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Research paper

Fast Algorithm for Selective Image Segmentation Model

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Abstract

Selective image segmentation model aims to separate a specific object from its surroundings. To solve the model, the common practice to deal with its non-differentiable term is to approximate the original functional. While this approach yields to successful segmentation result, however the segmentation process can be slow. In this paper, we showed how to solve the model without approximation using Chambolle's projection algorithm. Numerical tests show that good visual quality of segmentation is obtained in a fast-computational time.

Keywords: *Active Contour/Snake; Computational mathematics; Fast Algorithm; Image processing; Selective Segmentation.*

1. Introduction

Image segmentation is a fundamental task in image processing that try to extract objects from their surroundings. There are many image segmentation models in literature that aim to segment all objects in a given image. These particular models are referred to global image segmentation. The most celebrated global segmentation model is Chan-Vese model [1]. A new modification of Chan-Vese model [1] has been proposed by [2] which apply the Euler's elastica in the functional that make the modified model has data and shape driven properties.

To segment a specific object in an image, a convex selective segmentation model has been successfully developed in [3] that is used to segment an object in artificial geometrical image and an organ in CT scan image as well. The convex formulation of the model allows solution (global minimizer) to be found independently of initial guess.

In [4], we modified the selective model of [3], so that the new modified model called SC2 model is less sensitive to regularization and area parameters. We developed and solved the SC2 model using a fast algorithm called multilevel algorithm in [4]. This algorithm is a modification of the original multilevel algorithm [5] used to solve some image restoration problem such as image denoising and deblurring problems in [5-7].

To avoid singularity problem in solving SC2 model, we take the approximation of SC2 model by regularizing the non-differentiable term so that the functional is differentiable. The multilevel algorithm used to solve the approximated version of SC2 model produces a good quality of segmentation in an optimum computational time. There is another class of fast solver that can be used directly to solve convex optimization problem without approximation called Chambolle's projection algorithm (CPA) [8]. In this paper, we will apply the CPA to solve SC2 model. We are not aware of similar work that apply CPA to solve any selective segmentation model.

The rest of the paper is organized in the following way. In Section 2, we will review the SC2 model. In Section 3, we will show how to apply CPA to solve the SC2 model. The result is reported in Section 4 before concluding in Section 5.

2. SC2 Model

On 2-Dimensional image $z(x,y)$ of domain Ω, assume the availability of $n_1(\geq 3)$ points inside the target object that form a set $A=\left\{w_i=\left(x_i^*,y_i^*\right)\in\Omega, 1\leq i\leq n_1\right\}$ that defines a polygon P. The initial contour can start from P and the contour is penalized from moving further away from the polygon P. The function $P_d\left(x,y\right)$ is the Euclidean distance of each point $\left(x,y\right)\in\Omega$ from its nearest point in the polygon P made up of $\left(x_p,y_p\right)\in P$, constructed from the user input set, A: $P_d\left(x,y\right)=\sqrt{\left(x-x_p\right)^2+\left(y-y_p\right)^2}$. Then, the SC2 model is given as

$$\min_{u,w\in[0,1]} J\left(u,w\right)=\int_\Omega\left|\nabla u\right|_g d\Omega+\int_\Omega rw\, d\Omega +\theta\int_\Omega P_d w\, d\Omega+\frac{1}{2\rho}\int_\Omega\left(u-w\right)^2 d\Omega \tag{1}$$

Here, w is the dual variable, the right-most term enforces $w\approx u$ for sufficiently small $\rho>0$. The term $r=\left(c_1-z\right)^2-\left(c_2-z\right)^2$ is the fitting term where the intensity values of c_1 and c_2 are defined as $c_1(u)=\dfrac{\int_\Omega uz\, d\Omega}{\int_\Omega u\, d\Omega}$, $c_2(u)=\dfrac{\int_\Omega\left(1-u\right)z\, d\Omega}{\int_\Omega\left(1-u\right) d\Omega}$. The additional fitting term P_d is weighted with parameter θ. If the parameter θ is too strong, the final result will just be the polygon P which is undesirable. The weighted regularization term with edge detection functional is defined as

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$|\nabla u|_g = g(|\nabla z|)|\nabla u| = \dfrac{|\nabla u|}{1+\gamma|\nabla z(x,y)|^2}$. The edge detector function helps to stop the evolving curve on the edge of the objects in an image. The strength of detection is adjusted by a parameter γ.

The minimizer of J is computed by minimizing J with respect to u and w separately, and to iterate until convergence. Thus, the following minimization problems are considered:

Firstly, w being fixed, minimize the following functional J_1 with respect to u:

$$\min_u J_1(u,w) = \int_\Omega |\nabla u|_g \, d\Omega + \frac{1}{2\rho}\int_\Omega (u-w)^2 \, d\Omega \tag{2}$$

Secondly, u being fixed, minimize the following functional J_2 with respect to w:

$$\min_{w\in[0,1]} J_2(u,w) = \int_\Omega rw \, d\Omega + \theta\int_\Omega P_d w \, d\Omega + \frac{1}{2\rho}\int_\Omega (u-w)^2 \, d\Omega \tag{3}$$

The solution of (3) is given as $w = \min\left\{\max\left\{u(x) - \rho r - \rho\theta P_d, 0\right\}, 1\right\}$ [4]. In [4], we proposed the optimization multilevel algorithm to solve (2). Note that, (2) is non-differentiable where some solver may have difficulty to deal with singularity problem. To avoid such problem for our multilevel algorithm in [4], we proposed to solve the approximation of (2) that is

$$\min_u J_1(u,w) = \int_\Omega g(|\nabla z|)\sqrt{|\nabla u|^2 + \beta} \, d\Omega + \frac{1}{2\rho}\int_\Omega (u-w)^2 \, d\Omega \tag{4}$$

Here, the additional parameter β is introduced to ensure (4) is differentiable and the value of β chosen should be positive and small (approach to 0). This means that, we are solving

$$\begin{aligned}\min_{u,w\in[0,1]} J(u,w) = &\int_\Omega g(|\nabla z|)\sqrt{|\nabla u|^2 + \beta} \, d\Omega + \int_\Omega rw \, d\Omega \\ &+ \theta\int_\Omega P_d w \, d\Omega + \frac{1}{2\rho}\int_\Omega (u-w)^2 \, d\Omega\end{aligned} \tag{5}$$

instead of (1).

While the multilevel algorithm is successfully used to solve (5) in [4], there is another option to solve (1) directly in a fast way using CPA [8]. To the best our knowledge, we are not aware of similar work that apply CPA to solve our SC2 model in [4] or any selective segmentation model.

3. Solution of SC2 Model Using CPA

The CPA is introduced by [8] to solve image denoising problem and it is considered as powerful and fast method. There is application of CPA in global segmentation model as reported in [9]. To apply the CPA, first we define the gradient and divergence operator as follows:

The gradient operator:

$$(\nabla u)_{i,j} = \left((\nabla u)^1_{i,j}, (\nabla u)^2_{i,j}\right)$$

$$(\nabla u)^1_{i,j} = \begin{cases} u_{i+1,j} - u_{i,j} & \text{if } i<N \\ 0 & \text{if } i=N \end{cases}$$

$$(\nabla u)^2_{i,j} = \begin{cases} u_{i,j+1} - u_{i,j} & \text{if } j<N \\ 0 & \text{if } j=N \end{cases}$$

The divergence operator:

$$(\nabla\cdot p)_{i,j} = \begin{cases} p^1_{i,j} - p^1_{i-1,j} & \text{if } 1<i<N \\ p^1_{i,j} & \text{if } i=1 \\ -p^1_{i-1,j} & \text{if } i=N \end{cases} + \begin{cases} p^2_{i,j} - p^2_{i,j-1} & \text{if } 1<j<N \\ p^2_{i,j} & \text{if } j=1 \\ -p^2_{i,j-1} & \text{if } j=N \end{cases}$$

We proceed exactly as in [8, 9]. As shown in [8, 9], equation (2) can be written with the dual variable $p = (p_1, p_2)$:

$$\min_u \max_{|p|\le g} \int_\Omega u\nabla\cdot p + \frac{1}{2\rho}(u-w)^2 dx \tag{6}$$

One can now switch the min and max to obtain the equivalent

$$\max_{|p|\le g} \min_u \int_\Omega u\nabla\cdot p + \frac{1}{2\rho}(u-w)^2 dx \tag{7}$$

The inner minimization in (7) is point-wise in u. This gives:

$$\nabla\cdot p + \frac{1}{\rho}(u-w) = 0 \Rightarrow u = w - \rho\nabla\cdot p \tag{8}$$

Substituting (8) for minimal into problem (7) gives

$$\max_{|p|\le g} \int_\Omega (w-\rho\nabla\cdot p)\nabla\cdot p + \frac{\rho}{2}(\nabla\cdot p)^2 dx \tag{9}$$

After simplification, we arrived to

$$\max_{|p|\le g} \int_\Omega w\nabla\cdot p - \frac{\rho}{2}(\nabla\cdot p)^2 dx \tag{10}$$

Variation of energy (10) with respect to the vector field p give:

$$\int_\Omega \left(-\nabla w + \rho\nabla(\nabla\cdot p)\right)\cdot\delta p \, dx \tag{11}$$

Along with the point-wise constraint $|p|^2 - g^2 \le 0$, the optimality condition is given as:

$$-\nabla(\rho\nabla\cdot p - w) + \psi(x)p = 0 \tag{12}$$

Here, the Lagrange multiplier $\psi(x)\ge 0$ for all x. As Chambolle shows in [8], it can be determined and eliminated as follows: If the constraint is not active at a point x, that is if $|p(x)|^2 < g^2(x)$, then $\psi(x)=0$. Otherwise, if the constraint is active at a point x, that is if $|p(x)|^2 = g^2(x)$, then

$$\left|\nabla\left(\rho\nabla\cdot p-w\right)\right|^{2}-\psi^{2}g^{2}\left(x\right)=0 \tag{13}$$

In either case, the value of $\psi(\mathbf{x})$ is given by:

$$\psi=\frac{1}{g(x)}\left|\nabla\left(\rho\nabla\cdot p-w\right)\right| \tag{14}$$

Substituting (14) into (12) gives:

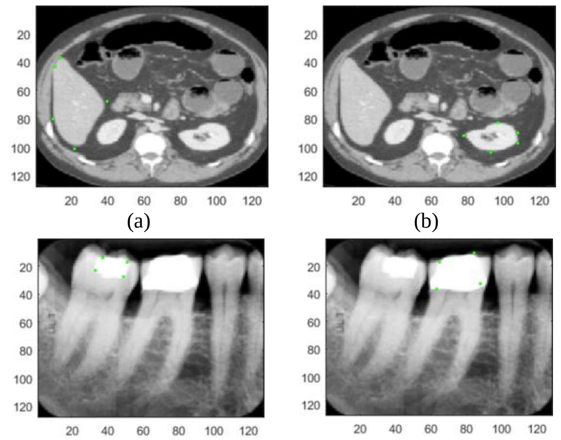
$$-\nabla\left(\rho\nabla\cdot p-w\right)+\frac{1}{g(x)}\left|\nabla\left(\rho\nabla\cdot p-w\right)\right|p=0 \tag{15}$$

We use semi-implicit gradient descent algorithm, as proposed by Chambolle in [8], to solve (15):

$$p^{n+1}=\frac{p^{n}+\delta t\nabla\left(\nabla\cdot p^{n}-w/\rho\right)}{1+\dfrac{\delta t}{g(x)}\left|\nabla\left(\nabla\cdot p^{n}-w/\rho\right)\right|} \tag{16}$$

4. Results and Discussion

In this section, we illustrate performance of CPA in segmenting some problems. The CT-scan test images are shown in Figure 1 that consist of different targeted object to be extracted for each test image. The green points are the set of markers used.



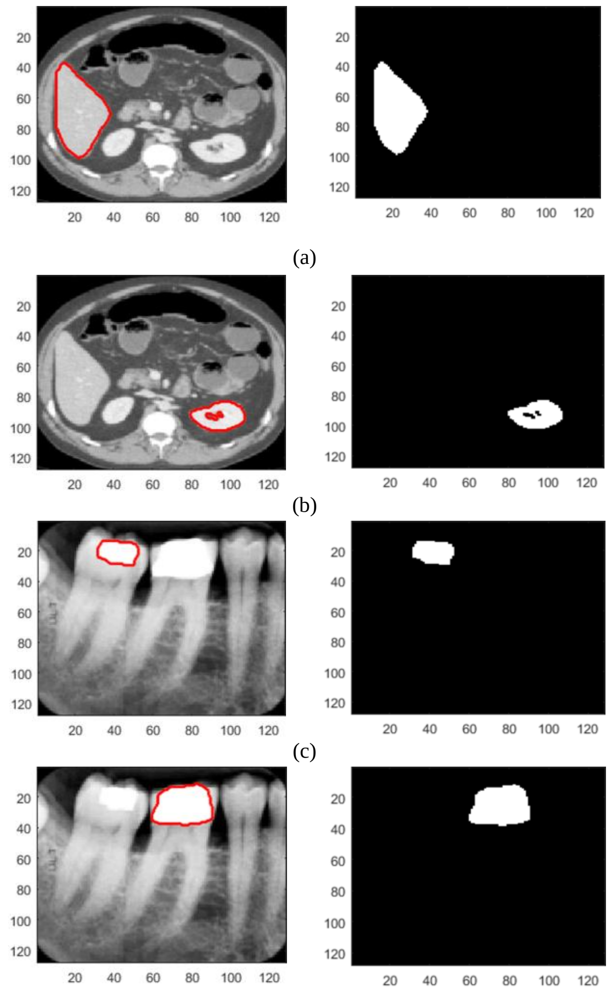
(a) (b) (c) (d)

Fig. 1: The test images used. The green points show the markers set used. All the images are size of 128×128 .

All the images are size of 128×128 . This implies, there are 16,384 unknown to be solved by CPA for each image.

Firstly, we segment all images in Figure 1 using CPA with a pre-defined stopping criterion;

$\max\left(\dfrac{\|u_{new}-u_{old}\|}{\|u_{old}\|},\dfrac{\|w_{new}-w_{old}\|}{\|w_{old}\|}\right)<tol$. We take the value of $\theta=5800$ and $tol=10^{-7}$ for image in Figure 1(a) and 1(b) while for Figure 1(c) and 1(d), the values of $\theta=2000$ and 3000 respectively and $tol=10^{-6}$. Figure 2 shows the segmentation results with its binary representation:



(a) (b) (c) (d)

Fig. 2: Segmentation result using CPA. The first column shows the segmentation contour while the second column shows the binary representation of the segmentation object that is useful for machine vision.

For Figure 2(a), the CPU time taken is 7.0 seconds while the result in Figure 2(b-d) need 10.2, 2.6, and 3.3 seconds respectively. All the results show that SC2 successfully segments the images in a fast way using CPA.

Finally, we compare the performance of CPA with multilevel algorithm that we developed in [4]. Here, we test the CPU time taken for each method for different values of tol . For this final comparison, we choose to segment Figure 1(d). Table 1 shows the CPU time with different stopping accuracy, tol .

Table 1: Comparison of Multilevel Algorithm with CPA

Algorithm	Stopping Accuracy, *tol*	CPU Time, (s)
SC2 with Multilevel Algorithm	5.5×10^{-7}	10.3
	4.5×10^{-7}	10.3
	3.5×10^{-7}	10.3
	2.5×10^{-7}	10.3
	1.0×10^{-7}	10.3
SC2 with CPA	5.5×10^{-7}	6.9
	4.5×10^{-7}	13.1
	3.5×10^{-7}	22.3
	2.5×10^{-7}	35.3
	1.0×10^{-7}	84.5

From Table 1, it shows that CPA give a fast solution when the values of tol used is larger (5.5×10^{-7}) . However, CPA gets slower than multilevel algorithm [4] when we set a smaller tol .

These findings illustrate the convergence speed of our multilevel algorithm proposed in [4] is faster than CPA when the values of tol decreasing.

5. Conclusion

We have successfully applied CPA to solve the original SC2 model proposed in [4]. Good visual segmentation quality with fast computation time is achieved. We also compare the performance of CPA and the multilevel algorithm developed in [4]. We remark that the multilevel algorithm is used to solve the approximated version of SC2 while CPA directly solved the SC2 model. Comparison between CPA with multilevel algorithm in [4] indicates that the convergence speed of multilevel algorithm is faster than CPA when the values of tol is getting smaller. In one hand, the CPA has an advantage where it can be used to solve the original optimization problem without any modification (by approximation) compare to multilevel algorithm. This property may contribute to accurate segmentation especially for difficult cases. On the other hand, the multilevel algorithm has a fast convergence speed property in solving large image data. Due to these properties, it is recommended for future research to combine the CPA in multilevel algorithm framework in order to achieve high segmentation accuracy result in an optimum computation time.

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