between adjacent flight paths, Figure 2. An unstable UAV may capture image will result discrepancies when the images are stitched together [5]. Figure 2 shows loss overlap and stable overlap during capturing images.

The discussions above show the need of quadrotor UAV platform with a better flight dynamic respond. Rather than relying solely to thrust differential, tilting thrusts can be added. Hence, the objective of this paper is to report development of a quadrotor UAV flight dynamic mathematical model based on thrust differential and rotor tilted angle. In this study, a multirotor with four tilted quadrotor UAV is used. Quadrotor can be cost effective to implement and with an appropriate controller design it can handle small disturbances and remains stable [6], The copters of this type have gained increasing research focus among scientists and engineers [7,8].

2. Tilting of Rotor

In order to achieve successful autonomous mission and dynamic robust response, many control techniques have been applied to the copter. The flight of quadrotor requires that each motor produces the propulsion force against the gravitational to lift the controller upward. The propulsion force will produce trust difference among the 4 motors. The better the performance of quadrotor requires that it should reach in steady state value smoothly without any disturbances or overshoot [9].

The challenge and limitation of thrust differential is the tendency of the UAV platform to be rotating, due to the moment with respect to the Centre of Gravity (C.G.) of the platform. This condition was mentioned by Luukkonen, and also by Hoffmann et al. Luukkonen said the challenge in controlling a quadcopter is that the copter has six degrees of freedom but there are only four control inputs [10]. Hoffmann et al. said current design of typical quadrotor UAV considered only nominal operating conditions for vehicle control design. This work seeks to address issues that arise when deviating significantly from the hover flight regime. Aerodynamic effects due to velocity, angle of attack, and airframe design can cause moments that affect attitude control, and thrust variation that affects altitude control [11].

A quadrotor is naturally unstable, has a complex dynamic model and six degrees of freedom. Even with four motors it is under actuated, and cannot move translative without rotating about one of its axes [12]. It is an inherently unstable system, having similar kinematics to a reversed swing. By maximizing the accuracy of the mathematical model the resultant motion of a quadrotor can be predicted. It can be used to increase the accuracy, stability and time response of the overall system [13].

Control technique using trust differential for stabilizing is not an ideal platform attitude. It is not a proper technique for aerial photogrammetry requirement. The mission almost rely on additional stabilizer to carry a camera for capturing accurate images. Development of an ideal platform with a better flight dynamic respond to improve stability is required.

Figure 3 shows comparison of forward movement of typical quadrotor system (based on thrust differential) and tilted based rotor. During flying, tilted rotor will be an ideal platform attitude which is straight horizontally. If there is any disturbance, the reaction to return to ideal platform attitude is faster due to more degree of freedom. It will go back to original location of way point as soon as possible due to fast response.

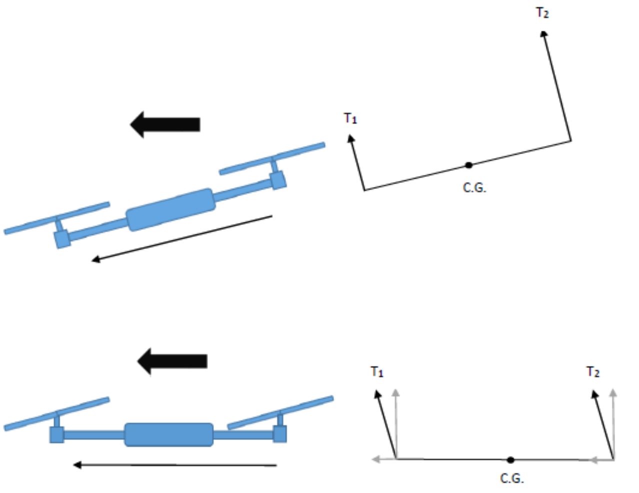

Fig. 3: Comparison of forward movement of typical rotor (top) and tilted rotor (bottom)

Thrust differential will produce the yawing, pitching and rolling motion. By having tilted rotor it is expected that this pitching and rolling of the platform can be alleviated. This tilted rotor requires the development of the flight dynamics equation.

Flight dynamics model equation of motion for existing fix axis quadrotor must be extended to include tilting effect to the flight dynamics. The degree of freedom of the platform will increase and change the flight behaviour.

Also needed is the control technique to ensure the stability during flight. By applying control not only through thrust differential but also tilting the rotor, the tilt quadrotor will be able to have a better stability in flight operation.

3. Tilted Quadrotor Design

The quad-rotor design reference is based on model DJI Flame Wheel 450 V2.2 with 450mm diagonal wheelbase frame [14]. It is an X-type flying configuration with two motors at the front and two motors at the rear. It is controlled by an angular speeds and tilted angles of four electrical motor as shown in Figure 4. M_1 and M_3 motors rotates clockwise while M_2 and M_4 motors rotate counter clock wise. With this configuration, gyroscopic effect and aerodynamics torques will terminate in trimmed flight

The quadrotor can change angular speed of every one of a four rotors and tilted angles its housing to acquire the roll and pitch control torques. Every rotor generates thrust and torque. With combination of pitching and rolling angles, all the rotors will produce the main thrust, the yaw, pitch and roll torque acting on the quadrotor. All the four rotors can have tilting with tilt angle pitch $= (\Theta + B)^{O}$ in Y direction for pitching movement and tilting with tilt angle roll $= (\Phi + C)^{O}$ in X direction for rolling movement.

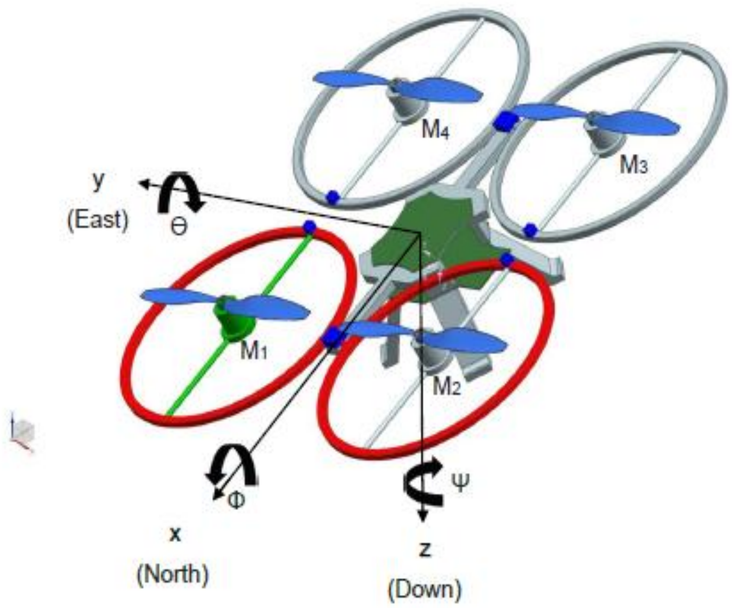

Fig. 4: Platform Design - Isometric View

4. Dynamic Model

The tilted quadrotor flight dynamic mathematical models are based on thrust differential and rotor tilted angle. All the four rotors are assumed to tilt synchronously with same degree in rolling (X) and pitching (Y) axis.

The tilted quadrotor dynamics model is developed by demonstrating the platform as a rigid body in a three dimensional space. The model is subject to the main thrust and three torque which are pitch, roll and yaw. Two fundamental equations of motion for rigid body in three dimension is used for the kinetics analysis.

$$F = ma \quad (4.1)$$

$$M = I\alpha \quad (4.2)$$

The dynamics model equations are created on 3 dimensional free body diagram, where the equations is developed in North-East-Down (NED) view initial and body-fixed coordinates. {N,E,D} represent the initial reference frame and {X,Y,Z} represent the body fix frame. The orientation vector of the aircraft with respect to initial frame, are:

Ψ = Yaw Angle	$\Psi + A$ = Tilted Yaw Angle
Θ = Pitch Angle	$\Theta + B$ = Tilted Pitch Angle
Φ = Roll Angle	$\Phi + C$ = Tilted Roll Angle

The mathematical model equations are developed by making use Figures 5, Figure 6 and Figure 7.

4.1. Right View based Mathematical Model Development

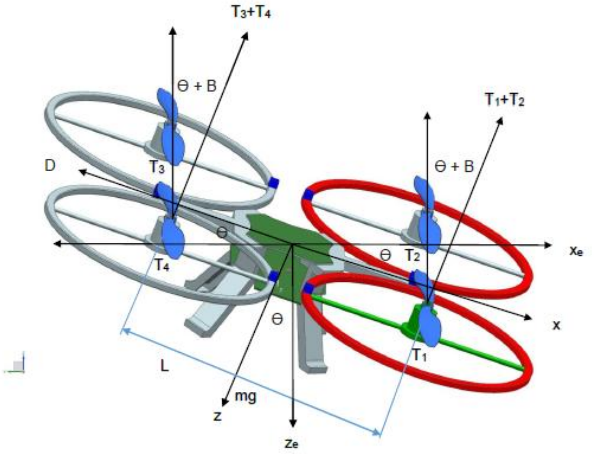

Fig. 5: Right view (from East)

$+ve \quad \sum F_{Xe} = (T_1 + T_2 + T_3 + T_4) \sin(\Theta + B)$
$- mg \sin\Theta - D \cos\Theta$

$= m\, a_x$ (4.3)

$\downarrow\ +ve \quad \sum F_{Ze} = -(T_1 + T_2 + T_3 + T_4) \cos(\Theta + B)$
$+ mg \cos\Theta - D \sin\Theta$

$= m\, a_z$ (4.4)

$\circlearrowleft\ +ve \quad \sum M_{Ye} = (T_1 + T_2) \cos(\Theta + B)$
$\times L/2 (\cos\Theta)$
$- (T_3 + T_4) \cos(\Theta + B)$
$\times L/2 (\cos\Theta)$

$= I_{yy}\, \alpha_y$ (4.5)

4.2. Right View based Mathematical Model Development

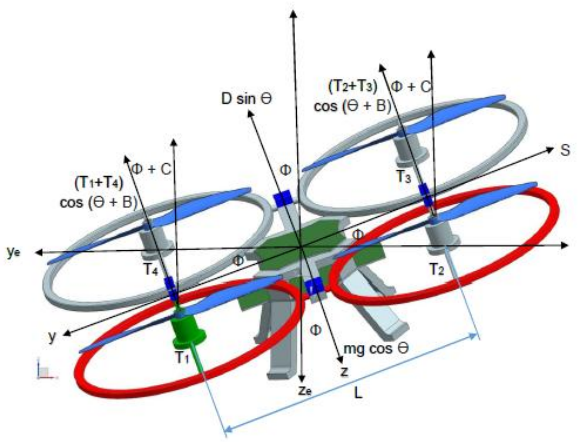

Fig. 6: Front view (from North)

$\leftarrow\ +ve \quad \sum F_{Ye} = (T_1+T_2+T_3+T_4) \cos(\Theta + B)$
$\sin(\Phi + C)$
$+ D \sin\Theta \sin\Phi - mg \cos\Theta \sin\Phi$
$- S \cos\Theta$

$= m\, a_y$ (4.6)

$\downarrow\ +ve \quad \sum F_{Ze} = -(T_1 + T_2 + T_3 + T_4) \cos(\Theta + B) \cos(\Phi + C)$
$- D \sin\Theta \cos\Phi + mg \cos\Theta \cos\Phi \quad - S \sin\Theta$

$= m\, a_z$ (4.7)

$\circlearrowleft\ +ve \quad \sum M_{Xe} = (T_1+T_4) \cos(\Theta + B) \sin(\Phi + C)$
$\times L/2 (\cos\Phi)$
$- (T_2+T_3) \cos(\Theta + B) \sin(\Phi + C)$
$\times L/2 (\cos\Phi)$

$= I_{xx}\, \alpha_x$ (4.8)

4.3. Bottom View based Mathematical Model Development

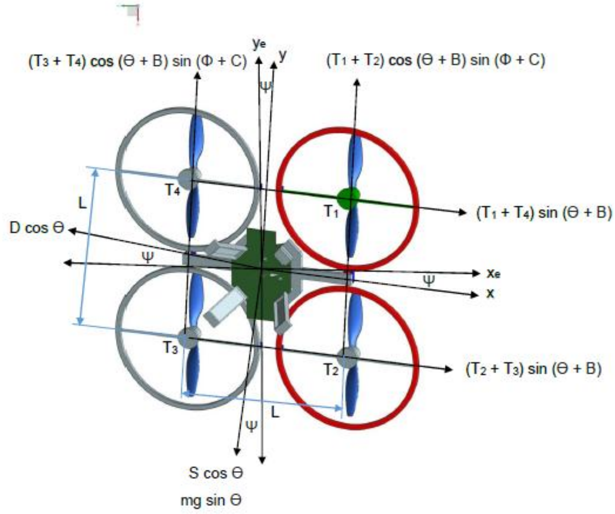

Fig. 7: Bottom view

$\rightarrow\ +ve \quad \sum F_{Xe} = (T_1 + T_2 + T_3 + T_4) \sin(\Theta + B)$
$\cos\Psi$
$+ (T_1 + T_2 + T_3 + T4) \cos(\Theta + B)$
$\sin(\Phi + C) \sin\Psi$
$- mg \sin\Theta \sin\Psi - D \cos\Theta \cos\Psi \quad -$
$S \cos\Theta \sin\Psi$

$= m\, a_x$ (4.9)

$\uparrow\ +ve \quad \sum F_{Ye} = (T_1 + T_2 + T_3 + T_4) \cos(\Theta + B)$
$\sin(\Phi + C) \cos\Psi$
$- (T_1 + T_2 + T_3 + T_4) \sin(\Theta + B)$
$\sin\Psi$
$- mg \sin\Theta \cos\Psi + D \cos\Theta \sin\Psi \quad -$
$S \cos\Theta \cos\Psi$

$= m\, a_y$ (4.10)

$\circlearrowleft\ +ve \quad \sum M_{Ze} = (T_1 + T_2) \cos(\Theta + B) \sin(\Phi + C)$
$\cos\Psi \times L/2 (\cos\Psi)$
$- (T_3 + T_4) \cos(\Theta + B)$
$\sin(\Phi + C) \cos\Psi \times L/2 (\cos\Psi)$
$= (T_1 + T_4) \sin(\Theta + B) \cos\Psi$
$\times L/2 (\cos\Psi)$
$- (T_2 + T_3) \sin(\Theta + B) \cos\Psi$
$\times L/2 (\cos\Psi)$
$= I_{zz}\, \alpha_z$ (4.11)

4.4. Tilted Quadrotor Dynamic Mathematical Model

The X-type tilted quadrotor dynamic model can be represented by:

$mx = (T_1 + T_2 + T_3 + T_4) \sin(\Theta + B) \cos\Psi$
$+ (T_1 + T_2 + T_3 + T4) \cos(\Theta + B) \sin(\Phi + C)$
$\sin\Psi$
$- mg \sin\Theta \sin\Psi - D \cos\Theta \cos\Psi$

$- S \cos\Theta \sin\Psi$ (4.12)

$my = (T_1 + T_2 + T_3 + T_4) \cos(\Theta + B) \sin(\Phi + C) \cos\Psi$
$- (T_1 + T_2 + T_3 + T_4) \sin(\Theta + B) \sin\Psi$
$- mg \sin\Theta \cos\Psi + D \cos\Theta \sin\Psi$

$- S \cos\Theta \cos\Psi$ (4.13)

$mz = - (T_1 + T_2 + T_3 + T_4) \cos(\Theta + B) \cos(\Phi + C)$
$- D \sin\Theta \cos\Phi + mg \cos\Theta \cos\Phi$

$- S \sin\Theta$ (4.14)

$\Psi = \tau_\Psi$
$= (T_1 + T_2) \cos(\Theta + B) \sin(\Phi + C) \cos\Psi$
$\times L/2 (\cos\Psi)$
$- (T_3 + T_4) \cos(\Theta + B) \sin(\Phi + C) \cos\Psi$

$\times L/2 (\cos\Psi)$ (4.15a)

$= (T_1 + T_4) \sin(\Theta + B) \cos\Psi \times L/2 (\cos\Psi)$
$- (T_2 + T_3) \sin(\Theta + B) \cos\Psi$

$\times L/2 (\cos\Psi)$ (4.15b)

$\Theta = \tau_\Theta$
$= (T_1 + T_2) \cos(\Theta + B) \times L/2 (\cos\Theta)$

$- (T_3 + T_4) \cos(\Theta + B) \times L/2 (\cos\Theta)$ (4.16)

$\Phi = \tau_\Phi$
$= (T_1 + T_4) \cos(\Theta + B) \sin(\Phi + C)$
$\times L/2 (\cos\Phi)$
$- (T_2 + T_3) \cos(\Theta + B) \sin(\Phi + C)$

$\times L/2 (\cos\Phi)$ (4.17)

5. Conclusion

X-tilted quadcopter design concept has been developed. Using three-view approach based on three dimensional kinetic analysis, the dynamic mathematical model were derived. These mathematical models will be used further for developing control laws dedicated to attitude stabilisation of X-tilted quadcopter.

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Nomenclature

F	Force
m	Mass
a	Acceleration
M	Moment
I	Moment of Inertia
α	Angular Acceleration
M_1	Motor 1
M_2	Motor 2
M_3	Motor 3
M_4	Motor 4
Ψ	Yaw Angle
Θ	Pitch Angle
Φ	Roll Angle
$\Psi + A$	Tilted Yaw Angle
$\Theta + B$	Tilted Pitch Angle
$\Phi + C$	Tilted Roll Angle
T_1	Thrust 1
T_2	Thrust 2
T_3	Thrust 3
T_4	Thrust 4
Ixx	Moment of Inertia of x axis
Iyy	Moment of Inertia of y axis
Izz	Moment of Inertia of z axis
τ	Torque
g	Gravity
L	Length