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Research paper

Universal study of Three Product Inventory Model With Production by Two-Unit System and Sales

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Abstract

This paper is dedicated to the analysis of two (s, S) inventory systems. In this paper we study three different products inventory systems when productions of three different products are done alternately. The products are produced by a two-unit system with exponential failure and repair times. The sales time starts when k sets of the three different products are produced or when the two unit system fails. The double Laplace transform of the probability density function of the time to start sales and sales time and their means are obtained. Numerical examples are presented with MATLAB ODE Tool.

Keywords: *Inventory systems; Production models; stochastic models; Two-unit systems.*

1. Introduction

Inventory management deals with physical goods or other products or materials used by a firm for the purpose of production and sale. Many of the usual models for production of products for sales do not consider failure and 1 repair times of machines involved. In this model, two-unit system produces two products for sale. The three products are produced one by one. The sale time starts when k sets of products are produced or when the two-unit system fails whichever occurs first. Assuming the production and sale time of products have general distribution and the failure and repair rates of the two unit are constants the joint distribution function of time to start sales and sales time has been derived. Gaver [1] has studied correlated models assuming two types of shocks that can occur to a device causing major and minor damages. Parvathi [2] have derived general analysis of two product inventory model with production by two unit system and sales. Ramanarayanan [3] has discussed general analysis of 1-out of-2: F system exposed to cumulative damage process. Thangaraj and Ramanarayanan [6] have studied such models with general lead time and general inter-arrival time of demand with two fixed ordering levels of the inventory. Taylor[5] has discussed optimal replacement under additive damage and other failure methods. Usha and Ramanarayanan [7] have derived general analysis of system in which a two-unit system is a sub system.

2. Model Description

2.1 Assumptions

The company three different products A, B and c and at a time only one type is produced. The production time of product A is random variable with cdf $G_X(.)$ the product B has cdf $G_Y(.)$ and that of product C has cdf $G_Z(.)$. Products A, B and C are produced is successive manner. The production time X+Y+Z of a triplet has cdf G (.).

3. The products are produced by a two-unit system which fails when the two-units are down and it works when at least one unit is

good. Let the probability that either of two-unit fails during (t, t+Δ

t) given that the two-unit are operating at time t, be $\lambda_1\Delta t + o(\Delta t)$

and the probability that one fixed unit fails during (t, t+Δt)

Sale time starts when k numbers of triplet of products are produced or when the two-unit system fails.

The products are sold in triplet and the selling time of a triplet is random variable with cdf S(.) and the selling time of product A is $S_A(.)$.

2.2 Analysis

The probability of n number of triplet produced in (0,t) is considered

$= G_n(t) - G_{n+1}(t)$ where $G_n(t)$ is the cdf of $\sum_{i=1}^{n}(X_i + Y_i + Z_i)$

When the nth triplet is produced at time x< t during t-x there are two possibilities. After the nth triplet production,

The production for product A is over but for B is not over

Their respective probabilities are given below. The probability of n number of triplet and one production of A is over before t

$= \int_0^t g_n(x) \int_0^{t-x} g_X(u)\overline{G}_Y(t-x-u)dudx.$ (1)

To find the distribution function of the time to failure of the two-unit system, the function $P_{0,0}(u) = P$, at time u the two-unit system is working is considered. The system does not fail during (0,u) \ at time 0 the two-unit of the system are working . $P_{0,1}(u) = P$ (at time u one unit is under repair, system does not fail during (0,u) \ (at time 0 the units are working). $P_{0,2}(u)$ be the pdf of time to failure of the two-unit system, $P_{0,2}(u) = P$ (the two-unit system fails during (u, u+du), it does not fail during (0,u) \ the two-unit are working at time 0).

The P….. (.) Functions are calculated. $P_{0,0}(x)$ Satisfies the following

$$P_{0,0}(x) = e^{-\alpha_1 x} + \int_0^x \alpha_1 e^{-\alpha_1 u} P_{1,0}(x-u)\,du$$

Where $P_{1,0}(x)$= P (at time x the two-unit are working \ at time 0 one unit is in failed state)

$$P_{1,0}(x) = \int_0^x \beta e^{-\beta u} e^{-\alpha_2 u} e^{-\alpha_1(x-u)}\,du + \int_0^x \int_0^v \beta e^{-\beta v} e^{-\alpha_2 u} e^{-\alpha_1(v-u)}\,du$$

$P_{1,0}(x-v)dv.$

The first term is the probability that the failed unit is repaired at u and no unit of the system fails during (0,x) and the second term is the probability that the failed unit is repaired at u and other unit does not fail during (0,u) , a unit fails at v> u and at x all the two-unit are working.

Laplace transforms of the above equations give

$P^*_{0,0}(S) = (\alpha_2 + \beta + s)[s^2 + s(\alpha_1 + \alpha_2 + \beta) + \alpha_1\alpha_2]$ (2)

Where * indicates Laplace transform.

Using a similar argument

$P_{0,1}(x) = \int_0^x \lambda e^{-\lambda u} e^{-\mu(x-u)} e^{-\lambda_2(x-u)}\,du + \int_0^x \int_0^v \lambda_1 e^{-\lambda_1 u} e^{-\mu(v-u)} e^{-\lambda_2(v-u)}\,du\ P_{0,1}(x-v)dv.$

Taking Laplace transform

$P^*_{0,1}(S) = \lambda_1[s^2 + s(\lambda_1 + \lambda_2 + \mu) + \lambda_1\lambda_2]$ (3)

The failure density $P_{0,2}(x)$ satisfies

$$P_{0,2}(x) = \int_0^x \lambda_1 e^{-\lambda_1 u} P_{1,2}(x-u)du$$

Where $P_{1,2}(x)dx = P$ (the two-unit system fails during $(x, x+dx)$\ at time 0 one unit of the system is under repair)

$P_{1,2}(x) = \lambda_2 e^{-\lambda_2 x} e^{-\mu x} + \int_0^x e^{-\lambda_2 u}\,\mu e^{-\mu u}\,P_{0,2}(x-u)du$

By Laplace transformation.

$P^*_{0,2}(s) = \lambda_1\lambda_2 \backslash [s^2 + s(\lambda_1 + \lambda_2 + \mu) + \lambda_1\lambda_2]$ (4)

Equation can be inverted easily

$$P_{0,0}(t) = (\frac{1}{2})e^{-at}[e^{bt} + e^{-bt}] + (\frac{1}{4b})(\lambda_2 - \lambda_1 + \mu)\,e^{-at}[e^{bt} + e^{-bt}]$$

$$P_{0,1}(t) = (\frac{\lambda_1}{2b})\,e^{-at}[e^{bt} + e^{-bt}]$$

$$P_{0,2}(t) = (\frac{\lambda_1\lambda_2}{2b})\,e^{-at}[e^{bt} + e^{-bt}]$$

Here p= $(\alpha_1 + \alpha_2 + \beta)/2$ and

q= $(\frac{1}{2})\sqrt{(\alpha_1 - \alpha_2)^2 + (\beta^2) + 2\beta(\alpha_1 + \alpha_2)}$

$P_{0,0}(t) + P_{0,1}(t)$ =Survival function of two-unit system

$= e^{-(p-q)t}\,[\frac{1}{2} + \frac{\alpha_2}{4q} - \frac{\alpha_1}{4q} + \frac{\beta}{4q}] + e^{-(p+q)t}\,[\frac{1}{2} - \frac{\alpha_2}{4q} + \frac{\alpha_1}{4q} - \frac{\beta}{4q}]$

$= r_1 e^{-(p-q)t} + r_2\ e^{-(p+q)t}$

$P_{0,2}(t) = \text{pdf of two} - \text{unit system}$

$= (\frac{\alpha_1\alpha_2}{2q})[\,e^{-(p-q)t} - e^{-(p+q)t}]$

$= r_3[\,e^{-(p-q)t} - e^{-(p+q)t}]$

Where $r_1 = [\frac{1}{2} + \frac{\alpha_2}{4q} - \frac{\alpha_1}{4q} + \frac{\beta}{4q}], r_2 = [\frac{1}{2} - \frac{\alpha_2}{4q} + \frac{\alpha_1}{4q} - \frac{\beta}{4q}], r_3 = (\frac{\alpha_1\alpha_2}{2q})$

The probability density function of T is

$$f_T(t) = g_k(t)\,r_1 e^{-(p-q)t} + r_2\ e^{-(p+q)t} + r_3(\,e^{-(p-q)t} - e^{-(p+q)t}) + \sum_{i=0}^{k-1}\int_0^t g_i(x)\int_0^{t-x} g_X(u)\overline{G_Y}(t-x-u)dudx + \sum_{i=0}^{k-1}\int_0^t g_i(x)\int_0^{t-x}\int_0^v g_X(u)g_Y(v-u)\overline{G_z}(t-x-v)dudvdx\Bigg\} \quad (5)$$

The first term of the right side of () is the part of the pdf that the time to produce k number of triplet is t and the two unit system has not failed upto time t. One part of the second term is the part of the pdf that the two unit system fails at time t, the time to produce I number of A and B triplet is x, the (i+1)th A product is produced at time x+u and the production of the (i+1)th B product is not over during (0,t-x-u) for o$\leq$ i $\leq k-1$. The other part of the second term is part of the pdf that the two unit system fails at time t, the time to produce i number of A and B triplets is x and the (i+1)th production of product A is not over during (0,t-x), o$\leq$ i $\leq k-1$.

This gives the joint pdf of time to start sales T and total sales time of triplest S as follows considering the sales time of the triplets.

$$f_{T,S}(x,y) = g_k(x)(r_1 e^{-(p-q)x} + r_2\ e^{-(p+q)x})\,s_k(y) + r_3(\,e^{-(p-q)x} - e^{-(p+q)x})\Bigg\{\sum_{i=0}^{k-1}\int_0^t g_i(u)\overline{G_X}(x-u)s_i(y)du + \sum_{i=0}^{k-1}\int_0^x g_i(u)\int_0^{x-u} g_X(v)\overline{G_Y}(x-u-v)dvdu\int_0^y s_i(p)s_A(y-p)dp + \sum_{i=0}^{k-1}\int_0^x g_i(u)\int_0^{x-u} g_X(v)g_y(w-v)\overline{G_z}(x-u-w)dwdudv \times \int_0^y s_i(p)s_A(y-p)s_B(y-q)dpdq\Bigg\} \quad (6)$$

The double Laplace transform of the pdf is given by

$f_{T,S}{}^{*}(\epsilon,\eta)=\int_0^\infty\int_0^\infty e^{-\varepsilon x}e^{-\eta y}f_{T,S}(x,y)dxdy$

$=\int_0^\infty\int_0^\infty e^{-\varepsilon x}e^{-\eta y}g_k(x)\left(r_1 e^{-(p-q)x}+r_2\ e^{-(p+q)x}\right)s_k(y)dxdy$

$+\int_0^\infty\int_0^\infty e^{-\varepsilon x}e^{-\eta y}r_3\left(e^{-(p-q)x}-e^{-(p+q)x}\right)\sum_{i=0}^{k-1}\int_0^t g_i(u)\overline{G_X}(x-u)s_i(y)dudxdy$

$+\int_0^\infty\int_0^\infty e^{-\varepsilon x}e^{-\eta y}r_3\left(e^{-(p-q)x}-e^{-(p+q)x}\right)\sum_{i=0}^{k-1}\int_0^x g_i(u)\int_0^{x-u}g_X(v)\overline{G_Y}(x-u-v)dvdu\int_0^y s_i(p)s_A(y-p)dp\,dxdy$

$+\int_0^\infty\int_0^\infty e^{-\varepsilon x}e^{-\eta y}\sum_{i=0}^{k-1}\int_0^x g_i(u)\int_0^{x-u}g_X(v)g_y(w-v)\overline{G_z}(x-u-w)dwdudv \times\int_0^y s_i(p)s_A(y-p)s_B(y-q)dpdq\,dxdy$

$f_{T,S}{}^{*}(\epsilon,\eta)=r_1 g^{*k}(\varepsilon+p-q)s^{*k}(\eta)+r_2 g^{*k}(\varepsilon+p+q)s^{*k}(\eta)$

$+r_3\sum_{i=0}^{k-1}g^{*i}(\varepsilon+p-q)\ \overline{G}^{*}{}_X(\varepsilon+p-q)\ s^{*k}(\eta)$

$r_3\sum_{i=0}^{k-1}g^{*i}(\varepsilon+p+q)\ \overline{G}^{*}{}_X(\varepsilon+p+q)\ s^{*k}(\eta)$

$+r_3\sum_{i=0}^{k-1}g^{*i}(\varepsilon+p-q)\,g^{*}{}_x(\varepsilon+p-q)\overline{G_Y{}^{*}}(\varepsilon+p-q)s^{*i}(\eta)s_A{}^{*}(\eta)$

$r_3\sum_{i=0}^{k-1}g^{*i}(\varepsilon+p+q)\ g^{*}{}_x(\varepsilon+p+q)\overline{G_Y{}^{*}}(\varepsilon+p+q)s^{*i}(\eta)s_A{}^{*}(\eta)$

$+r_3\sum_{i=0}^{k-1}g^{*i}(\varepsilon+p-q)\,g^{*}{}_x(\varepsilon+p-q)\,g^{*}{}_Y(\varepsilon+p-q)\overline{G^{*}}{}_Z(\varepsilon+p-q)s^{*i}(\eta)s_A{}^{*}(\eta)s_B{}^{*}(\eta)$

$-r_3\sum_{i=0}^{k-1}g^{*i}(\varepsilon+p+q)\,g^{*}{}_x(\varepsilon+p+q)\,g^{*}{}_Y(\varepsilon+p+q)\overline{G^{*}}{}_Z(\varepsilon+p+q)s^{*i}(\eta)s_A{}^{*}(\eta)s_B{}^{*}(\eta)$

This gives on simplification

$f_{T,S}{}^{*}(\epsilon,\eta)=r_1 g^{*k}(\varepsilon+p-q)s^{*k}(\eta)+r_2 g^{*k}(\varepsilon+p+q)s^{*k}(\eta)+r_3\left[\frac{1-\left(g^{*}(\varepsilon+p-q)s^{*}(\eta)\right)^k}{1-g^{*}(\varepsilon+p-q)s^{*}(\eta)}\right]$

$[\overline{G}^{*}{}_X(\varepsilon+p-q)+\ g^{*}{}_x(\varepsilon+p-q)\overline{G_Y{}^{*}}(\varepsilon+p-q)s_A{}^{*}(\eta)+g^{*}{}_x(\varepsilon+p-q)\,g^{*}{}_Y(\varepsilon+p-q)\overline{G^{*}}{}_Z(\varepsilon+p-q)\ s_A{}^{*}(\eta)s_B{}^{*}(\eta)]$
$-r_3\left[\frac{1-\left(g^{*}(\varepsilon+p+q)s^{*}(\eta)\right)^k}{1-g^{*}(\varepsilon+p+q)s^{*}(\eta)}\right]$

$[\overline{G}^{*}{}_X(\varepsilon+p+q)+\ g^{*}{}_x(\varepsilon+p+q)\overline{G_Y{}^{*}}(\varepsilon+p+q)s_A{}^{*}(\eta)+g^{*}{}_x(\varepsilon+p+q)\,g^{*}{}_Y(\varepsilon+p+q)\overline{G^{*}}{}_Z(\varepsilon+p+q)\ s_A{}^{*}(\eta)s_B{}^{*}(\eta)]$ (7)

The Laplace transform of T is

$f^{*}(\varepsilon,0)=r_1 g^{*k}(\varepsilon+p-q)+r_2 g^{*k}(\varepsilon+p+q)+r_3\left[\frac{1-\left(g^{*}(\varepsilon+p-q)\right)^k}{1-g^{*}(\varepsilon+p-q)}\right]$

$[\overline{G}^{*}{}_X(\varepsilon+p-q)+\ g^{*}{}_x(\varepsilon+p-q)\overline{G_Y{}^{*}}(\varepsilon+p-q)+$

$g^{*}{}_x(\varepsilon+p-q)\,g^{*}{}_Y(\varepsilon+p-q)\overline{G^{*}}{}_Z(\varepsilon+p-q)]\ -r_3\left[\frac{1-\left(g^{*}(\varepsilon+p+q)\right)^k}{1-g^{*}(\varepsilon+p+q)}\right]$

$[\overline{G}^{*}{}_X(\varepsilon+p+q)+\ g^{*}{}_x(\varepsilon+p+q)\overline{G_Y{}^{*}}(\varepsilon+p+q)+$

$g^{*}{}_x(\varepsilon+p+q)\,g^{*}{}_Y(\varepsilon+p+q)\overline{G^{*}}{}_Z(\varepsilon+p+q)]$

On differentiation of equation $\frac{\partial}{\partial\varepsilon}f^{*}(0,0)=-E(T)$

$E(T)=-r_1 g^{*k-1}(p-q)g^{*\prime}(p-q)-r_2 g^{*k-1}(p+q)g^{*\prime}(p+q)+r_3\left[\frac{k\,g^{*k-1}(p-q)g^{*\prime}(p-q)}{1-g^{*}(p-q)}\right]$

$[\overline{G}^{*}{}_X(p-q)+\ g^{*}{}_x(p-q)\overline{G_Y{}^{*}}(p-q)+\ g^{*}{}_x(p-q)\,g^{*}{}_Y(p-q)\overline{G^{*}}{}_Z(p-q)]$

$-r_3\left[\frac{1-\left(g^{*}(p-q)\right)^k}{\left(1-g^{*}(p-q)\right)^2}\right]g^{*\prime}(p-q)[\overline{G}^{*}{}_X(p-q)+\ g^{*}{}_x(p-q)\overline{G_Y{}^{*}}(p-q)$

$+\ g^{*}{}_x(p-q)\,g^{*}{}_Y(p-q)\overline{G^{*}}{}_Z(p-q)]$

$-r_3\left[\frac{1-\left(g^{*}(p-q)\right)^k}{1-g^{*}(p-q)}\right]$

$[\{\overline{G}^{*\prime}{}_X(p-q)+g^{*\prime}(p-q)\overline{G}^{*}{}_Y(p-q)+g^{*}{}_X(p-q)\overline{G}^{*\prime}{}_Y(p-q)g^{*}{}_Y(p-q)\overline{G}^{*}{}_Z(p-q)+\ g^{*}{}_X(p-q)\overline{G}^{*\prime}{}_Y(p-q)\overline{G}^{*}{}_Z(p-q)+g^{*}{}_X(p-q)g^{*}{}_Y(p-q)\overline{G}^{*\prime}{}_Z(p-q)\}$

$-r_3\left[\frac{k\,g^{*k-1}(p+q)g^{*\prime}(p+q)}{1-g^{*}(p+q)}\right]$

$[\overline{G}^{*}{}_X(p+q)+\ g^{*}{}_x(p+q)\overline{G_Y{}^{*}}(p+q)+\ g^{*}{}_x(p+q)\,g^{*}{}_Y(p+q)\overline{G^{*}}{}_Z(p+q)]$

$+r_3\left[\frac{1-\left(g^{*}(p+q)\right)^k}{\left(1-g^{*}(p+q)\right)^2}\right]g^{*\prime}(p+q)[\overline{G}^{*}{}_X(p+q)+\ g^{*}{}_x(p+q)\overline{G_Y{}^{*}}(p+q)+\ g^{*}{}_x(p+q)\,g^{*}{}_Y(p+q)\overline{G^{*}}{}_Z(p+q)]$

$+r_3\left[\frac{1-\left(g^{*}(p+q)\right)^k}{1-g^{*}(p+q)}\right][\{\overline{G}^{*\prime}{}_X(p+q)+g^{*\prime}(p+q)\overline{G}^{*}{}_Y(p+q)+g^{*}{}_X(p+q)\overline{G}^{*\prime}{}_Y(p+q)g^{*}{}_Y(p+q)\overline{G}^{*}{}_Z(p+q)+\ g^{*}{}_X(p+q)\overline{G}^{*\prime}{}_Y(p+q)\overline{G}^{*}{}_Z(p+q)+g^{*}{}_X(p+q)g^{*}{}_Y(p+q)\overline{G}^{*\prime}{}_Z(p+q)\}]$ (8)

Similarly $\frac{\partial}{\partial\eta}f^{*}(0,0)=-E(S)$

$E(S)=r_1\,g^{*k}(p-q)kE(s_1)+r_2 g^{*k}(p+q)kE(s_1)-r_3 kE(s_1)\left[\frac{g^{*k}(p-q)}{1-g^{*}(p-q)}\right]$

$[\overline{G}^{*}{}_X(p-q)+\ g^{*}{}_x(p-q)\overline{G_Y{}^{*}}(p-q)+\ g^{*}{}_x(p-q)\,g^{*}{}_Y(p-q)\overline{G^{*}}{}_Z(p-q)]$

$+r_3\left[\frac{1-g^{*k}(p-q)}{\left(1-g^{*}(p-q)\right)^2}\right]g^{*\prime}(p-q)\,[\,\overline{G}^{*}{}_X(p-q)+\ g^{*}{}_x(p-q)\overline{G_Y{}^{*}}(p-q)$

$+\ g^{*}{}_x(p-q)\,g^{*}{}_Y(p-q)\overline{G^{*}}{}_Z(p-q)]$

$+\ r_3\left[\frac{1-g^{*k}(p-q)}{1-g^{*}(p-q)}\right]\ \{g^{*}{}_X(p-q)g^{*}{}_Y(p-q)\overline{G}^{*}{}_Z(p-q)\left(E(S_A)\right)+g^{*}{}_X(p-q)g^{*}{}_Y(p-q)\overline{G}^{*}{}_Z(p-q)\left(E(S_B)\right)\}\ +\ r_3 kE(s_1)\left[\frac{g^{*k}(p+q)}{1-g^{*}(p+q)}\right]$

$[\ \overline{G}^{*}{}_X(p+q)+\ g^{*}{}_x(p+q)\overline{G_Y{}^{*}}(p+q)+\ g^{*}{}_x(p+q)\ g^{*}{}_Y(p+q)\overline{G^{*}}{}_Z(p+q)]$

$+r_3\left[\frac{1-g^{*k}(p+q)}{\left(1-g^{*}(p+q)\right)^2}\right]g^{*\prime}(p+q)\,[\,\overline{G}^{*}{}_X(p+q)+\ g^{*}{}_x(p+q)\overline{G_Y{}^{*}}(p+q)$

$+\ g^{*}{}_x(p+q)\,g^{*}{}_Y(p+q)\overline{G^{*}}{}_Z(p+q)]$

$$+ \; r_3\left[\frac{1-g^{*k}(p+q)}{1-g^{*}(p+q)}\right] \; \{g^*{}_X(p+q)g^*{}_Y(p+q)\bar{G}^*{}_Z(p+q)\big(E(S_A)\big) + g^*{}_X(p+q)g^*{}_Y(p+q)\bar{G}^*{}_Z(p+q)\big(E(S_B)\big)\} \quad (9)$$

Considering the special case in which X and Y are exponential random variables with parameters λ ,μ and ν .

$$g^*{}_X(p-q) = \frac{\lambda}{\lambda+p-q};\; g^{*\prime}{}_X(\lambda) = \frac{-\lambda}{(\lambda+p-q)^2}$$

$$g^*{}_Y(p-q) = \frac{\mu}{\mu+p-q};\; g^{*\prime}{}_Y(p-q) = \frac{-\mu}{(\mu+p-q)^2}$$

$$g^*{}_Z(p-q) = \frac{\nu}{\nu+p-q};\; g^{*\prime}{}_Z(p-q) = \frac{-\nu}{(\nu+p-q)^2}$$

$$\bar{G}^*{}_X(p-q) = \frac{1}{\lambda+p-q};\; \bar{G}^{*\prime}{}_X(p-q) = \frac{-1}{(\lambda+p-q)^2}$$

$$\bar{G}^*{}_Y(p-q) = \frac{1}{\mu+p-q};\; \bar{G}^{*\prime}{}_Y(p-q) = \frac{-1}{(\mu+p-q)^2}$$

$$\bar{G}^*{}_Z(p-q) = \frac{1}{\nu+p-q};\; \bar{G}^{*\prime}{}_Z(p-q) = \frac{-1}{(\nu+p-q)^2} \quad (10)$$

$$E(T) = \left[1-\left(\frac{\lambda\mu\nu}{(\lambda+p-q)(\mu+p-q)(\nu+p-q)}\right)^k\right]\left(\frac{p+q}{(p-q)2q}\right) + k\left(\frac{\lambda\mu\nu}{(\lambda+p-q)(\mu+p-q)(\nu+p-q)}\right)^k \left(\frac{3p^2+p(2\lambda+2\mu+2\nu)+(\lambda\mu+\mu\nu+\nu\lambda)}{(\lambda+p-q)(\mu+p-q)(\nu+p-q)}\right)\left(\frac{-\alpha_1}{2q}\right)$$
$$+ \left[1-\left(\frac{\lambda\mu\nu}{(\lambda+p+q)(\mu+p+q)(\nu+p+q)}\right)^k\right]\left(\frac{p-q}{(p+q)2q}\right) + k\left(\frac{\lambda\mu\nu}{(\lambda+p+q)(\mu+p+q)(\nu+p+q)}\right)^k \left(\frac{3p^2+p(2\lambda+2\mu+2\nu)+(\lambda\mu+\mu\nu+\nu\lambda)}{(\lambda+p+q)(\mu+p+q)(\nu+p+q)}\right)\left(\frac{\alpha_1}{2q}\right) \quad (11)$$

And

$$E(S) = \left[1-\left(\frac{\lambda\mu\nu}{(\lambda+p-q)(\mu+p-q)(\nu+p-q)}\right)^k\right] \frac{\lambda\mu\nu E(S)+(p-q)[(\lambda\mu+\lambda\nu+\lambda(p-q))E(S_A)+\lambda\mu E(S_B)]}{p-q[(p-q)^2+(p-q)(\lambda+\mu+\nu)+(\lambda\mu+\mu\nu+\nu\lambda)]}$$
$$+k\left(\frac{\lambda\mu\nu}{(\lambda+p-q)(\mu+p-q)(\nu+p-q)}\right)^k E(S)\left(\frac{-\alpha_1}{2q}\right)$$
$$+\left[1-\left(\frac{\lambda\mu\nu}{(\lambda+p+q)(\mu+p+q)(\nu+p+q)}\right)^k\right] \frac{\lambda\mu\nu E(S)+(p+q)[(\lambda\mu+\lambda\nu+\lambda(p+q))E(S_A)+\lambda\mu E(S_B)]}{p-q[(p+q)^2+(p+q)(\lambda+\mu+\nu)+(\lambda\mu+\mu\nu+\nu\lambda)]}$$
$$+ k\left(\frac{\lambda\mu\nu}{(\lambda+p+q)(\mu+p+q)(\nu+p+q)}\right)^k E(S)\left(\frac{\alpha_1}{2q}\right) \quad (12)$$

3. Numerical Examples

This To illustrate the applications of the above result different values for $\alpha_1, \alpha_2, \beta$ and k are taken E (T), E(S) are the following table for, E(S)=5, $E(S_A)$=10, $E(S_B) = 15$.

Using formula (6.3.11) and (6.3.12), taking the values p=6,q=4.3875, k=1 to 8, $\lambda = 1, \mu = 2, \nu = 3$. E(T) and E(S) are found and the corresponding graphs are drawn.

3.1 Numerical tabulation for obtaining of E (T) and E(S) values for

Different values of k

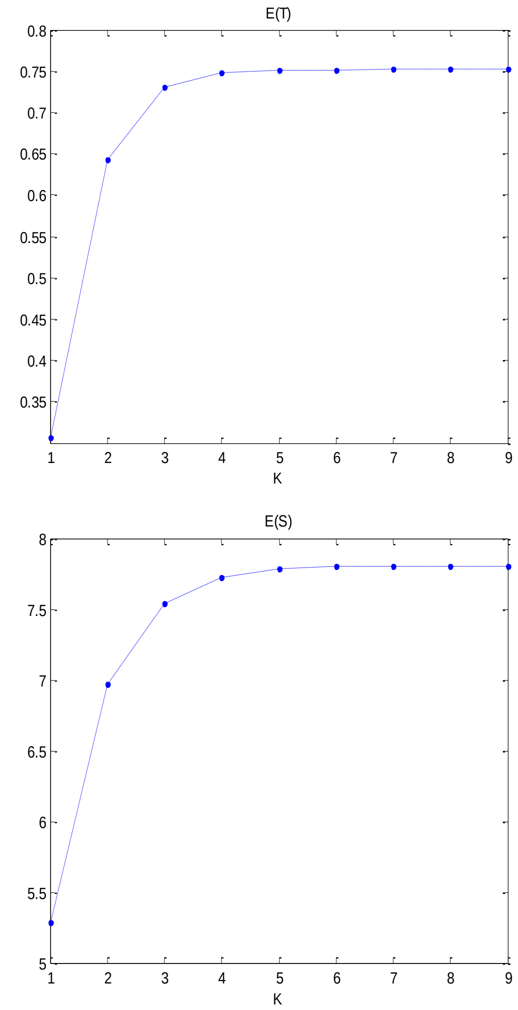


Fig.1: Graphs of E (T) and E(S) for table 3.1

3.1 Inference

As k increases from 1 to 8, E (T) increases and E(S) increases.

Taking p=6, q=4.3875, λ =1 to 6 and vary k=1 to 6,

Table 1: Numerical tabulation for obtaining of E (T) values

k	p	q	E(T)	E(S)
1	6	4.3875	0.3062	5.289
2	6	4.3875	0.6429	6.9672
3	6	4.3875	0.73025	7.5421
4	6	4.3875	0.74793	7.7256
5	6	4.3875	0.75114	7.7819
6	6	4.3875	0.75169	7.7987
7	6	4.3875	0.75178	7.8036
8	6	4.3875	0.7518	7.805

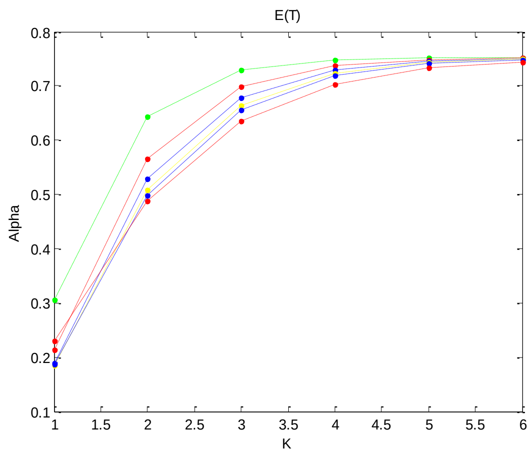
Table 2: Numerical tabulation for obtaining of E (T) values

	E(T)					
k	λ =1	λ =2	λ =3	λ =4	λ =5	λ =6
1	0.30609	0.21309	0.18945	0.18534	0.18764	0.22923
2	0.64285	0.5661	0.52851	0.50921	0.49876	0.48793
3	0.73023	0.69918	0.67807	0.66461	0.65586	0.63515
4	0.74793	0.7382	0.72951	0.72303	0.71831	0.70372
5	0.75114	0.74847	0.7454	0.74277	0.74068	0.73283
6	0.75169	0.75101	0.75003	0.74906	0.74823	0.74453

Table 3: Numerical tabulation for obtaining of E (S) values

	E(S)					
k	λ = 4	λ = 5	λ = 6	λ = 7	λ = 8	λ = 9
1	5.289	5.9182	6.5153	7.0984	7.677	8.2557
2	6.9672	7.7633	8.4902	9.1782	9.844	10.4972
3	7.5421	8.4241	9.2192	9.9623	10.6736	11.365

4	7.7256	8.6461	9.4729	10.2426	10.9762	11.6866
5	7.7819	8.7179	9.5582	10.3394	11.083	11.802
6	7.7987	8.7405	9.5861	10.372	11.1198	11.8425



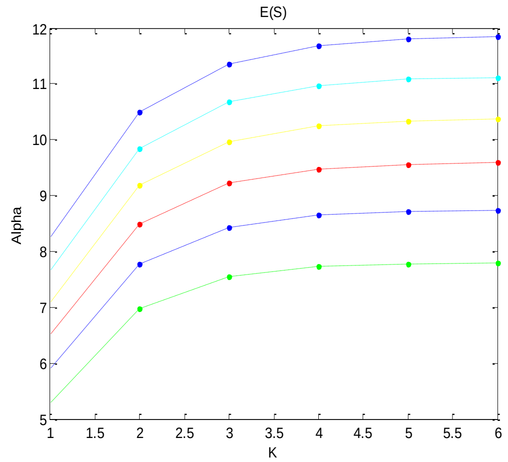


Fig. 2: Graphs of E (T) and E(S) for table 3.2 and 3.3

3.2 Inference

As k increases and the parameter λ is fixed, the expected values of E (T) and E(S) increase.

As k increases and the parameter λ increases, the expected values of E (T) and E(S) decrease.

4. Conclusion

As increases from 1 to 6, E (T) increases and E(S) increases.

(Seen in the Table 3.1)

As k increases and the parameter λ is fixed, the expected values of time to sales E (T) and sales time E(S) increase. As k is fixed and

the parameter λ increases, the expected values of time to sales E (T) decreases and sales time E(S) increases.(Seen in Table 3.2 and 3.3)

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