

A Mixed Approach for Approximation of Higher Order Linear Time Invariant Systems

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Abstract

This work suggests a technique for order reduction of larger order mathematical model into lower order by combining Modified Inverse Distance Measure (MIDM) and time moment matching criterion. The constant coefficients of the denominator of reduced model are attained by proposed algorithm named MIDM and numerator coefficients of the same are obtained by using the suitable number of time moments and Markov parameters in the set of equations of Improved Pade Approximations (IPA). The suggested method of order reduction is equally useful for both Single-Input Single-Output (SISO) and Multi-Input Multi-Output (MIMO) dynamic systems. The simplicity of the proposed method has been validated through various linear mathematical models taken from literature. To get into the touch of a researcher, the qualitative measure and dynamic analysis of the proposed reduced model output has been elaborated via error index and time/frequency response comparisons respectively.

Keywords: Modified Inverse Distance Measure; Improved Pade Approximations; Pole Clusters; Performance Indices.

1. Introduction

Higher order differential equations obtained from linear dynamic systems are not suitable either for analysis or in economical manner perspective. So to examine such higher order systems, one may change it into required reduced order mathematical model using Model Order Reduction (MOR) techniques. The reduced order models have many advantages like reduction of mathematical complexity, easy controller design, reduction of hardware complexity etc. Hence, MOR Techniques play an essential job to converting higher order mathematical model into reduced order model in suitable way and provide easy realization and designing of controllers.

Recently, a new method for reducing Linear Time Invariant (LTI) systems based on modified pole clustering and factor division is proposed by Sikander and Prasad [1]. They have obtained the denominator polynomial using modified pole clustering and numerator of the same is obtained by factor division algorithm. Based on balanced truncation method Ghosh and Senroy [2] also proposed a technique of reduced order modeling. Desai and Prasad [3] also suggested a method for reducing LTI systems in which denominator polynomial is synthesized by Routh Approximation [4] for preserving the stability of the system and Big Bang - Big Crunch algorithm [5] is used to obtain the numerator polynomial. Another combined approach of order reduction of both Single-Input Single-Output (SISO) and Multi-Input Multi-Output (MIMO) systems is proposed by Parmar et al. [6] by utilizing the concept of Eigen spectrum analysis and factor division algorithm. They utilize the technique only for real poles and method is not compared. In addition, Mittal [7] proposed a method for order reduction utilizing Error minimization i.e. Integral of Square of Error

(ISE). Further, some other researchers also proposed the mixed technique of two frequency domain approaches for reducing the models [8-10]. Sometimes joint methods give suitable results and sometimes follow the non-minimum phase tendency which results in system realization problem.

In this paper two powerful frequency domain methods have been mixed to decrease the order of the LTI Systems. One of the methods proposes by using the concept of pole clustering techniques [8, 11] and Inverse Distance Measure (IDM) and the other is Improved Pade Approximation (IPA) [12]. Further, Vishwakarma [13] also proposed a method based on pole clustering and IDM which uses seven iterations to get the dominant poles. Here, method [13] is lengthy and computationally difficult as iteration depends upon the number of order of the reduced model. Sometimes data feeding and storing through IDM [13] may not results in most dominant pole and it may extends the system towards mismatching results, i.e. not follow original system characteristics patterns. Therefore, authors have proposed a method without any iteration to get a most dominant pole cluster center which holds the characteristics of original higher order system. Also, the suggested method uses a logarithmic approximation to extract the cluster poles towards the dominance of respective cluster. To elaborate the proposed method, the paper includes three different cases of real and imaginary poles. The simulation results of original and reduced models have been incorporated into MATLAB environment. The performance indices, i.e. Integral of Absolute Magnitude of Error (IAE) and Integral of Square of Error (ISE) comparison of approximated models obtained using suggested technique and existing techniques between the transient portion of the original higher order and reduced order systems are also prepared. The present work is organized into three sections, section 1 introduces the literature review, section 2 contains the proposed method and section 3 includes the three different cases of numerical problems.

2. Description of the Method

In order to find the denominator and numerator polynomial coefficients of the lower order model, the MIDM and IPA techniques respectively have been described separately as follows.

2.1. Proposed Method to Obtain Coefficients of Denominator Polynomial [MIDM]

Let the original n^{th} order system is mathematically symbolized as

$$G_n(s) = \frac{N_{n-1}(s)}{D_n(s)} = \frac{\sum_{\zeta=0}^{n-1} a_{\zeta} s^{\zeta}}{\sum_{\zeta=0}^n b_{\zeta} s^{\zeta}} \tag{1}$$

Where, a_{ζ} and b_{ζ} are the constant coefficients of numerator and denominator polynomials respectively. Let p_1, p_2, \dots, p_n are the poles of this system and lie in such manner that $|p_1| < |p_2| < \dots < |p_n|$.

After calculating the unknown coefficients of numerator and denominator the r^{th} ($r < n$) order reduced model represented as

$$G_r(s) = \frac{N_{r-1}(s)}{D_r(s)} = \frac{\sum_{\zeta=0}^{r-1} c_{\zeta} s^{\zeta}}{\sum_{\zeta=0}^r d_{\zeta} s^{\zeta}} \tag{2}$$

To get the denominator polynomial the reduction process as follows:

Step-I:

- Select the cluster of poles of the original higher order system.
- Pole cluster must be detached for left and right half of s-plane.
- Poles at the origin and imaginary axis should remain to retain for lower order system.
- For complex poles, there should be detached clusters for real and imaginary poles.

Step-II:

To obtain denominator of r^{th} - order reduced model, 'r' number of pole are required and obtained from pole clusters such that each cluster center is the dominant pole of that specific cluster. To obtain 'r' number of poles of the approximated model, the computer oriented algorithm is given as follows:

- Let there are 'w' poles i.e. p_1', p_2', \dots, p_w' in the v^{th} cluster such that $|p_1'| < |p_2'| < \dots < |p_w'|$
- Set $v = 1$
- Calculate the pole cluster center using
-

$$c_v = \left[\sum_{k=1}^w \left(\frac{1}{|p_k'|} \right) \div w \right]^{-1} \tag{3}$$

- Set $v = v + 1$
- Now find the most dominant pole cluster center from the equation given as

$$p_{ev} = -\lambda - \left\{ \log(1 + c_v) \right\} \div (r \times n) \tag{4}$$

Where, λ = dominant pole in each cluster

- Check, is $v = r$? If no go to step (iv)
- Choose the most dominant pole cluster center of the r^{th} - order lower model as $p_{er} = p_{ev}$

Step-III: While synthesizing the denominator polynomial $D_r(s)$ of reduced model, three different cases may occur as follows:

Case (1)- If original system is consisting of real cluster centers only and $p_{e1}, p_{e2}, \dots, p_{er}$ are the dominant poles obtained from equation (4). Then denominator polynomial can be attained as

$$D_r(s) = (s - p_{e1})(s - p_{e2}) \dots (s - p_{er}) \tag{5}$$

Case (2) – If original system is consisting of real and complex cluster centers both such that $p_{e1}, p_{e2}, \dots, p_{e(r-2)}$ are dominant real poles cluster centers and one complex pole cluster center with ϕ_{e1}^* and ϕ_{e1}^{\bullet} are real and imaginary parts of reduced system obtained from equation (4), then

$$D_r(s) = (s - p_{e1})(s - p_{e2}) \dots (s - p_{e(r-2)})(s - \phi_{e1}^*)(s - \phi_{e1}^{\bullet}) \tag{6}$$

Case (3) – If original system consisting of only complex poles and $\phi_{e1}^*, \phi_{e2}^*, \dots, \phi_{e(r/2)}^*$ dominant real parts and $\phi_{e1}^{\bullet}, \phi_{e2}^{\bullet}, \dots, \phi_{e(r/2)}^{\bullet}$ dominant imaginary parts of reduced system obtained from equation (4), then

$$D_r(s) = (s - \phi_{e1}^*)(s - \phi_{e1}^{\bullet}) \dots (s - \phi_{e(r/2)}^*)(s - \phi_{e(r/2)}^{\bullet}) \tag{7}$$

So, denominator polynomial $D_r(s)$ of reduced order system is written as

$$D_r(s) = \sum_{\zeta=0}^r d_{\zeta} s^{\zeta} = d_0 + d_1 s + d_2 s^2 + \dots + d_r s^r \tag{8}$$

2.2 Synthesizing the Numerator Polynomial Using [IPA]:

Numerator polynomial of the reduced model is obtained by choosing the suitable number of time moments and Markov parameters and constant coefficients of $D_r(s)$

In terms of time moments and Markov parameters, original n^{th} order system may be represented in series expansion form as:

$$G_n(s) = - \sum_{i=0}^{\infty} T_i s^i \quad (\text{Abouts} = 0) \tag{9}$$

$$= \sum_{i=0}^{\infty} M_i s^{-i-1} \quad (\text{Abouts} = \infty) \tag{10}$$

Where T_i the i^{th} is time moment and M_i is the i^{th} Markov parameter.

Now choosing α = suitable number of time moments and β = suitable number of Markov parameters such that

$$\alpha + \beta = r \tag{11}$$

The numerator coefficients $(c_0, c_1, \dots, c_{(r-1)})$ of lower order system can be determined from the set of equations as follows [12].

$$\left. \begin{aligned} c_0 &= T_0 d_0 \\ c_1 &= T_1 d_0 + T_0 d_1 \\ c_2 &= T_2 d_0 + T_1 d_1 + T_0 d_2 \\ &\dots \\ &\dots \\ c_{(\alpha-1)} &= T_{(\alpha-1)} d_0 + T_{(\alpha-2)} d_1 + \dots \\ &+ T_1 d_{(\alpha-2)} + T_0 d_{(\alpha-1)} \\ c_{(r-\beta)} &= M_{(\beta-1)} d_r + M_{(\beta-2)} d_{(r-1)} + \dots \\ &+ M_1 d_{(r-\beta+2)} + M_0 d_{(r-\beta+1)} \\ c_{(r-\beta+1)} &= M_{(\beta-2)} d_r + M_{(\beta-3)} d_{(r-1)} + \dots \\ &+ M_1 d_{(r-\beta+3)} + M_0 d_{(r-\beta+2)} \\ &\dots \\ &\dots \\ c_{(r-2)} &= M_1 d_r + M_0 d_{(r-1)} \\ c_{(r-1)} &= M_0 d_r \end{aligned} \right\} \quad (12)$$

So solutions of equations in (12) give the suitable numerator coefficients i.e. $(c_0, c_1, \dots, c_{(r-1)})$ of reduced order system and $N_{(r-1)}(s)$ can be written as

$$N_{(r-1)}(s) = \sum_{\zeta=0}^{r-1} c_{\zeta} s^{\zeta} \quad (13)$$

3. Numerical Cases

To better understand the suggested method three different types of numerical cases have been selected from the literatures [5, 11 and 14]. After obtaining the reduced model, it has been compared with existing reduction methods [1, 5-7&13-18] via performance indices i.e. ISE and IAE between the transient portion of step responses of reduced and original systems. Let $\Psi_r(t)$ is the step response of obtained reduced system using proposed method and $\Psi(t)$ is the step response of given system, performance indices ISE and IAE can be written as

$$ISE = \int_0^{\infty} |\Psi_r(t) - \Psi(t)|^2 dt \quad (14)$$

$$IAE = \int_0^{\infty} |\Psi_r(t) - \Psi(t)| dt \quad (15)$$

Numerical Case-1: Let an 8th order model consisting of real poles only taken from G. Parmar [5].

$$G_8(s) = \frac{40320 + 185760s + 222088s^2 + 122664s^3 + 36380s^4 + 5982s^5 + 514s^6 + 18s^7}{40320 + 109584s + 118124s^2 + 67284s^3 + 22449s^4 + 4536s^5 + 546s^6 + 36s^7 + s^8}$$

Determination of denominator polynomial for the approximated model

The original given system is of order 8th, and let 2nd order reduced system is required.

The real poles of the original system are: $(-1, -2, -3, -4, -5, -6, -7, -8)$ Now to determine the 2nd order approximated model, two pole clusters are mandatory and taken as $(-1, -2, -3, -4)$ and $(-5, -6, -7, -8)$.

Using equations (3) and (4) the dominant pole cluster centers are obtained as

$$p_{e1} = -1.029 \text{ And } p_{e2} = -5.0539$$

Therefore, denominator polynomial of approximated 2nd order system can be achieved as

$$D_2(s) = (s - p_{e1})(s - p_{e2}) = (s + 1.029)(s + 5.0539);$$

$$D_2(s) = s^2 + 6.0829s + 5.2004$$

Determination of numerator polynomial for the approximated model For the realization of numerator polynomial of lower order system few time moments and Markov parameters are required, from equations (9) time moments and Markov parameters can be obtained as $T_0 = 1, T_1 = 1.8904, T_2 = -2.5592$ and $M_0 = 18, M_1 = -1339520, M_2 = 975.8720$

With the help of equation (12) coefficients can be calculated as

$$c_0 = T_0 d_0 = 1 \times 5.2004 = 5.2004$$

$$c_1 = T_1 d_0 + T_0 d_1 = (1.8904 \times 5.2004) + (1 \times 6.0829) = 15.9137$$

Therefore, by chosen suitable α and β numerator polynomial of 2nd order reduced system can be written as follows

$$N_2(s) = 5.2004 + 15.9137s \quad [\alpha = 2, \beta = 0]$$

So, 2nd order reduced model transfer function can be obtained as

$$G_2(s) = \frac{5.2004 + 15.9137s}{5.2004 + 6.0829s + s^2} \quad (16)$$

Similarly, 3rd order reduced model transfer function $G_3(s)$ is obtained by selecting the pole clusters $(-1, -2), (-3, -4, -5)$ and $(-6, -7, -8)$ as

$$G_3(s) = \frac{18.5632 + 62.5799s + 14.5339s^2}{18.5632 + 27.4881s + 10.0774s^2 + s^3}; \quad (17)$$

$[\alpha = 3, \beta = 0]$

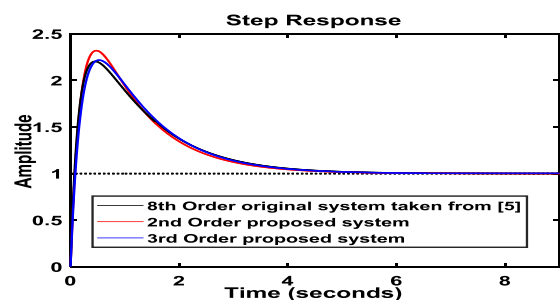


Fig1: Comparisons of step responses of original and approximated lower order models for numerical case 1

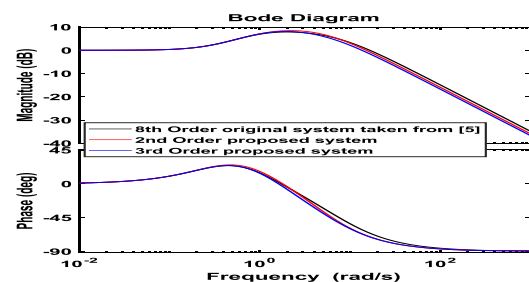


Fig2: Comparisons of frequency responses of original and lower order models for example 1.

For numerical case-1 step response and frequency response comparison between reduced and original models have been shown in Fig. 1 and Fig. 2 correspondingly. Also performance keys comparison i.e. ISE and IAE between the transient

responses of original model taken from [5] and proposed models obtained in equations (16) and (17) with existing methods [1, 5-7, 13-15 and 18] have been given in the Table 1.

Table 1 : Performance keys comparisons of proposed and existing techniques for numerical case-1

Reduction Method	Reduced (Approximated) Model	$ISE = \int_0^{\infty} \Psi_r(t) - \Psi(t) ^2 dt$	$IAE = \int_0^{\infty} \Psi_r(t) - \Psi(t) dt$
Proposed Reduced models	$G_2(s) = \frac{5.2004 + 15.9137s}{5.2004 + 6.0829s + s^2}$	0.0087	0.1522
	$G_3(s) = \frac{18.563 + 62.579s + 14.533s^2}{18.563 + 27.488s + 10.077s^2 + s^3}$	0.00881	0.1181
C. N. Singh [1]	$G_2(s) = \frac{4.35076 + 13.4498s}{4.35076 + 5.22997s + s^2}$	0.0152	0.169
CBV. [13]	$G_2(s) = \frac{5.45971 + 16.51145s}{5.45971 + 6.19642s + s^2}$	0.0140	0.1971
G. Parmar [5]	$G_2(s) = \frac{8 + 24.11429s}{8 + 9s + s^2}$	0.0481	0.3007
A. K. Mittal [7]	$G_2(s) = \frac{1.9906 + 7.0908s}{2 + 3s + s^2}$	0.2689	0.8054
Mukherjee [15]	$G_2(s) = \frac{4.4357 + 11.3909s}{4.4347 + 4.2122s + s^2}$	0.0568	0.4572
Krishnamu. [18]	$G_2(s) = \frac{40320 + 15565861s}{40320 + 75600s + 65520s^2}$	1.6533	0.4572
R. Prasad [14]	$G_2(s) = \frac{500 + 17.98955s}{500 + 13.2457s + s^2}$	1.4584	1.000
M. F. Hutton [6]	$G_2(s) = \frac{0.43184 + 1.98955s}{0.43184 + 41.17368s + s^2}$	1.9171	10.0702

Numerical Case-2: Let an 8th order model consisting of real and complex poles both taken from A.K. Sinha [11].

$$G_8(s) = \frac{N(s)}{D(s)},$$

Where

$$N(s) = 84259795 + 18904431s + 90581205s^2 + 24154475s^3 + 45575892s^4 + 48438098s^5 + 42926156s^6 + 19.82s^7$$

$$D(s) = 37752826 + 14917219s + 1733835s^2 + 67556983s^3 + 18110567s^4 + 29138638s^5 + 3584295s^6 + 30.41s^7 + s^8$$

The real and complex poles of this model are (-0.46, -0.75, -8.5, -15.6) and (-0.35 ± j6.8, -2.2 ± j3.6) respectively.

To determine the 4th order approximated model, three pole clusters are selected as (-0.46, -0.75), (-8.5, -15.6) and (-0.35 ± j6.8, -2.2 ± j3.6).

So from equation (3) and (4) pole cluster centers are obtained as: $p_{e1} = -0.4661$, $p_{e1} = -8.5325$, $\phi_{e1}^* = -0.3564$ and $\phi_{e1} = -3.6236$

Therefore, denominator polynomial of reduced 4th order system can be found as

$$D_4(s) = (s + 0.4661)(s + 8.5325)(s + 0.3564 + j3.6236)(s + 0.3564 - j3.6236);$$

$$D_4(s) = 52.7250 + 122.1337s + 23.6487s^2 + 9.7114s^3 + s^4$$

Consequently from section 2.2, the numerator polynomial can be obtained as

$$N_3(s) = 117659 + 717.318s + 18.9829s^2 + 19.8240s^3; [\alpha = 2, \beta = 2]$$

Therefore, 4th order reduced model can be written as

$$G_4(s) = \frac{117659 + 717.318s + 18.9829s^2 + 19.8240s^3}{52.7250 + 122.1337s + 23.6487s^2 + 9.7114s^3 + s^4} \quad (18)$$

For numerical case-2 step response comparison between reduced and original models has been shown in Fig. 3. Also performance indices i.e. ISE and IAE comparison between the transient responses of original model taken from [11] and proposed models obtained in equations (18) with existing methods [5, 16] have been given in the Table 2.

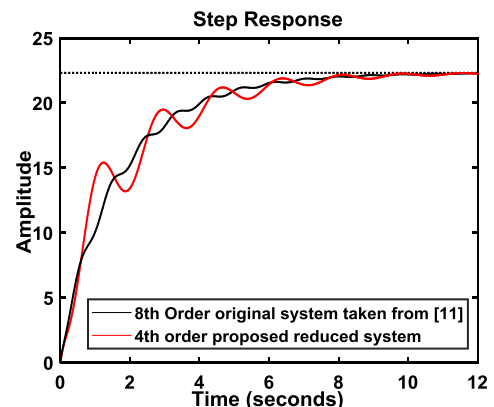


Fig3: Comparisons of step responses of original and approximated 4th order model for numerical case 2

Numerical Case-3: Let a 4th order model consisting of complex poles only taken from Prasad [14].

$$G_4(s) = \frac{2400 + 1800s + 496s^2 + 28s^3}{240 + 360s + 204s^2 + 36s^3 + 2s^4}$$

The poles of this system are: $(-7.8033 \pm j1.3576)$ and $(-1.1967 \pm j0.6934)$

To determine the 4th order reduced model, three pole clusters are selected as $(-1.1967, -7.8033)$ and $(-0.6934, -1.3576)$.

Therefore, reduced model obtained from the proposed method is

$$G_2(s) = \frac{21.1273 + 9.3036s}{2.1125 + 2.5152s + s^2}; \quad (19)$$

$[\alpha = 2, \beta = 0]$

For numerical case-3 step response comparison between reduced and original models has been shown in Fig. 4. Also performance indices comparison i.e. ISE and IAE between the transient responses of original model taken from [14] and proposed model obtained in equations (18) with existing methods [1, 13, 17 and 18] have been given in the Table 3.

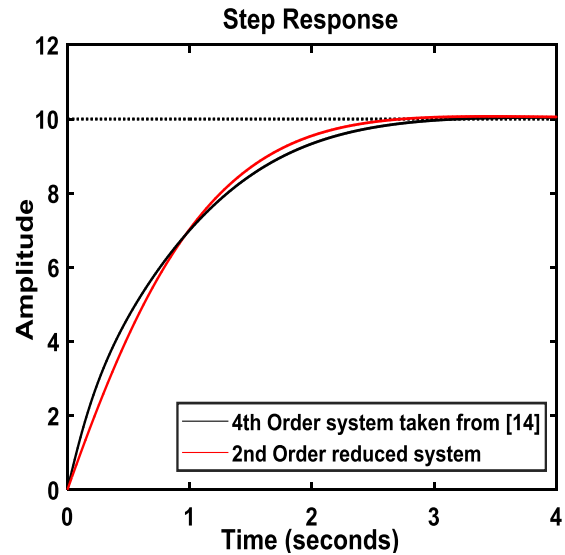


Fig4: Comparisons of step responses of original and approximated 2ndorder model for numerical case 3

Table 2 : Performance comparison of propos for and existing methods for numerical case-2

Method of Reduction	Model Reduced	$ISE = \int_0^{\infty} \Psi_r(t) - \Psi(t) ^2 dt$	$IAE = \int_0^{\infty} \Psi_r(t) - \Psi(t) dt$
Proposed Reduced model	$G_4(s) = \frac{117659 + 717318s + 189829s^2 + 19.8240s^3}{527250 + 1221337s + 23.6487s^2 + 9.7114s^3 + s^4}$	12.6	7.089
J. Singh [16]	$G_4(s) = \frac{11705 + 7248086s + 18.7966s^2 + 19.82s^3}{5245 + 122s + 23.5s^2 + 9.702s^3 + s^4}$	13.29	7.385
G. Parmar [5]	$G_4(s) = \frac{3153 + 2390s + 2428s^2 + 61.27s^3}{1413 + 2682s + 4277s^2 + 12.78s^3 + s^4}$	41.54	12.47

Table 3 : Performance comparison of propos for and existing methods for numerical case-3

Method of Reduction	Model Reduced	$ISE = \int_0^{\infty} \Psi_r(t) - \Psi(t) ^2 dt$	$IAE = \int_0^{\infty} \Psi_r(t) - \Psi(t) dt$
Proposed Reduced model	$G_2(s) = \frac{21.1273 + 9.3036s}{2.1125 + 2.5152s + s^2}$	0.2549	0.7607
C. N. Singh [1]	$G_2(s) = \frac{27.345 + 9.5852s}{2.7345 + 3.0094s + s^2}$	0.6739	0.1958
Lucas [17]	$G_2(s) = \frac{40 + 30s}{4 + 6s + 3s^2}$	0.2061	0.7626
Krishnamurthy [18] (Routh Hurwitz Array)	$G_2(s) = \frac{13.043478 + 9.046283s}{1.304348 + 1.701323s + s^2}$	1.208	2.265
C.B. Vishwakarma [13]	$G_2(s) = \frac{2400 + 9.043478s}{240 + 317.1498s + 201s^2}$	1.763	2.597

Table 4 : Comparisons of dynamic response components of Original and proposed models

Step Response Information	Example-1			Example-2		Example-3	
	Original 8 th Order Model [5]	Reduced 2 nd Order Model	Reduced 3 rd Order Model	Original 8 th Order Model [11]	Reduced 2 nd Order Model	Original 4 th Order Model [14]	Reduced 2 nd Order Model
Rise Time	0.0569	0.0622	0.0696	3.895	4.0307	1.6915	1.5307
Settling Time	4.8201	4.4969	4.7156	7.5749	8.9337	2.5524	2.285
Settling Min	0.9712	0.9447	1.0038	20.1	20.1002	9.0084	9.089
Settling Max	2.2035	2.3197	2.2177	22.3041	22.3195	10.0443	10.0723
Overshoot	1203496	1319707	121.7691	0	0.0174	0.4428	0.7121
Undershoot	0	0	0	0	0	0	0
Peak	2.2035	2.3197	2.2177	22.3041	22.3195	10.0443	10.0723
Peak Time	0.4490	0.4738	0.5195	14.2569	13.2874	3.9847	3.4547

4. Performance Evaluation Practices

An order reduction method has been proposed using MIDM and IPA where most dominant frequency has been calculated by proposed MIDM technique and selection of time moments and Markov parameters decide the number of reduced order models to select best one for designing, analysis etc.. Error indexes i.e. ISE and IAE have also been measured between the transient portions of higher order original model and reduced approximated models. In addition to this few analysis of the proposed method are given as follows:

- The approximated models obtained by proposed technique have very closed transient and steady-state value to the original model.
- The Reduced models obtained through proposed technique always free from any steady state error.
- Dominant frequencies and numerator polynomials obtained using proposed method eradicate the limitations of [14], where reduced model is obtained via stability equation technique.
- Zero input response components of the approximated models are nearly same as original models and diminish as time approaches infinity.
- Dynamic response components of the approximated models are also very close to that of original models as given in Table 4, means proposed reduction technique preserves the original characteristics of the systems.
- Reduction technique provides decent approximations of original higher order systems for a better reduced order model instead of approximation taken by [10, 14, and 19-21].
- The proposed method of order reduction is applicable for all types of linear dynamic systems having complex poles, real-complex or real poles only, which overcome the drawback of [10, 22].

5. Conclusion

The composite technique of MOR has been suggested by combining MIDM and IPA Technique. MIDM is being used to generate most dominant poles for reduced order model whereas appropriate time moments and Markov parameters are generated by IPA technique to obtain the zeros of the reduced systems. The suggested technique has been explained with three different kinds of problems having real poles only, real-complex both and complex poles only. The reduction algorithm is straight forward, uneven and takes little calculation time to reduce the model. From the Fig. 1, Fig. 3 and Fig. 4, it can be observe that step responses of approximated models are quite good as well as follow the pattern of original system responses. The frequency response of the approximated model for numerical case -1 is also shown in Fig. 2 and almost similar to that of original system. The proposed approach is equally applicable in both SISO and MIMO systems. The error index comparisons i.e. ISE and IAE are given in Tables 1-3 respectively via MATLAB platform. Using obtained reduced models; compact controllers can be designed and compared with original model designed controllers.

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