

# Wake Modifications in Confined Flows Due to the Presence of a Downstream Cylinder in Staggered Arrangement

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## Abstract

In the present study, confined flows around two square cylinders in staggered arrangement were numerically investigated. Cross-flow and streamwise center-to-center spacings of one- and three-cylinder diameters, respectively, were considered. Simulations were carried out at Reynolds numbers  $Re = 50, 100, 150$  and  $180$ , where the resulting wakes are laminar and periodic. Results indicate that the presence of the downstream cylinder tends to reduce the Strouhal number, amplitude and the time-averaged lift coefficient of the upstream cylinder relative to the single cylinder cases. Furthermore, the time variations of upstream cylinder's lift coefficient behave similar to that of a single cylinder.

**Keywords:** Drag coefficient; Lift coefficient; Square cylinder; Strouhal number; Vortex shedding.

## 1. Introduction

The flow over a bluff body has been the topic of extensive investigation over the years due to its practical application and also because of the interesting fluid dynamic characteristics. Examples of such applications are electronic cooling, design of probes and sensors, vortex flow meters and flow dividers, cooling towers, heat exchange systems and etc. At a certain Reynolds number larger than the critical value, vortices are shed from the bluff body. The primary reason for the formation of these vortices is the frictional shear stress arising within the boundary layer, which denotes a very thin layer in the neighborhood of the body. It has been reported that the frequency of the vortex shedding increases with the increase of Reynolds number [1].

In some cases, the vortex shedding phenomenon can have beneficial effects, while in other cases, it can result in adverse or even detrimental effects. In liquid cooling blanket, for example, cylinders have been used to enhance the heat transport from hot wall to the adjacent cold wall. The oscillating wake flows of a cylinder reduce the thermal boundary layer adjacent to the hot wall, increasing the temperature gradient and thus the heat transfer [2].

Cylinders with a square shape have received special attention amongst researchers since it sharp edges can trigger boundary layer separation [3]–[5]. Bhattacharyya and Maiti [6] reported that the vortex shedding frequency from a single square cylinder near a wall is higher for a shear flow than the uniform flow. Other configurations investigated include flow around two cylinders in tandem, side-by-side and staggered arrangement. The flow past two cylinders in various configurations is much more complex compared to a single cylinder case [7]. A. Etminan, M. Moosavi and N. Ghaedsharafi [8] have reported the characteristics of aerodynamics forces acting on two square cylinders in tandem arrangement and its wake patterns. They found that the wake becomes unsteady at Reynolds number larger than 40. They also found that

the amplitude of lift and drag force fluctuations on the downstream cylinder is larger compared to the upstream cylinder due to the flow interaction between upstream and downstream cylinder. Furthermore, the drag coefficient of the downstream cylinder is always lower than the upstream cylinder.

T.K Prasanth and S. Mittal [7] performed a numerical investigation of freely vibrating two circular cylinders placed in tandem arrangement at low Reynolds number. The lift coefficient signals revealed a similar response between the upstream cylinder and the single cylinder. However, the downstream cylinder experienced relatively larger oscillation amplitude in both transverse and streamwise directions.

Despite its wide applications, few studies have been reported on staggered configuration, particularly in a confined channel with relatively high blockage ratio. In this paper, we investigate the characteristics of wakes produced by two square cylinders in the abovementioned configuration. The study focuses on a laminar regime which produces periodic wakes.

## 2. Methodology

The geometry of the problem under consideration is presented in Fig. 1. The computational domain consists of a long channel with a uniform rectangular cross-section. The origin of the coordinate system of the computational domain is located at the center of the first cylinder and all lengths are non-dimensionalized by the side length of the square cylinder,  $d$ . The width and length of the channel is  $5d$  and  $10d$ , respectively. The inlet boundary is placed  $2.5d$  upstream of the upstream cylinder centerline, while outlet boundary is placed  $7.5d$  downstream of the upstream cylinder centerline. The distances between the centerlines of the two cylinders are  $d$  and  $3d$  in the streamwise and transverse directions, respectively.

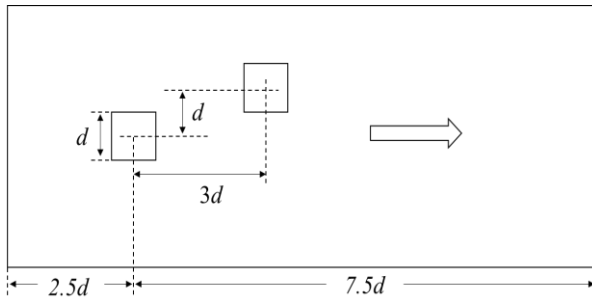


Fig. 1: Schematic diagram of computational domain. The arrow represents the direction of flow.

### 2.1. Solver Validation

Both the continuity and momentum equations were solved using a commercial CFD software FLUENT. The validity of numerical results is established by comparing Strouhal number of a square cylinder wakes with published previous data. The comparisons are presented in Fig. 2.

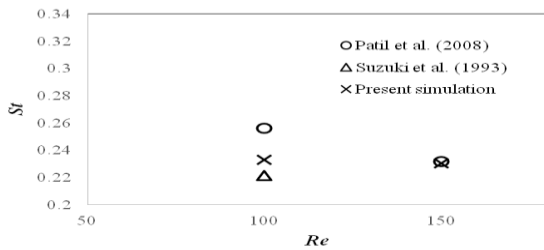


Fig. 2: Strouhal number comparison between present data and previous findings.

The comparisons yielded percentage difference of less than 10% between present data and previous findings for  $Re = 100$ , and approximately 0.2% for  $Re = 150$ . Furthermore, the CFD solver was also validated in previous study by [4].

### 2.2. Grid Independence Study

The quality of the mesh elements used in the present investigation was analyzed by comparing the errors of time-averaged drag coefficients and the time-averaged absolute lift coefficients of both the upstream and downstream cylinders relative to the finest mesh, i.e. mesh M4. A demanding case with  $Re = 180$  was chosen for the analysis. The results are presented in Table 1.

Table 1: Grid convergence of domain for staggered cylinders configuration

Mesh	Nodes	Errors (%)			
		$C_{D,up}$	$C_{D,down}$	$(C_{L,up})_{abs}$	$(C_{L,down})_{abs}$
	33074	0.92	1.14	1.52	3.51
M2	47229	0.32	0.49	0.66	1.27
M3	56464	0.14	0.21	0.12	0.53
M4	66509	0	0	0	0

A rapid convergence was observed when the number of nodes is increased. Mesh M3 achieves at most a 0.53% error while incurring an acceptable computational cost and is therefore has been chosen in the present investigation.

## 3. Results and Discussion

### 3.1. Lift Coefficient

In this section, time histories of lift coefficients for single cylinder and staggered cylinders cases are presented, as shown in Fig. 3. It

should be noted that the signals of lift coefficients presented in Fig. 3 are scaled to a reference value of 1 m for length.

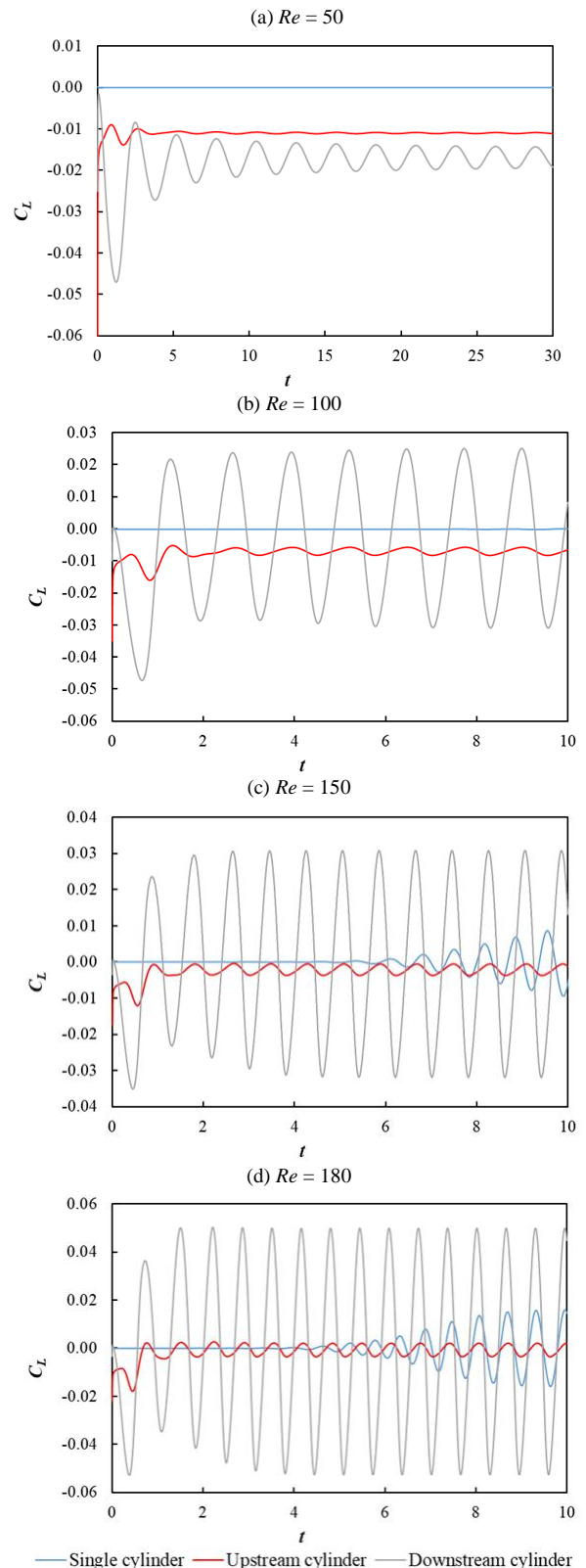
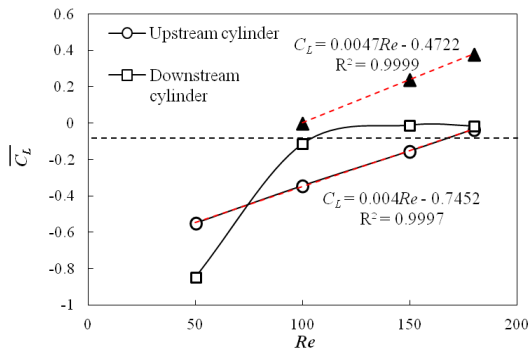


Fig. 3: Lift coefficient time history of square cylinders for  $Re$  as indicated. It should be noted that the length is scaled to a unit meter for these plots.

The lift force induced on the cylinder in a cross flow is developed due to the pressure and viscous forces acting on the body. It is observed from Fig. 3 that the oscillation of lift coefficient signals is sinusoidal in nature for all configurations and Reynolds numbers, indicating laminar periodic wakes downstream of the cylinder.

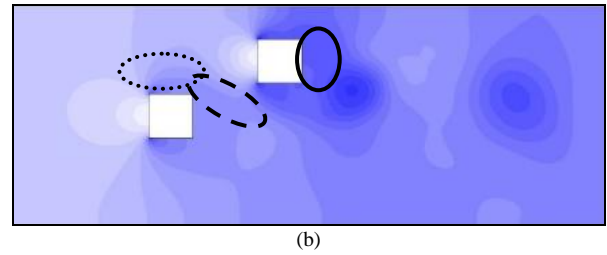
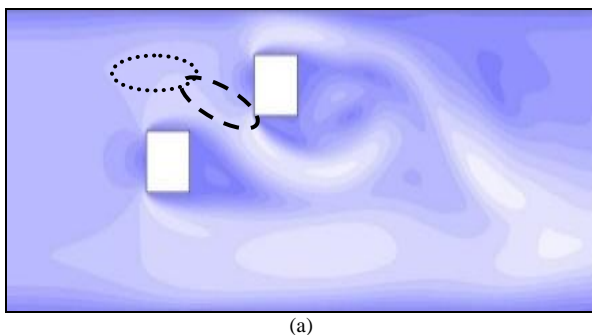
der. It is also observed from Fig. 3 that the amplitude of lift force increases as Reynolds number is increased. Furthermore, for all Reynolds numbers, the presence of the secondary cylinder reduces the amplitude of the lift force on the upstream cylinder relative to the case of a single cylinder, while the amplitude of the lift force on the downstream cylinder is larger than on the single cylinder case. This is because the flow interaction between the wakes of the upstream cylinder and the downstream cylinder causes the magnitude of forces and thus their fluctuations to be amplified.

It is also noted that for single cylinder cases, the lift coefficient fluctuates around zero, indicating symmetrical wakes behind the cylinder. This is expected since the geometry and flow conditions of the problem are symmetrical. However, for staggered cases, the analysis of the time-averaged lift coefficient revealed that the lift coefficients of both cylinders fluctuate around negative values (as indicated in Fig. 4).



**Fig. 4:** Time-averaged lift coefficient for a single cylinder (solid symbols) and staggered cylinder array (open symbols). The time-averaged lift coefficients for single cylinder cases were calculated based on its absolute values. Time-averaged lift coefficients were calculated over a range where the flow has reached a fully-developed periodic state, and that it has been re-scaled to a reference length of the cylinder diameter, i.e. 0.02 m.

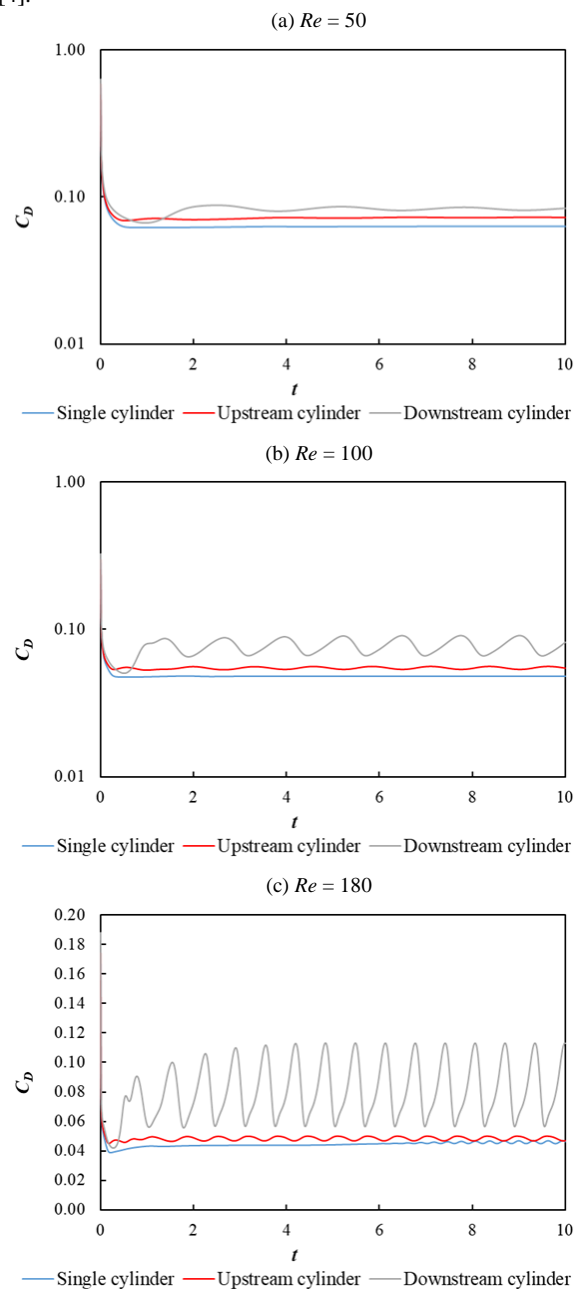
This observation can be explained as follows: the downstream cylinder, which is located above the upstream cylinder, tends to inhibit the flow in the stream direction. Thus, there is a localized low velocity-field region on the top of the upstream cylinder (as indicated by the region enclosed by dotted line in Fig. 5(a)), which leads to a local high-pressure region, and thus resulting a net downforce (i.e. negative lift coefficient)- as indicated by the region enclosed by dotted line in Fig. 5(b)). For the downstream cylinder, the gap between both cylinders accelerates the flow (as indicated by the region enclosed by dashed line in Fig. 5(a)). This leads to a low local pressure region at the bottom of the downstream cylinder (as indicated by the region enclosed by dashed line in Fig. 5(b)), which results in the net negative lift force. However, interestingly at higher Reynolds number, the time-averaged lift coefficient asymptotes to zero. Furthermore, the time-averaged lift coefficient of the upstream cylinder increases almost linearly with Reynolds number. Interestingly, the slope of the linear curve fit of the data is almost similar to the slope of linear curve fit of the lift coefficient magnitude of single cylinder cases data (as evidenced by dashed lines in Fig. 4).



**Fig. 5:** Contour plots of instantaneous (a) velocity magnitude and (b) static pressure of square cylinders in staggered arrangement at  $Re = 100$ . The velocity and pressure contour plots range from -0.2 to 0.2 m/s (blue to white) and from -0.015 to 0.015 Pa (blue to white), respectively.

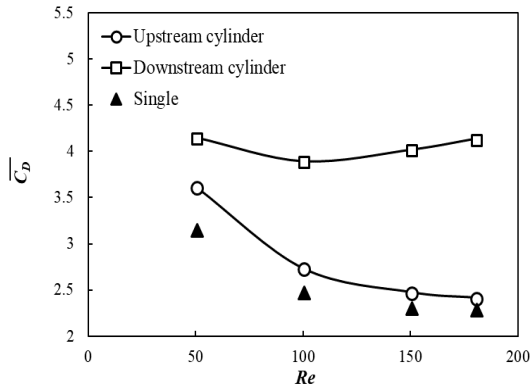
### 3.2. Drag Coefficient

The variations of drag coefficients for various Reynolds number are presented in Fig. 6. It was observed that the time variations of the upstream cylinder's drag coefficient behave similar to that of a single cylinder, which is in agreement with findings reported in [4].



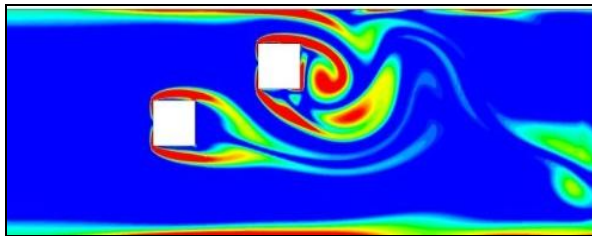
**Fig. 6:** Time history of drag coefficient for single and staggered cylinder cases.

The presence of secondary cylinder does not significantly affect the drag coefficient of the upstream cylinder, although the fluctuation magnitude is slightly larger. However, the time-averaged drag coefficient for the downstream cylinder is much larger compared to the upstream cylinder, as indicated in Figure 7. This is due to the fact that the fluid pressure upstream of the cylinder much higher compared at the downstream of the cylinder (as indicated by the region enclosed by solid line in Figure 5(b)). This observation is in contrast with the case of two square cylinders in tandem arrangement, as reported in [8].



**Fig. 7:** Time-averaged drag coefficient for a single cylinder (solid symbols) and staggered cylinder array (open symbols). Time-averaged drag coefficients were calculated over a range where the flow has reached a fully-developed periodic state, and that it has been re-scaled to a reference length of the cylinder diameter, i.e. 0.02 m.

It is also noted from Fig. 6 that the magnitude of drag coefficient time signal fluctuation of the downstream cylinder is relatively much larger than of the upstream cylinder and single cylinder case, which is in agreement with the case of tandem square cylinders [8]. This is likely due to the strong interaction between the shear layer that is formed near the upper surface of the upstream cylinder and the downstream cylinder (as evidenced in Fig. 8).

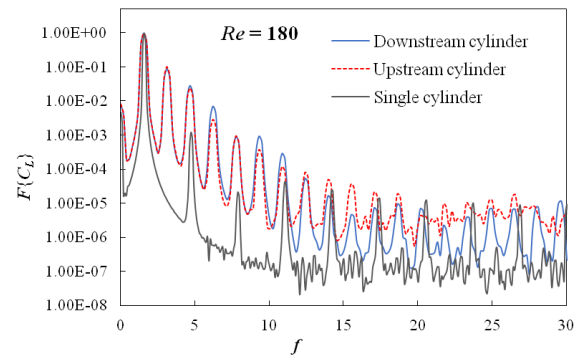


**Fig. 8:** Instantaneous contour of vorticity from  $Re = 180$ . Vorticity contour ranges from  $10 \text{ s}^{-1}$  (blue) to  $40 \text{ s}^{-1}$  (red).

Furthermore, the amplitudes of drag force fluctuation of the downstream cylinder are increased with increasing Reynolds number, also likely due to the increased in the strength of the abovementioned interaction. It is also noted from Fig. 8 that the presence of the downstream cylinder results in the formation of secondary vortices. The secondary vortices were formed due to the separation of a boundary layer adjacent to the upper wall of the channel.

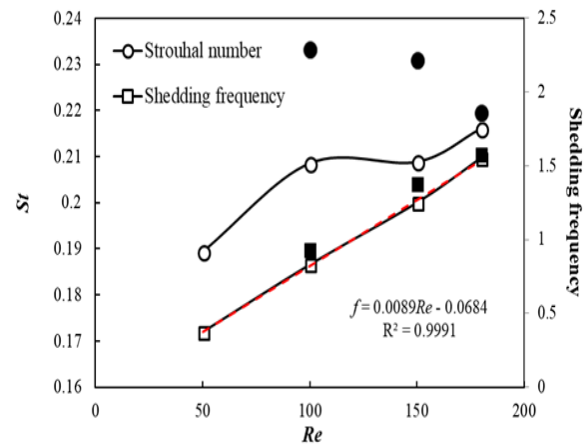
### 3.3. Strouhal Number

Strouhal number,  $St$  is a dimensionless parameter characterizing frequency of oscillation, and is useful for analyzing oscillating unsteady fluid flow dynamics problems. Strouhal number is defined as  $St = f \times d/U$ , where  $f$  is frequency of vortex shedding,  $d$  is cylinder diameter and  $U$  is mean flow velocity. The frequency of vortex shedding is measured from the time evolution data of the cylinder's lift coefficient. In all cases, only one dominant frequency is observed, as evidenced in Fig. 9.



**Fig. 9:** Fourier spectra of the lift coefficient signals in Fig. 3 (d).

Hence, Strouhal numbers are calculated based on the dominant frequency of the lift coefficient signals. The analysis of vortex shedding frequency revealed that the shedding frequency (and thus the Strouhal number) is the same for upstream and downstream cylinders for the staggered arrangement.



**Fig. 10:** Strouhal number (circular symbols on primary axis) and Shedding frequency (square symbols on secondary axis) for both single (solid symbols) and staggered cylinder (open symbols) cases for  $Re$  as indicated.

As can be seen in Fig. 10, the frequency of vortex shedding increases almost linearly for both single cylinder and staggered cylinder cases. The Strouhal number for staggered cylinders shows a similar trend to that of shedding frequency, although the increment is not linear. However, the Strouhal number trend for single cylinder cases is opposite to that of staggered cylinder cases, where Strouhal number decreases with increasing Reynolds number. This observation can be explained as follows: the increment rate of shedding frequency with Reynolds number is lower than that of mean flow velocity, and since Strouhal number is proportional to shedding frequency and inversely proportional to mean flow velocity, the Strouhal number decreased with Reynolds number.

## 4. Conclusion

Confined flows over bluff bodies in staggered arrangement have been studied. Flows with low Reynolds numbers were considered, in order to concentrate on cases with two-dimensional laminar periodic vortex shedding. Results suggested that the presence of a secondary downstream cylinder placed in staggered arrangement significantly alter the lift force characteristics of the upstream cylinder. The Strouhal number and the fluctuation magnitude and the time-averaged values of the upstream cylinder's lift coefficient are reduced due to the presence of the downstream cylinder. However, the drag coefficients and vortex shedding frequency remained almost unaltered.

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